Quantifying the contribution of a subpopulation to inequality

An application to Mozambique

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Motivation

- The analysis of inequality by **subpopulations**: key element for understanding **inequality levels and trends** across countries.
  - To identify **sources** of inequality and **dynamics**.
    - E.g. Rural vs. urban areas or regions in China or India; Racial groups in South Africa, Castes in India, race/ethnicity in LAC, …; Education; Formal vs. informal sectors, …
    - Growing relevance of **regression-based techniques** to analyze distributional changes.
  - We have:
    - **group inequality** analyses, **gap** between groups, or
    - **aggregate decompositions** (total inequality into between-group and within-group components).
  - … but, in general, **no explicit contribution of each group** to total inequality or each component.

Example (Mean Log Deviation) for Mozambique by area of residence

	% Population		Rel. mean		Inequality	
	2008	2014	2008	2014	2008	2014
Rural	70	68	88	79	0.240	0.243
Urban	30	32	126	146	0.402	0.541
All	100	100	100	100	**0.303**	**0.381**

Inequality

Inequality	2008	%	2014	%	Change	%
Between	0.014	5	0.043	11	0.029	38
Within	0.289	95	0.338	89	0.049	62
Total	0.303	100	0.381	100	**0.078**	100

What´s the contribution of rural and urban areas to the level and change of inequality?

Aim

- Proposing a **detailed decomposition** of inequality by subpopulations:
    - Contribution of each subpopulation to **overall inequality**.
        - + to **between-group** and **within-group** inequality (**additively decomposable indices**)
  - The **sum of the contributions of its members**
    - The impact that a marginal increase in the proportion of people with a specific income would have on total inequality using the **Recentered Influence Function** (RIF).
      - Consistent with **RIF regressions**.
      - Various **good properties**.

Contribution of rural and urban areas to the level and change of inequality?

Inequality (%)	*Total*	*Between*	*Within*
Rural	48.3	4.7	43.6
Urban	51.7	6.7	44.9
All	**100**	**11.4**	**88.6**

Urban areas contributed to 52% of inequality

Change (%)	*Total*	*Between*	*Within*
Rural	15.7	16.5	-0.9
Urban	84.3	21.2	63.1
All	**100**	**37.7**	**62.3**

Urban areas contributed to 84% of the increase in inequality (100% of the within-area component)

Aim (cont.)

- **Alternative approaches** adapted from the factor inequality decomposition literature (esp. marginal and Shapley factor decompositions)
  - **Mean Log Deviation (M)**, with best additive decomposability properties: these approaches are ‘almost’ equivalent.
- **Empirical illustration: Mozambique**
  - Low-income sub-Saharan African country, increase in inequality in recent years.
  - Disproportional contributions of **affluent groups** to inequality and its increase over time:
    - top percentiles, urban areas, especially Maputo, and families with heads with higher education.

Index of my presentation

- The RIF detailed decomposition of inequality by subpopulations:
  - General case.
  - Additively decomposable indices.
- Other approaches: factor decomposition.
- Empirical analysis: Mozambique.
- Conclusions.

The RIF detailed decomposition of inequality by subpopulations: general case

- Exhaustive partition, $K \geq 1$ disjoint groups
  - **Population:** $\boldsymbol{y} = (\boldsymbol{y}^{\boldsymbol{1}}, \dots, \boldsymbol{y}^{\boldsymbol{K}})$, size n, mean income μ
  - **Group k:** $\boldsymbol{y}^{\boldsymbol{k}} = (y_1^k, . . , y_{n_k}^k)$, size n^k, mean income μ^k.

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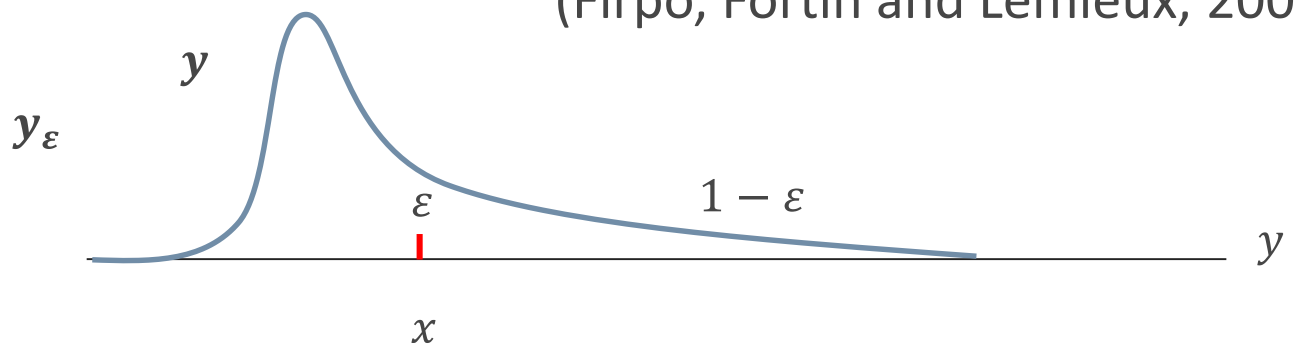
Influence function

- Impact on $I(\boldsymbol{y})$ of marginally increasing the population mass at x.
  - $\boldsymbol{y}_{\boldsymbol{\varepsilon}}$ is a mixture distribution assigning a probability $1-\varepsilon$ to the original distribution $\boldsymbol{y}$ and ε to x (Hampel, 1974):

$$IF(x;I(\boldsymbol{y})) = \frac{\partial}{\partial\varepsilon} I(\boldsymbol{y}_{\boldsymbol{\varepsilon}})|_{\varepsilon=0}\ ; \qquad \text{with } E\big(IF\big(x;I(\boldsymbol{y})\big)\big) = 0$$

$$RIF(x;\mathrm{I}(\boldsymbol{y})) = I(\boldsymbol{y}) + IF(x;\mathrm{I}(\boldsymbol{y}))\ ; \text{with } E\big(RIF\big(x;I(\boldsymbol{y})\big)\big) = I(\boldsymbol{y})$$

(Firpo, Fortin and Lemieux, 2007, 09)



Contribution to inequality

- Contribution of the jth **individual** of group k to $I(\boldsymbol{y})$:

$$S_j^k = \frac{1}{n} RIF\left(y_j^k; I(\boldsymbol{y})\right).$$

- Contribution of **group** k to $I(\boldsymbol{y})$:

$$S^k = \sum_{j=1}^{n^k} S_j^k.$$

Relationship with RIF regressions?

Contribution and RIF regression

$$RIF(y; \mathrm{I}(\boldsymbol{y})) = X'\beta + \varepsilon$$

- Group membership **dummies** λ^k used as explanatory variables:

$$RIF\left(y_j^k; I(\boldsymbol{y})\right) = \sum_{k=1}^{K} \hat{\beta}^k \lambda^k;$$

→ **Oaxaca-Blinder Decomposition**

where $\hat{\beta}^k = \frac{1}{n^k} \sum_{j=1}^{n^k} RIF\left(y_j^k; I(\boldsymbol{y})\right).$

Average contribution of group members

$$I(\boldsymbol{y}) = E\left[RIF\left(x; I(\boldsymbol{y})\right)\right] = \sum_{k=1}^{K} \underbrace{\frac{n^k}{n}}_{s^k} \hat{\beta}^k$$

RIF regression-based decomposition

$I(\boldsymbol{y}) = \bar{X}'\beta$

Characteristics effect (explained)
Change in population shares

Coefficients effect (unexplained)
Change in average contribution

$$I^1 - I^0 = \bar{X}^1\beta^1 - \bar{X}^0\beta^0 = (\bar{X}^1 - \bar{X}^0)\beta^1 + \bar{X}^0(\beta^1 - \beta^0)$$

Adding/subtracting $\bar{X}^0\beta^1$ **(counterfactual distribution)**

Properties

- Invariant to replications of the entire population (**population principle**) → population shares:

$$S^k(I(\boldsymbol{y}))= S^k\big(I(\boldsymbol{y}')\big) \text{ for any replication } \boldsymbol{y}' = (\boldsymbol{y}, \dots, \boldsymbol{y}).$$

- Invariant to the multiplication of all incomes in the population by the same factor (**scale invariance**) → income shares:

$$S^k(I(\boldsymbol{y}))= S^k(I(\lambda\boldsymbol{y})) \text{ for any } \lambda > 0.$$

- Asymmetric **U-pattern** with respect to income, reflecting the specific degree of **sensitivity to income transfers** that occur at different points of the distribution.

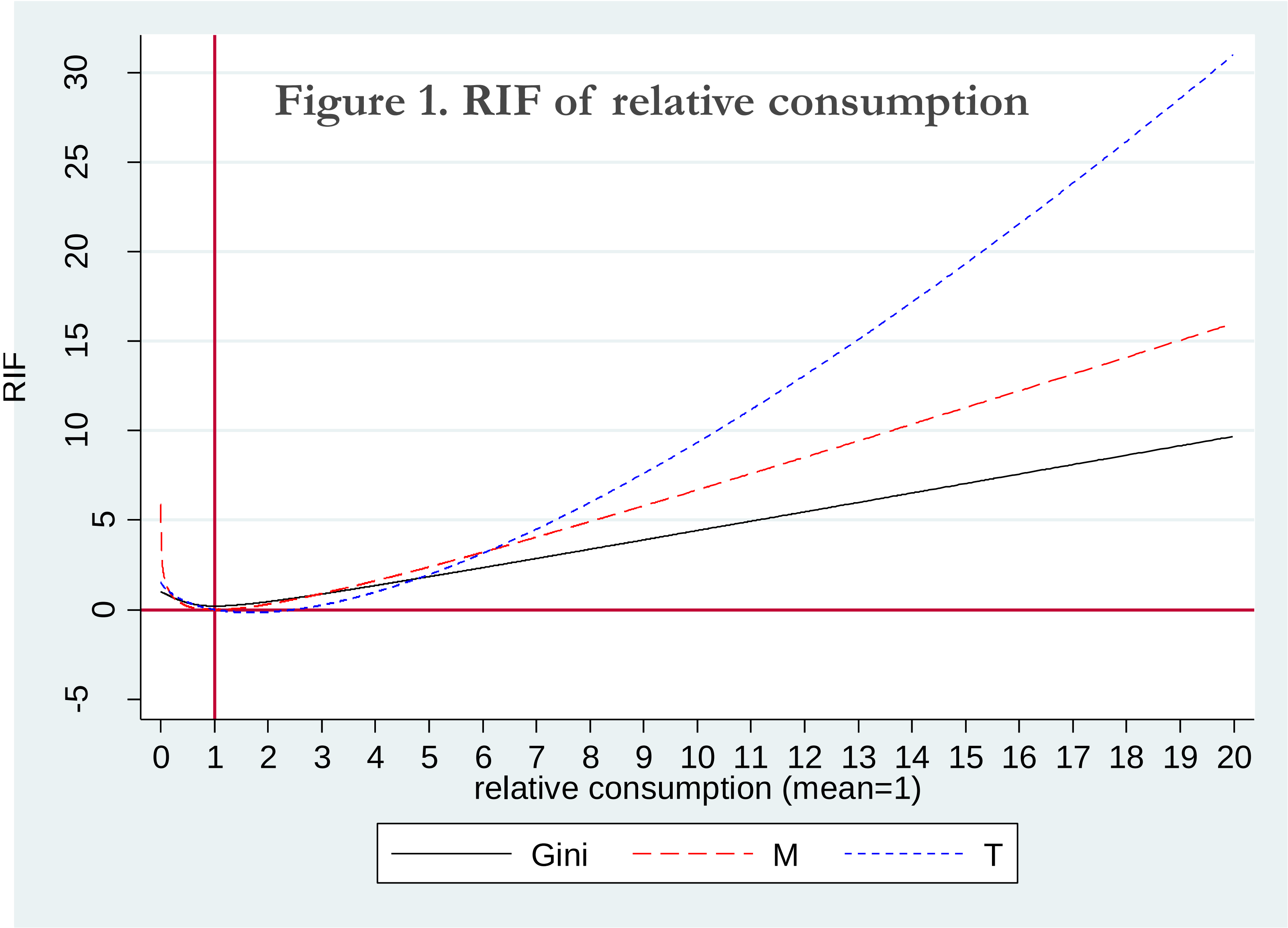


Figure 1. RIF of relative consumption
RIF
30
25
20
15
10
5
0
-5
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
relative consumption (mean=1)
Gini
M
T

Properties (cont.)

- **Consistency**: $I(\boldsymbol{y}) = \sum_{k=1}^{K} S^k = \sum_{k=1}^{K} \sum_{j=1}^{n^k} S_j^k$.

  → $s^k = S^k / I(\boldsymbol{y})$ (relative contribution)

- **Path independence** (order of groups)
- Invariant to the **level of aggregation** of groups.
- **Normalization** property (Gen. Entropy family):
  - $S^k = 0$ if $y_j^k = \mu, \forall\, j = 1, \ldots, n^k$;
  - $S^1 = I(\boldsymbol{y})$ if K=1.
- **Range** property (*M*): S^k will always fall between 0 and $I(\boldsymbol{y})$.

The case of additively decomposable indices

- $I(\boldsymbol{y}) = I_B + I_W$;

If $\boldsymbol{\mu^k} = (\mu^1 \mathbf{1}_{\boldsymbol{n}^1}, \dots, \mu^K \mathbf{1}_{\boldsymbol{n}^K})$

Each y_j^k is replaced by μ^k

- $I_W = I(\boldsymbol{y}) - I(\boldsymbol{\mu^k}) = \sum_{k=1}^{K} I(\boldsymbol{y^k}) w_I^k$;
- $I_B = I(\boldsymbol{\mu^k})$;
- This (+ scale and replication invariance) defines the **Generalized Entropy class** (Shorrocks, 1984), including limit cases $\boldsymbol{\alpha = 0, 1}$:

$$I_\alpha(\boldsymbol{y}) = \frac{1}{\alpha(\alpha-1)} \left[\frac{1}{n}\sum_{i=1}^{n} \left(\frac{y_i}{\mu}\right)^\alpha - 1\right];$$

$$I_0 = \frac{1}{n}\sum_{i=1}^{n} ln \frac{\mu}{y_i}$$

$$I_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{\mu} ln \frac{y_i}{\mu}$$

with $w_{I_\alpha}^k = \frac{n^k}{n}\left(\frac{\mu^k}{\mu}\right)^\alpha$

$$w_{I_0}^k = \frac{n^k}{n}$$

$$w_{I_1}^k = \frac{n^k}{n}\frac{\mu^k}{\mu}$$

Mimicking aggregate decomposition

- $S^k = S_B^k + S_W^k.$
- $S_W^k = S^k(I(\boldsymbol{y})) - S^k\left(I(\boldsymbol{\mu^k})\right),$

  with $I_W = \sum_{k=1}^{K} S_W^k$
- $S_B^k = S^k\left(I(\boldsymbol{\mu^k})\right) = \frac{n_k}{n} RIF\left(\mu^k; I(\boldsymbol{\mu^k})\right),$

  with $I_B = \sum_{k=1}^{K} S_B^k.$

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For limit cases, M and T

	$M \equiv I_0$	$T \equiv I_1$
I	$\frac{1}{n}\sum_{i=1}^{n} ln\frac{\mu}{y_i}$	$\frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\mu} ln\frac{y_i}{\mu}$
S^k	$\frac{n^k}{n}\left[M^k + \frac{\mu^k - \mu}{\mu} + ln\frac{\mu}{\mu^k}\right]$	$\frac{n^k}{n}\left[\left(\frac{\mu - \mu^k}{\mu}\right)(T+1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu} + \frac{\mu^k}{\mu}T^k\right]$
S_B^k	$\frac{n^k}{n}\left(\frac{\mu^k - \mu}{\mu} + ln\frac{\mu}{\mu^k}\right)$	$\frac{n^k}{n}\left[\left(\frac{\mu - \mu^k}{\mu}\right)(T_B+1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu}\right]$
S_W^k	$\frac{n^k}{n}M^k$	$\frac{n^k}{n}\left[\frac{\mu^k}{\mu}T^k + T_W\left(\frac{\mu - \mu^k}{\mu}\right)\right]$

M, compared with *T: +* sensitivity to transfers at the **bottom** and better decomposability properties (**independent of the path** for defining BG and WG terms).

Other approaches: factor decomposition

- **Marginal and Shapley factor decomposition (zero or equalizing approaches)**
  - **Marginal:** change after removing a factor (e.g. Kakwani, 1977)
    - Inconsistent decomposition + not invariant with the level of aggregation of the target factor + path dependence
  - **Shapley:** average marginal contribution over all possible sequences (Chantreuil and Trannoy, 2013; Shorrocks, 2013)
    - Consistent decomposition + path independence + not invariant with the level of aggregation of groups + cumbersome to compute.
- **Natural decomposition rules of some inequality indices** (Shorrocks, 1982, Morduch and Sicular, 2002)
  - Index-specific (CV, Gini, Theil) and does not fully account for the contribution of a factor.

Equalizing subpopulations

Reasonable?

$\tilde{\delta}^k = \frac{\delta_a^k}{\delta_a^k + \delta_a^{-k}}$

$\tilde{\delta}^k + \tilde{\delta}^{-k} = 1$

Marginal: $\delta^k = I(\boldsymbol{y}) - I\left(\left(\boldsymbol{y}^{-k}, \mu\mathbf{1}_{\boldsymbol{n}^k}\right)\right)$

$I(\boldsymbol{y}) \quad \boldsymbol{y} = \left(\boldsymbol{y}^{-k}, \boldsymbol{y}^k\right)$

δ_a^k: $\left(\boldsymbol{y}^{-k}, \mu\mathbf{1}_{\boldsymbol{n}^k}\right)$ → (δ_b^{-k}) → $\left(\mu\mathbf{1}_{\boldsymbol{n}^{-k}}, \mu\mathbf{1}_{\boldsymbol{n}^k}\right)$

δ_a^{-k}: $\left(\mu\mathbf{1}_{\boldsymbol{n}^{-k}}, \boldsymbol{y}^k\right)$ → (δ_b^k) → $\left(\mu\mathbf{1}_{\boldsymbol{n}^{-k}}, \mu\mathbf{1}_{\boldsymbol{n}^k}\right)$

$I(\boldsymbol{\mu}) = 0$

$\delta_a^k + \delta_a^{-k} \neq 1$

$\delta_a^k \neq \delta_b^k$

Shapley: $\delta'^k = \frac{1}{2}\left(\delta_a^k + \delta_b^k\right) = \frac{1}{2}\left(I(\boldsymbol{y}) - I\left(\left(\boldsymbol{y}^{-k}, \mu\mathbf{1}_{\boldsymbol{n}^k}\right)\right) + I\left(\left(\mu\mathbf{1}_{\boldsymbol{n}^{-k}}, \boldsymbol{y}^k\right)\right)\right)$

$$\delta_W^k(M) = \delta'^k_W(M) = S_W^k(M),$$

$$\delta_B^k(M) = \frac{n^k}{n} ln \frac{\mu}{\mu^k} - ln\left(1 + \theta^k\right) \approx S_B^k(M)$$

$$\delta'^k_B(M) = \frac{n^k}{n} ln \frac{\boldsymbol{\mu}}{\boldsymbol{\mu}^k} + \frac{1}{2} ln\left(\frac{1+\theta^k}{1-\theta^k}\right) \approx S_B^k(M)$$

If small

$$\theta^k = \frac{n^k}{n}\frac{\mu^k - \mu}{\mu}$$

Empirically similar

Zero subpopulation

- **Marginal:** $\gamma^k = I(\boldsymbol{y}) - I(\boldsymbol{y}^{-\boldsymbol{k}})$;
  - Inconsistent decomposition, hard to attribute inequality to groups:

    Ex. if groups have the same inequality and mean, the contributions to *M* or *T* are all zero.

- **Shapley:** $\gamma'^k = \frac{1}{2}\left(I(\boldsymbol{y}) + I(\boldsymbol{y}^{\boldsymbol{k}}) - I(\boldsymbol{y}^{-\boldsymbol{k}})\right)$.

- **Limitations**:
  - Potentially large impact of tiny groups:

$$\gamma^k - \gamma^{-k} = \gamma'^k - \gamma'^{-k} = I(\boldsymbol{y}^{\boldsymbol{k}}) - I(\boldsymbol{y}^{-\boldsymbol{k}}).$$

Natural decomposition rules of some inequality indices

If an index can be expressed as a weighted sum of incomes:

- $I(\boldsymbol{y}) = \sum_{i=1}^{n} a(\boldsymbol{y}) y_i,$

… the contribution of a group (factor) can be defined by:

- $\tau^k(I) = \sum_{j=1}^{n^k} a(\boldsymbol{y}) y_j^k.$

Example: $T = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\mu} ln \frac{y_i}{\mu},$ → $a(\boldsymbol{y}) = \frac{1}{\mu} ln \frac{y_i}{\mu}$

- $S^k = \underbrace{\frac{n^k}{n} \left[\frac{\mu^k}{\mu} ln \frac{\mu^k}{\mu} + \frac{\mu^k}{\mu} T^k \right]}_{\tau^k(T)} + \frac{n^k}{n} \left(\frac{\mu - \mu^k}{\mu} \right) (T+1)$

Empirical analysis: Mozambique

- **Data**: 2 most recent Household Budget Surveys.
  - *Inquéritos ao Orçamentos Familiares* (IOF 2008/09 and 2014/15, INE)
- **Wellbeing**: Daily **real per capita consumption** (MEF/DEEF, 2016).
  - Corrects for variability in prices across geographical regions and over time.
- **Sample:** about 11,000 households (>50,000 ind.) interviewed once in 2008/2009; similar but interviewed 1-3 times in 2014/15 (pool).
- **Subpopulations**
  - **consumption percentile groups,**
  - **area of residence (rural or urban),**
  - **province of residence,**
  - **head's attained education.**

Table 2: Consumption inequality

Index	2008/09	20014/15
Gini	0.415	0.468
I_{-1}	0.409	0.532
I_0=M	0.303	0.381
I_1=T	0.367	0.520
I_2	0.887	2.242

Lorenz dominance

C. Gradín and F. Tarp, "Investigating growing inequality in Mozambique", SAJE forthcoming

Figure 2: Relative contributions to inequality (Gini, *M* and *T*) by percentile group, 2014/15

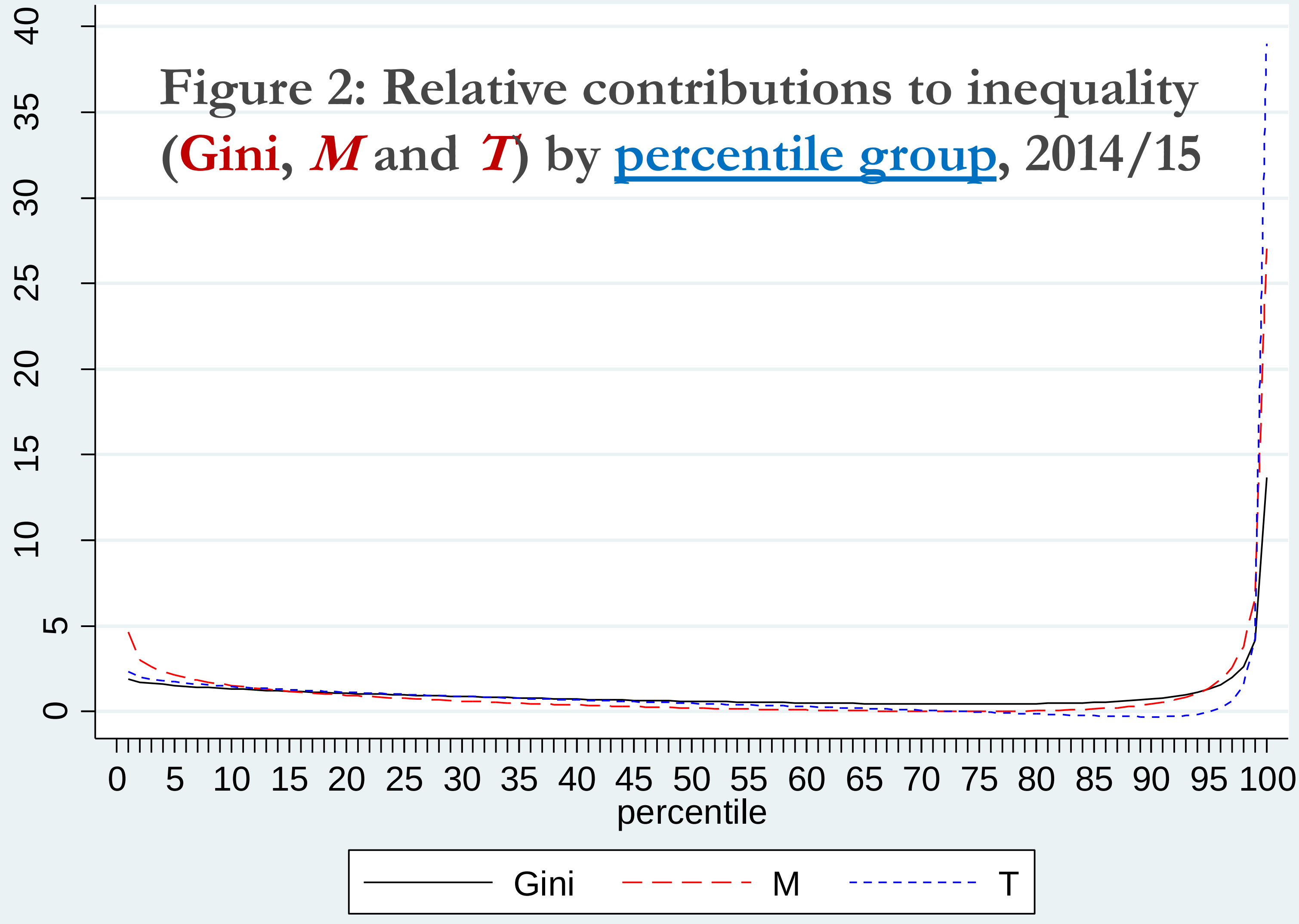


Figure 2b: Relative contributions to inequality (Gini, *M* and *T*) by percentile group, 2014/15

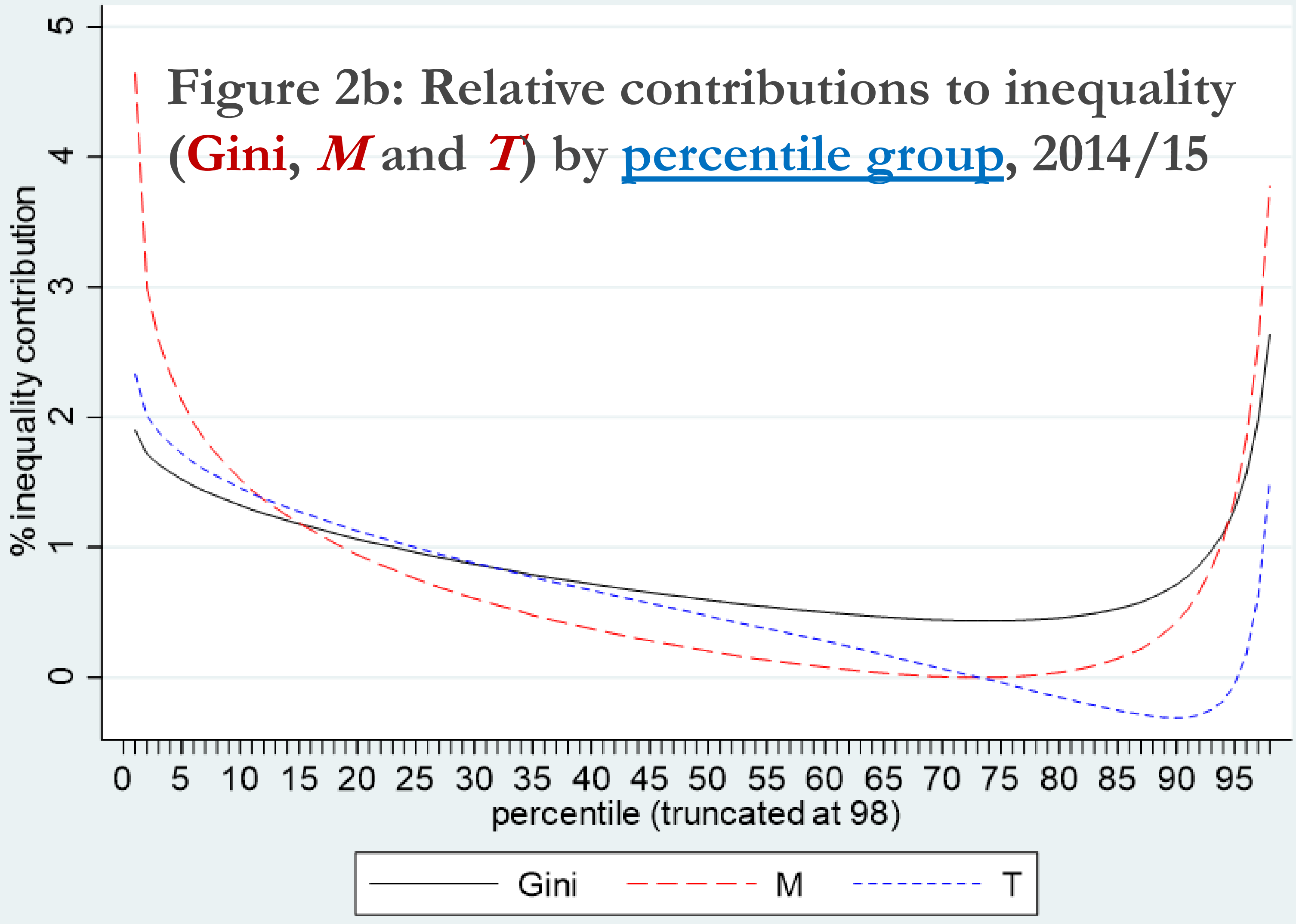
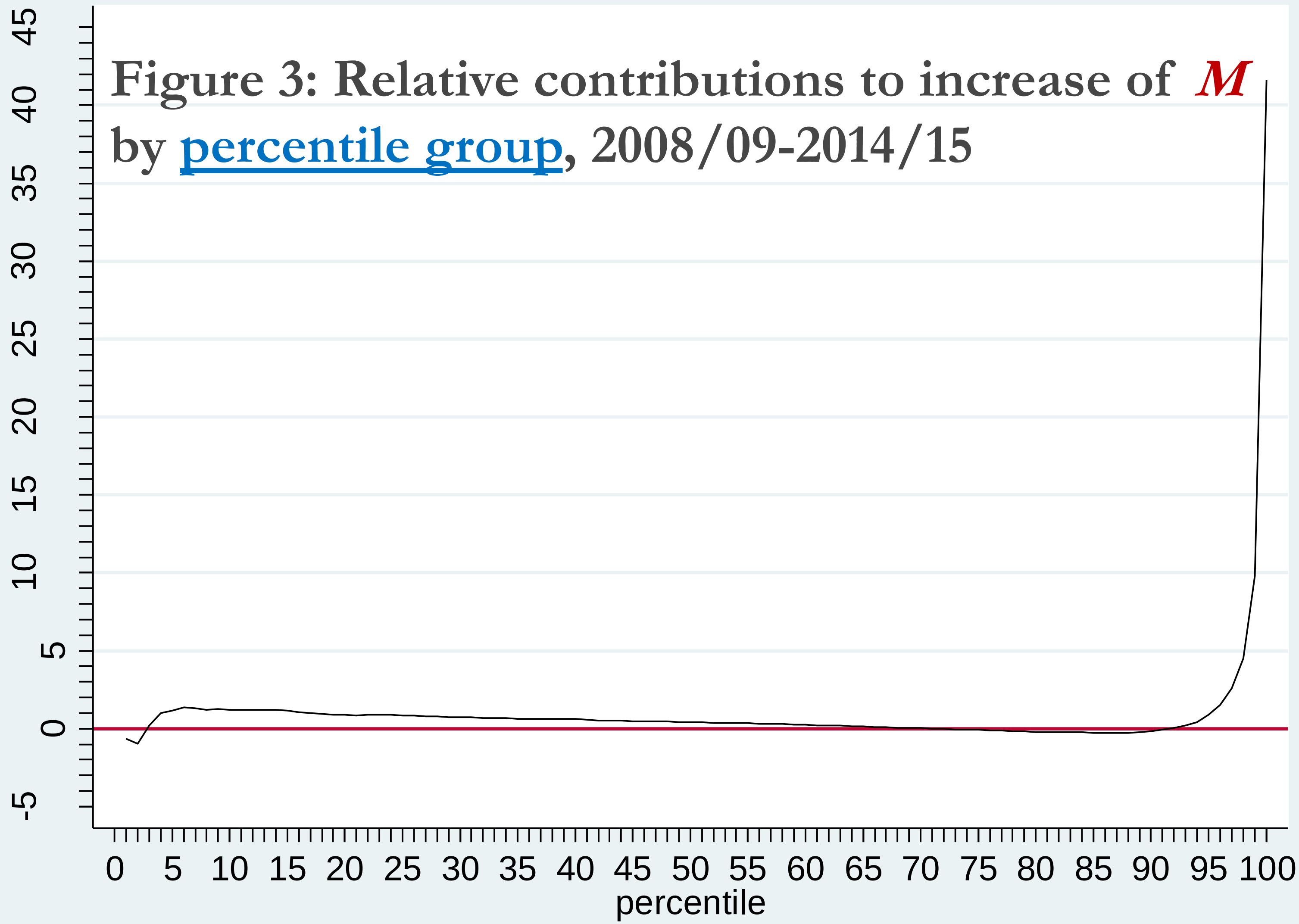


Figure 3: Relative contributions to increase of *M* by percentile group, 2008/09-2014/15



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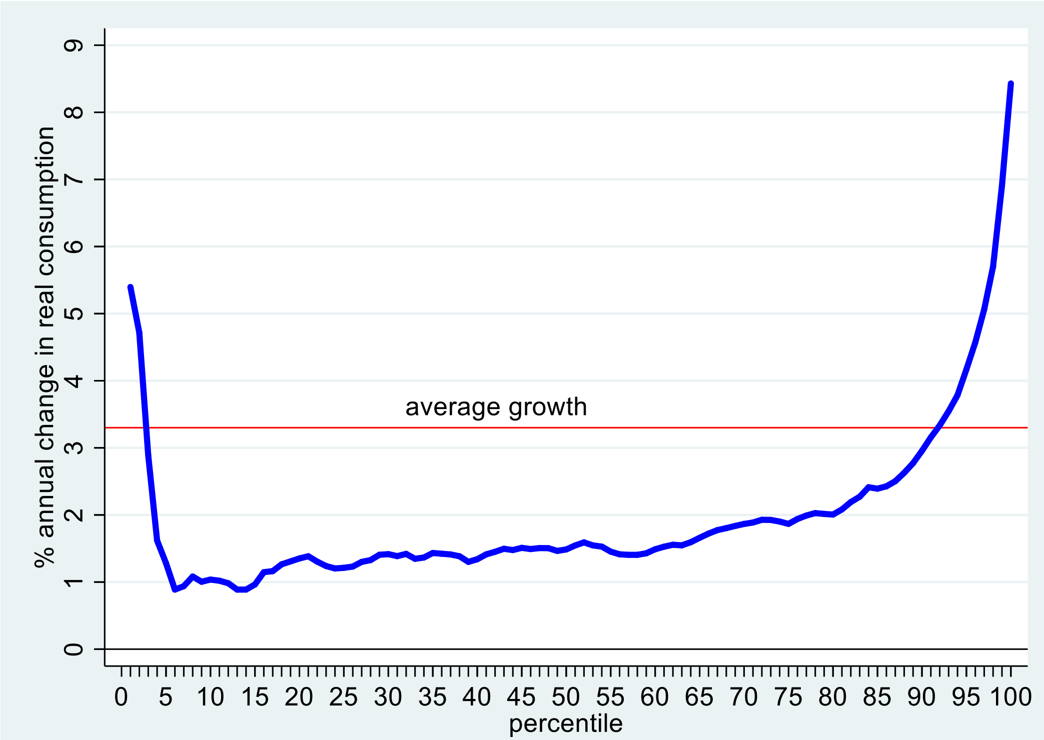
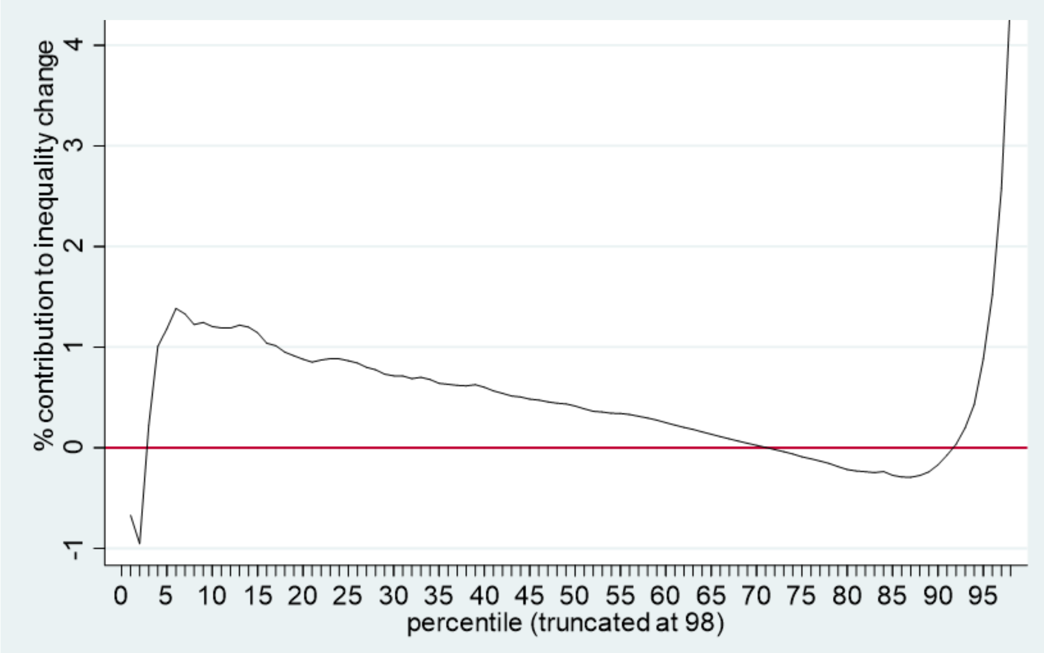


Growth incidence curve by percentile group, 2008/09-2014/15



Relative contributions to increase of *M* by percentile group, 2008/09-2014/15

Table 3: Relative RIF contributions to inequality by percentile, 2014/15

Range	%pop	%y	Gini	I_0=M	I_1=T
Bottom 5	5	0.8	8.4	14.7	9.7
6-25	20	6.5	23.7	24.5	25.6
26-75	50	34.3	31.1	12.5	23.2
76-95	20	30.0	12.9	6.5	-4.1
Top 5	5	28.5	24.0	41.9	45.6
Total	**100**	**100**	**100**	**100**	**100**

… to inequality increase between 2008/09 and 2014/15

Range	%pop	%y	Gini	I_0=M	I_1=T
Bottom 5	5	0.7	-0.1	0.8	4.5
6-25	20	5.6	15.1	21.5	21.6
26-75	50	30.6	39.9	19.5	32.0
76-95	20	29.1	7.1	-1.8	-8.7
Top 5	5	34.0	38.1	60.1	50.7
Total	**100**	**100**	**100**	**100**	**100**

Table 5a: RIF decomposition of M by province and area in 2014/15

Province	%pop.	μ_k/μ	M^k	s^k%	s_B^k%	s_W^k%
Niassa	6.4	66.1	0.267	5.7	1.3	4.5
Cabo Delgado	7.4	87.8	0.243	4.8	0.2	4.7
Nampula	19.5	77.7	0.304	17.0	1.5	15.5
Zambezia	18.8	76.0	0.291	16.0	1.7	14.3
Tete	9.8	97.6	0.247	6.3	0.0	6.3
Manica	7.5	93.2	0.259	5.1	0.0	5.1
Sofala	7.9	102.7	0.382	7.9	0.0	7.9
Inhambane	5.8	95.0	0.340	5.2	0.0	5.2
Gaza	5.5	89.8	0.345	5.1	0.1	5.0
Maputo province	6.6	169.4	0.376	9.3	2.9	6.5
Maputo City	4.9	280.1	0.583	17.3	9.8	7.5
All	**100**	**100**	**0.381**	**100**	**17.5**	**82.5**
Area						
Rural	68.3	78.8	0.243	48.3	4.7	43.6
Urban	31.7	145.7	0.541	51.7	6.7	44.9
All	**100**	**100**	**0.381**	**100**	**11.4**	**88.6**

Table 6a: RIF decomposition of ΔM by province and area, 2008/09-2014/15

Province	Δ%pop	$\Delta\mu_k/\mu$	ΔM^k	$\%\Delta S^k/\Delta M$	$\%\Delta S_B^k/\Delta M$	$\%\Delta S_W^k/\Delta M$
Niassa	0.5	-68.9	-0.078	-2.0	2.3	-4.3
Cabo Delgado	-0.5	-20.6	0.046	3.6	0.4	3.2
Nampula	0.3	-22.9	0.001	8.8	7.4	1.5
Zambezia	-0.2	-2.3	0.060	15.2	1.6	13.6
Tete	0.8	0.3	0.039	7.0	0.0	7.0
Manica	0.5	7.9	0.049	5.3	-0.8	6.1
Sofala	-0.2	8.3	-0.038	-5.2	-0.1	-5.1
Inhambane	-0.3	-3.5	0.082	5.2	0.1	5.1
Gaza	-0.8	5.7	0.013	-3.0	-0.7	-2.3
Maputo P.	0.3	74.6	0.125	25.3	14.0	11.3
Maputo C.	-0.4	95.2	0.148	39.8	32.5	7.3
All	**0.0**	**0.0**	**0.078**	**100**	**56.5**	**43.5**
Area						
Rural	-1.3	-9.6	0.003	15.7	16.5	-0.9
Urban	1.3	19.3	0.139	84.3	21.2	63.1
All	**0.0**	**0.0**	**0.078**	**100**	**37.7**	**62.3**

Table 5b: RIF decomposition of M by education in 2014/15

Education	%pop.	μ_k/μ	M^k	$s^k\%$	$s_B^k\%$	$s_W^k\%$
Less than primary	30.5	72.4	0.285	26.6	3.8	22.8
Lower Primary	43.9	82.1	0.247	30.5	2.1	28.4
Upper Primary	13.9	105.9	0.300	11.0	0.1	11.0
Lower Secondary	4.1	139.8	0.338	4.3	0.7	3.6
Upper Secondary	3.3	207.1	0.432	6.8	3.0	3.8
Technical	0.7	250.9	0.470	2.0	1.1	0.9
Some college	2.5	469.1	0.574	17.8	14.0	3.8
Unknown	1.1	94.7	0.334	0.9	0.0	0.9
All	**100**	**100**	**0.381**	**100**	**24.8**	**75.2**

Table 6b: RIF decomposition of ΔM by education, 2008/09-2014/15

Education	Δ%pop	$\Delta\mu_k/\mu$	ΔM^k	$\%\Delta S^k/\Delta M$	$\%\Delta S_B^k/\Delta M$	$\%\Delta S_W^k/\Delta M$
Less than primary	5.4	-10.2	0.032	43.1	12.9	30.1
Lower Primary	-11.4	-5.7	0.018	-18.9	4.5	-23.4
Upper Primary	1.3	-6.9	0.026	8.5	-0.9	9.4
Lower Secondary	1.1	-21.6	0.015	3.2	-2.0	5.1
Upper Secondary	1.8	-24.5	0.057	16.2	5.2	11.0
Technical	-0.1	12.3	0.138	0.8	-0.1	0.8
Some college	1.3	-8.5	0.023	44.2	34.4	9.8
Unknown	0.6	32.0	0.152	3.1	-0.5	3.6
All	**0.0**	**0.0**	**0.078**	**100**	**53.5**	**46.5**

Table 7a: Relative Decomposition of *M* and *T* by percentile, 2014/15

	M			T		
	RIF	Marginal	Shapley	RIF	Marginal	Shapley
Range	s^k	$\tilde{\delta}^k$	δ'^k	s^k	$\tilde{\delta}^k$	δ'^k
Bottom 5%	14.7	12.9	14.5	9.7	8.0	7.3
6-25	24.5	23.1	23.8	25.6	20.2	17.5
26-75	12.5	13.3	11.7	23.2	18.5	13.5
76-95	6.5	6.8	7.0	-4.1	-2.9	1.8
Top 5%	41.9	43.7	43.0	45.6	56.3	59.9
All	**100**	**100**	**100**	**100**	**100**	**100**

Note: Marginal, normalized to add up to 100

Table 7b: Relative Decomposition of M and T by province, 2014/15

	M			T		
	RIF	Marginal	Shapley	RIF	Marginal	Shapley
Province	S^k	$\widetilde{\delta}^k$	δ'^k	S^k	$\widetilde{\delta}^k$	δ'^k
Niassa	5.7	5.7	5.7	5.3	5.1	4.1
Cabo D.	4.8	4.8	4.8	4.7	4.5	4.2
Nampula	17.0	17.0	17.0	20.0	18.9	17.8
Zambezia	16.0	16.0	16.0	14.8	14.0	12.5
Tete	6.3	6.2	6.3	5.2	5.0	5.1
Manica	5.1	5.0	5.1	4.5	4.4	4.3
Sofala	7.9	7.8	7.9	7.3	7.2	7.4
Inhambane	5.2	5.1	5.2	4.7	4.6	4.5
Gaza	5.1	5.0	5.1	4.6	4.5	4.3
Maputo P.	9.3	9.4	9.4	7.8	8.2	10.1
Maputo C.	17.3	18.0	17.4	21.2	23.5	25.6
All	**100**	**100**	**100**	**100**	**100**	**100**

Table 7c: Relative Decomposition of M and T by area, 2014/15

	M			T		
	RIF	Marginal	Shapley	RIF	Marginal	Shapley
Area	S^k	$\widetilde{\delta}^k$	δ'^k	S^k	$\widetilde{\delta}^k$	δ'^k
Rural	48.3	48.2	48.1	46.6	39.5	38.7
Urban	51.7	51.8	51.9	53.4	60.5	61.3
All	**100**	**100**	**100**	**100**	**100**	**100**

Table 7d: Relative Decomposition of *M* and *T* by education, 2014/15

	M			T		
	RIF	Marg.	Shapley	RIF	Marg.	Shapley
Education	S^k	$\tilde{\delta}^k$	δ'^k	S^k	$\tilde{\delta}^k$	δ'^k
Less than primary	26.6	26.7	26.5	27.5	25.8	23.1
Lower Primary	30.5	30.4	30.5	28.7	26.9	24.6
Upper Primary	11.0	10.7	11.0	9.1	9.1	9.6
Lower Secondary	4.3	4.2	4.3	3.2	3.2	4.0
Upper Secondary	6.8	6.8	6.8	6.8	7.2	8.7
Technical	2.0	2.0	2.0	1.9	1.9	2.4
Some college	17.8	18.4	17.9	22.1	25.1	26.8
Unknown	0.9	0.9	0.9	0.8	0.8	0.8
All	**100**	**100**	**100**	**100**	**100**	**100**

Conclusions (1/3)

- A **detailed decomposition of inequality indices by subpopulations** based on RIF.
  - **Overall inequality** can be decomposed into the contribution of the distinct groups making up the population.
  - Additively decomposable indices: further decomposed into their **between-group and within-group components**.
  - Consistent with **RIF regressions**.
  - Verifies several **appealing properties** (e.g. consistency, path independence, and independence on the level of aggregation) and easy to compute.

Conclusions (2/3)

- Other natural **alternatives**,
  - Especially, **marginal and Shapley decomposition** using the equalizing subpopulation approach,
    - more appropriate for attributing the contribution of each group, especially with additive decomposable indices.
  - All three approaches are **approximately equal** in the case of the Mean Log Deviation (best additively decomposable index).

Conclusions (3/3)

- **Empirical analysis** of consumption inequality in Mozambique
  - Choice of approach is not empirically relevant (Mean Log Deviation)
    - Non-negligible differences with very extreme groups
  - The **richest groups**, such as people living in Maputo or in other urban areas, with higher educational level, or in the top of the consumption distribution are responsible for the largest shares of inequality and for its increasing trend over time.
    - **Even higher** contributions with Shapley decomposition of the **Theil index**, qualitative results are very similar.

Thanks!

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RIF regression-based decomposition

$RIF(y; \mathrm{I}(\boldsymbol{y})) = X'\beta + \varepsilon, \qquad \neq \ln y = X'\beta + \varepsilon$ (Fields, 1998 ...)

$$E\big(RIF(y; \mathrm{I}(\boldsymbol{y}))\big) = E_X[RIF(y; \mathrm{I}(\boldsymbol{y}))|X)] = E(X)'\beta$$

$$I(\boldsymbol{y}) = \frac{1}{n}\sum_{i=1}^{N} RIF(y_i; \mathrm{I}(\boldsymbol{y})) = \bar{X}'\beta$$

$$I^1 - I^0 = \bar{X}^1\beta^1 - \bar{X}^0\beta^0 = (\bar{X}^1 - \bar{X}^0)\beta^1 + \bar{X}^0(\beta^1 - \beta^0)$$

Characteristics effect (explained)

Coefficients effect (unexplained)

counterfactual distribution combining average characteristics of the initial distribution with the impact on inequality in the final one, $\bar{X}^0\beta^1$

- $S^k = \frac{1}{n}\sum_{i=1}^{n} \hat{\beta}^k \lambda_i^k = \frac{1}{n}\sum_{j=1}^{n_k} \hat{\beta}^k \lambda_j^k = \frac{n_k}{n}\hat{\beta}^k.$

Figure 3b: Relative contributions to increase of *M* by percentile group, 2008/09-2014/15