A Framework for Measuring Inclusive Growth

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Motivation

Why measure inclusive growth?

- Growth has potential to improve the lives of all people
- However, it is also possible that this potential may not, in fact, be realized – it is an empirical question hence measurement
- Consider the following growth scenarios:
  - Growth with growing inequality
  - Growth with modest or no improvements in poverty
  - Growth that leaves out certain ethnic groups, regions, or sectors
  - Growth without improvements in the other dimensions of wellbeing
  - Growth that leads to choking pollution
- These are cases that are contingent
  - Not all policymakers would agree that “growth is good”
  - Tradeoffs

Motivation

Alternatively, consider the following growth scenarios:

- Growth with falling inequality
- Growth with strong improvements in poverty
- Growth that includes all ethnic groups, regions, and sectors
- Growth with strong improvements in the other dimensions
- Growth with lower pollution levels

These are cases without disagreement

- Where policymakers with very different goals also can agree that "growth is good"
- No need for tradeoffs A. Sen (2009) *The Idea of Justice*

Motivation

A broad definition of inclusive growth

Growth that simultaneously achieves other important ends

Note

Must specify the “ends” one is interested in achieving with the “means” of income growth.

Use to construct measures of inclusive growth

Motivation

- How to implement?
  - As practical methodology that can help monitor progress and guide policy
- How to understand and measure the extent to which growth is inclusive?
  - Encompassing other outcomes and objectives besides growth of mean income
  - Giving broader policy traction to the growth agenda
- At beginning stages
  - Appreciate input and references

Basic Model

Definitions

- Let μ denote the average income or "means"
- Let e denote some other outcome or "ends" (cardinally measured)

Data

- Period 1 observations (μ_1, e_1)
- Period 2 observations (μ_2, e_2)

Note

- Could have more ends than one

Growth

- $(\%\Delta\mu, \%\Delta e) = ((\mu_2-\mu_1)/\mu_1 , (e_2-e_1)/e_1)$ percentage change

Absolute Measure

An absolute measure of inclusive growth

- $A = \%\Delta e$
- Measures the extent to which e grows
- Ignores growth in the means.
- Lower growth in means has no effect on measure
- All that matters is ends
- Ex
  - e = mean income of lowest 40%
  - e = P_1 poverty gap
  - e = mean earnings of women
  - e = MPI poverty

Relative Measure

A relative measure of inclusive growth

- $R = \%\Delta e / \%\Delta \mu$
- Measures the ‘productivity’ with which the means achieves the ends
- Elasticity of ends with respect to means
- Lower growth in means with the same growth rate for ends raises the relative measure.
- Ex
  - e = mean income of lowest 40%
  - e = P_1 poverty gap
  - e = mean earnings of women
  - e = MPI poverty

Benchmarked Measures

A benchmarked measure of inclusive growth

Ex

Rate that a similar country or set of countries experienced; obtained empirically

Rate that would have arisen if growth had been equally distributed among the population; e_c obtained via a thought experiment (What might otherwise be possible)

Apply absolute or relative measure of inclusive growth to counterfactual

$B = A/A_c$ (or R/R_c)

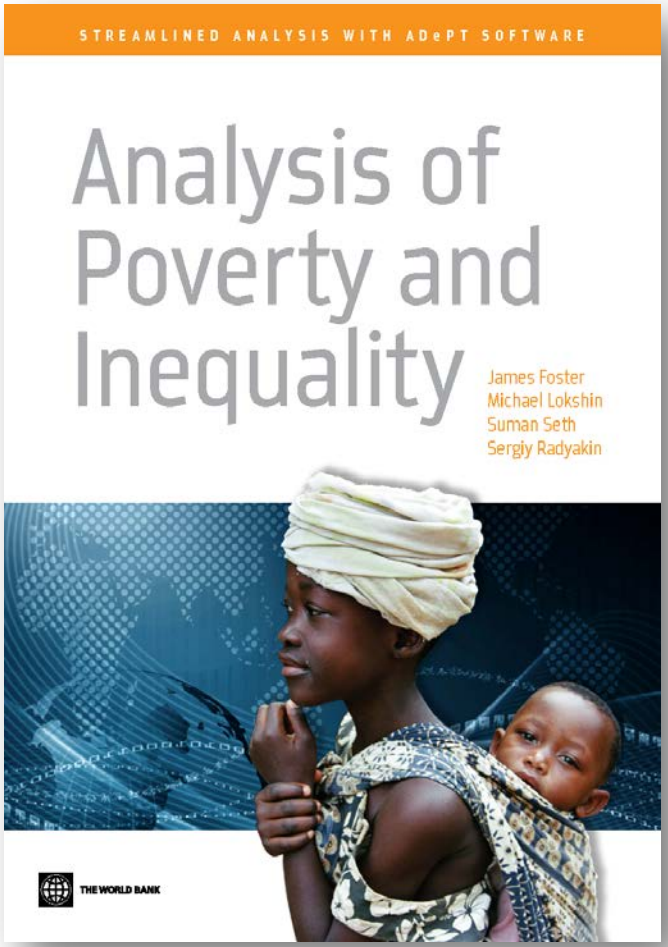
Idea Contrast actual to counterfactual

Q/Other forms of measures?

Three Varieties of Inclusive Growth

- Vertical
  - Capturing the impacts on income poverty, inequality or size
- Horizontal
  - Capturing the differential impacts across groups in society
- Dimensional
  - Capturing the impacts on different dimensions of wellbeing
- Note
  - Depends on “ends” variable
- Focus here
  - Two forms of variables: income standards, multidimensional poverty

Income Standards, Inequality, and Poverty



https://openknowledge.worldbank.org/handle/10986/13731

Income Standards, Inequality, and Poverty

Idea

An *income standard* summarizes entire distribution x in a single ‘representative income’ s(x)

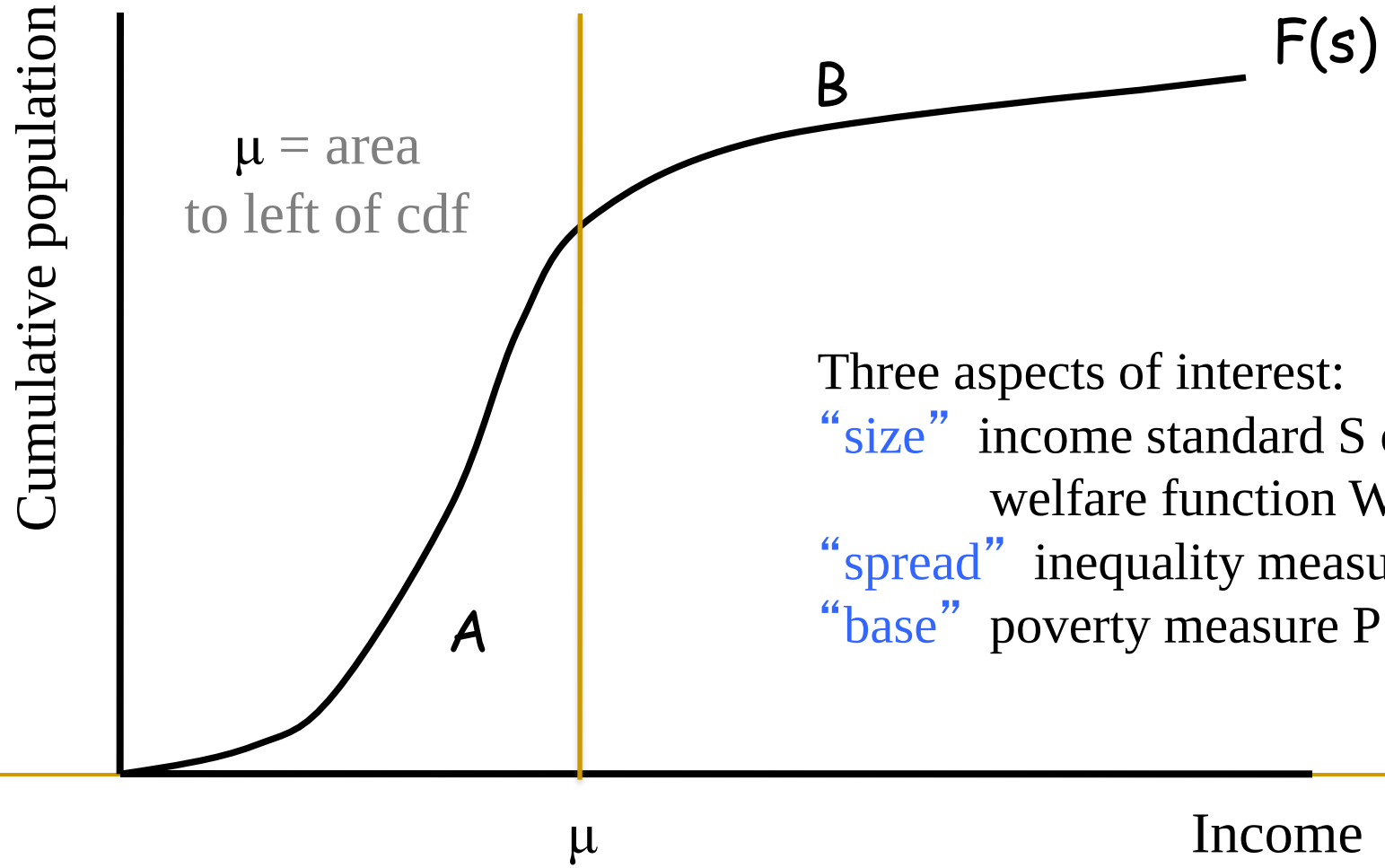
Ex

Mean, median, income at 90th percentile, mean of top 40%, Sen’s, Atkinson’s …

- Measures ‘size’ of the distribution
- Can have normative interpretation Atkinson’s
- Are basis of measures of inequality and poverty

Income Variable

Cumulative distribution function cdf



Three aspects of interest:

"size" income standard S or welfare function W

"spread" inequality measure I

"base" poverty measure P

Income Standards

Income standard s: D → R

Properties

Symmetry If x is a permutation of y, then s(x) = s(y)

Replication Invariance If x is a replication of y, then s(x) = s(y)

Linear Homogeneity If x = ky for some scalar k > 0, then s(x) = ks(y)

Normalization If x is completely equal, then s(x) = x_1

Continuity s is continuous on each n-person set D_n

Weak Monotonicity If x > y, then s(x) > s(y).

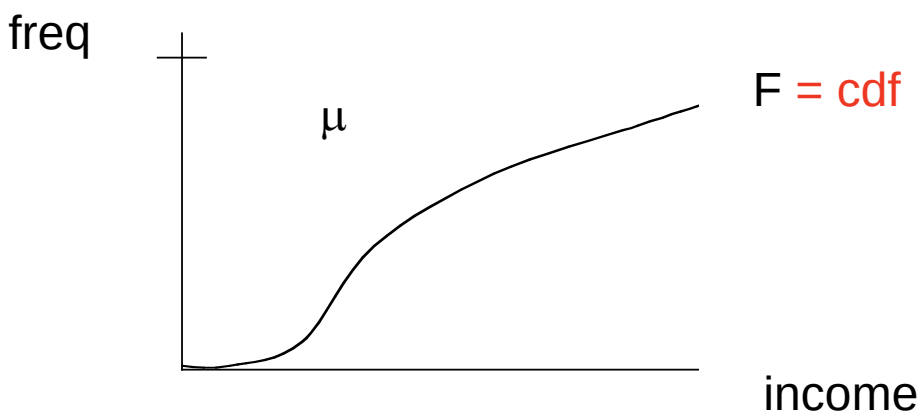
Note

Satisfied by all examples given above and below

Income Standards

Examples

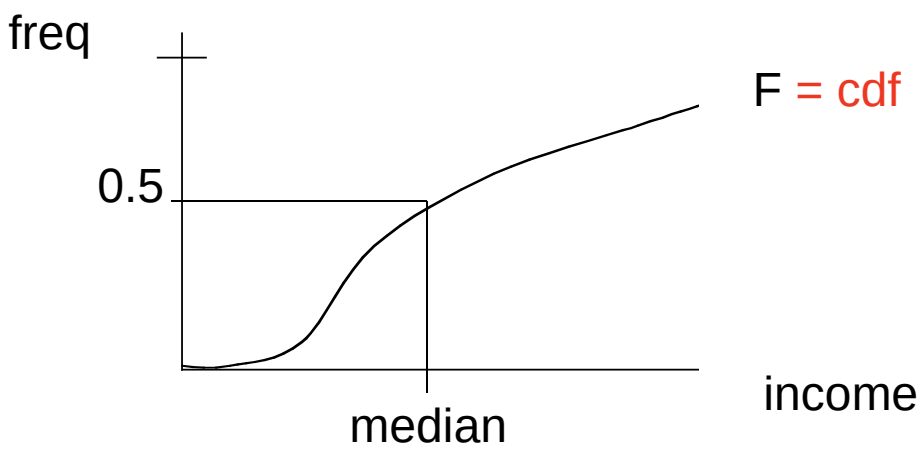
Mean $s(x) = \square(x) = (x_1+\ldots+x_n)/n$



Income Standards

Examples

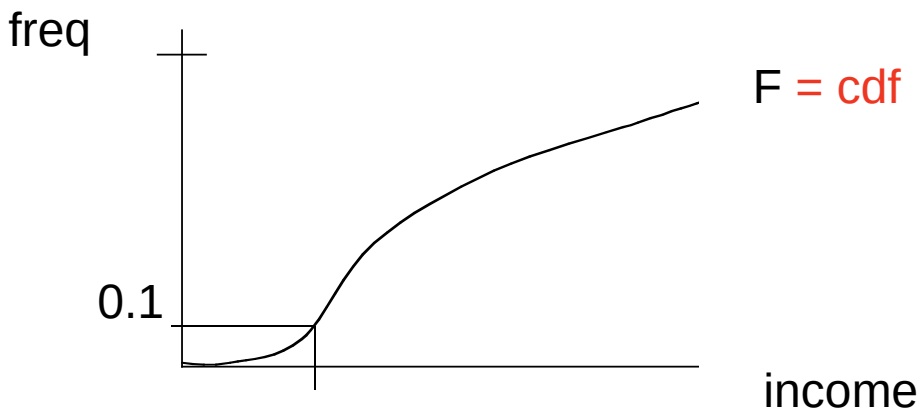
Median $x = (3, 8, 9, 10, 20)$, $s(x) = 9$



Income Standards

Examples

10th percentile income



s = Income at10th percentile

Income Standards

Examples

Mean of bottom 40%

$x = (3, 5, 6, 6, 8, 9, 15, 17, 23, 25)$

$s(x) = 5$

Income Standards

Examples

Mean of top 40%

x = (3, 5, 6, 6, 8, 9, 15, 17, 23, 25)

s(x) = 20

Income Standards

Examples

Sen Mean or Welfare Function

$$S(x) = E \min(a,b)$$

Ex/ $x = (1,2,3,4)$

$$s(x) = \square \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix} = 30/16 < \square(1,2,3,4) = 40/16$$

Income Standards

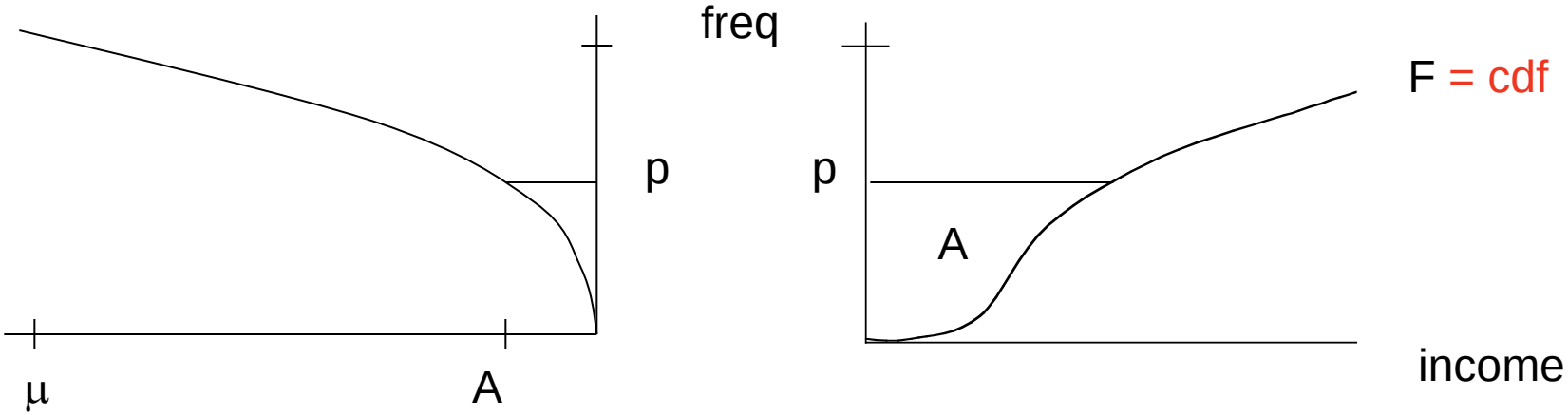
Examples

Sen Mean or Welfare Function

$$S(x) = E \min(a,b)$$

Another view

Generalized Lorenz



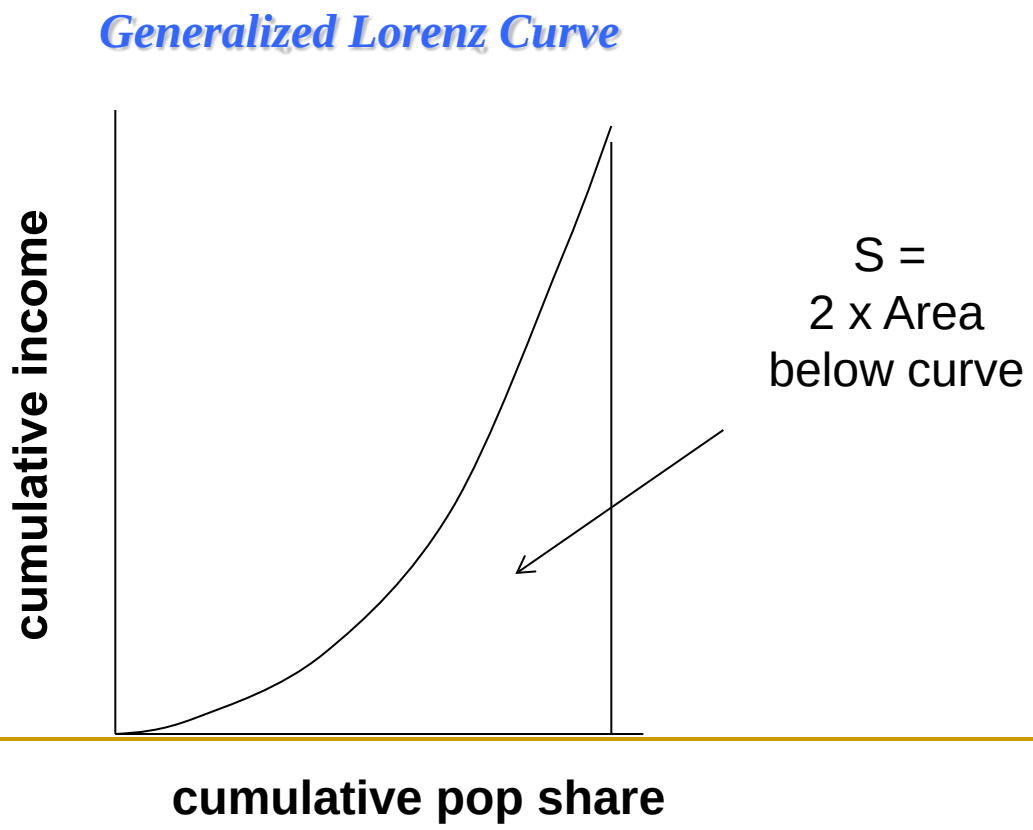
Income Standards

Examples

Sen Mean or Welfare Function

$$S(x) = E \min(a,b)$$

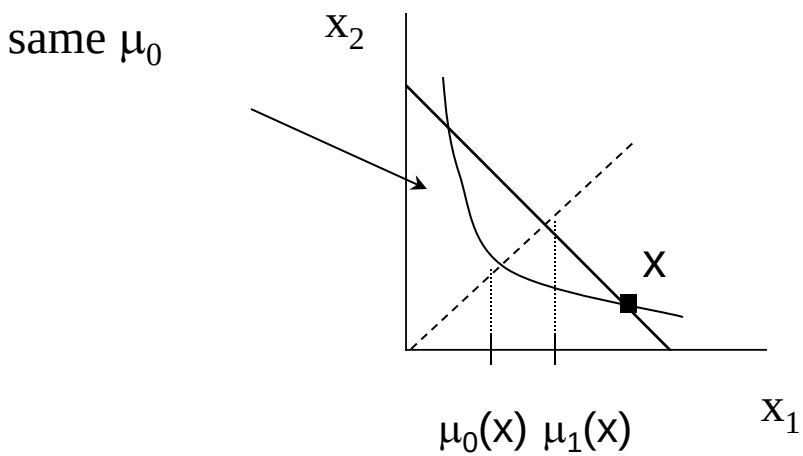
Another view



Income Standards

Examples

Geometric Mean $s(x) = \square_0(x) = (x_1 x_2 \ldots x_n)^{1/n}$



Thus $s(x) = \square_0$

- emphasizes lower incomes
- is lower than the usual mean

Unless distribution is completely equal

Income Standards

Examples

General Means

$$\square_{\square}(x) = \begin{cases} [(x_1^{\square} + \ldots + x_n^{\square})/n]^{1/\square} & \text{for all } \square\square 0 \\ (x_1 \cdots x_n)^{1/n} & \text{for } \square = 0 \end{cases}$$

Hardy Littlewood Polya 1952; Kolm 1969; Atkinson 1970

$\alpha = 1$	arithmetic mean
$\alpha = 0$	geometric mean
$\alpha = 2$	Euclidean mean
$\alpha = -1$	harmonic mean

For $\alpha < 1$: Distribution sensitive

Lower α implies greater emphasis on lower incomes

Income Standards and Inequality

Inequality

A wide array of measures

- Gini Coefficient
- Coefficient of Variation
- Mean Log Deviation
- Variance of logarithms
- Generalized Entropy Family
- 90/10 ratio
- Decile Ratio
- Atkinson Family

What do these measures have in common?

Income Standards and Inequality

Inequality

Framework for Population Inequality

- One income distribution x
- Two income standards:
- Lower income standard $a = s_L(x)$
- Upper income standard $b = s_U(x)$
- Note: $s_L(x) < s_U(x)$ for all x

Inequality

- $I = (b - a)/b$ or some function of ratio a/b

Observation

- Framework encompasses all common inequality measures
- Theil, variance of logs in limit

Income Standards and Inequality

Inequality in a Population

Measure	Twin Income Standards	
	s_L	s_U
Gini Coefficient	**Sen**	**mean**
Coefficient of Variation	**mean**	**euclidean**
Mean Log Deviation	**geometric mean**	**mean**
Generalized Entropy Family	**general**	**mean**
or	**mean**	**general**
90/10 ratio	**10th pc income**	**90th pc income**
Decile Ratio	**mean**	**top 10% mean**
Atkinson Family	**general**	**mean**
Palma or Kuznets	**bottom 40% mean**	**top 10% mean**

Back to Inclusive Growth

Each of the first two varieties of inclusive growth (Vertical and Horizontal) is fundamentally related to income standards

Example: Geometric mean g as a stylized welfare fcn

Absolute measure of inclusive growth: $\%\Delta g$

- "Growth of what?" Sen
  - Specify an alternative objective and maximize its growth

It could be a very useful case study in inclusive growth to repeat the Growth Report analysis with the geometric mean or another

Inclusive Growth

Relative measure of inclusive growth: $R = \%\Delta g / \%\Delta \mu$

Note

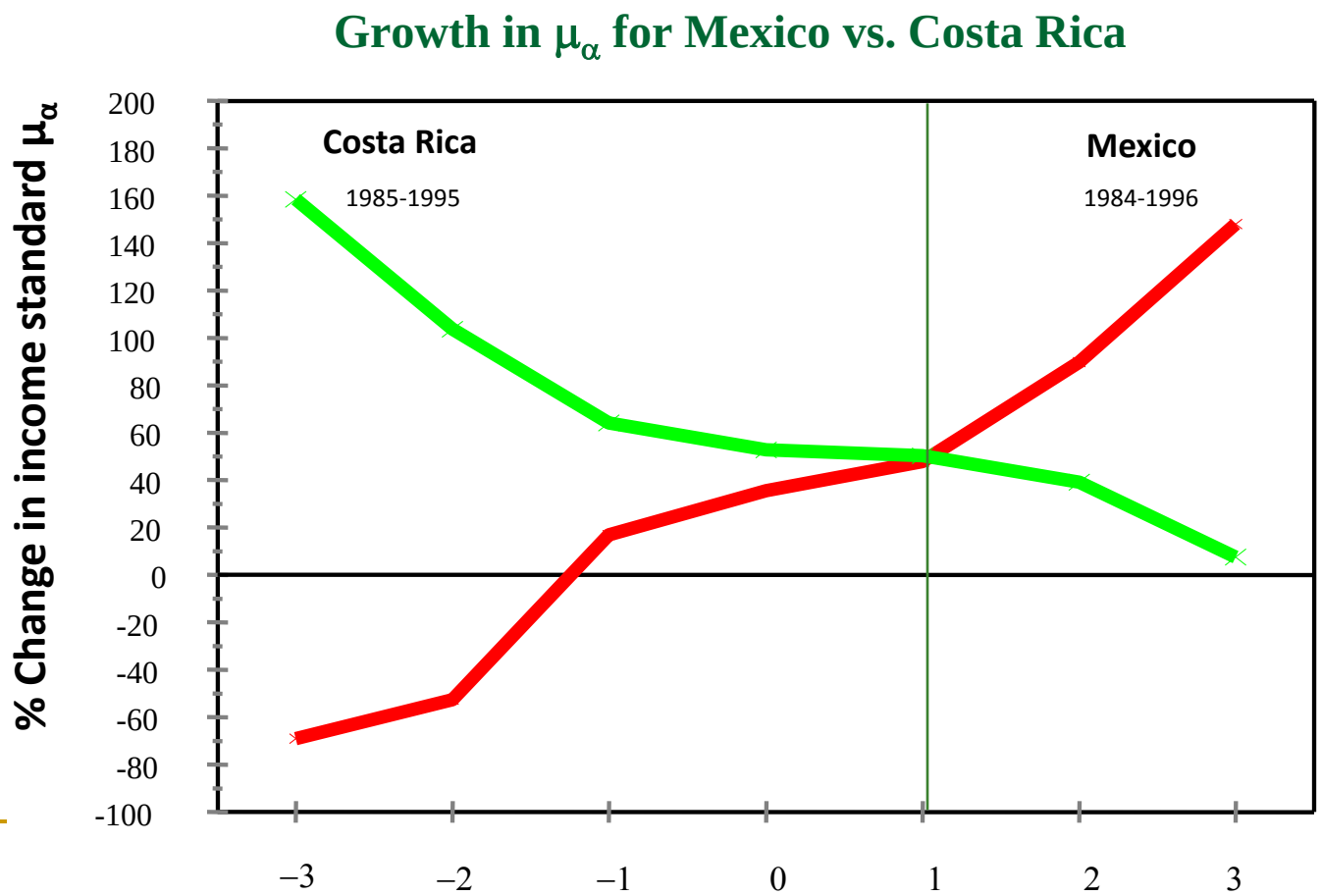
- Simply gauges progress in lowering Atkinson's inequality measure (or the mean log deviation)

Alternative standards yield different measures of inclusive growth and are linked to different inequality measures

Growth and Inequality

Inequality as Twin Standards

Application: Growth and Inequality



Foster and Szekely (2008)

Inclusive Growth

- Benchmarked measure of inclusive growth is the same as the relative measure here
  - Since income standards are linearly homogeneous.

- Pro-poor growth
  - Poverty measures have income standards censored at the poverty line.
- Horizontal inclusive growth
  - Concentrate purely on between group term
  - An income standard applied to a smoothed distribution that removes all within group inequality

Dimensional Inclusive Growth

If single dimensional non-income variables, can use above

If many, how to aggregate?

For size or spread, HDI, IHDI or other multidimensional measures of size can be used

Note – Serious assumptions needed

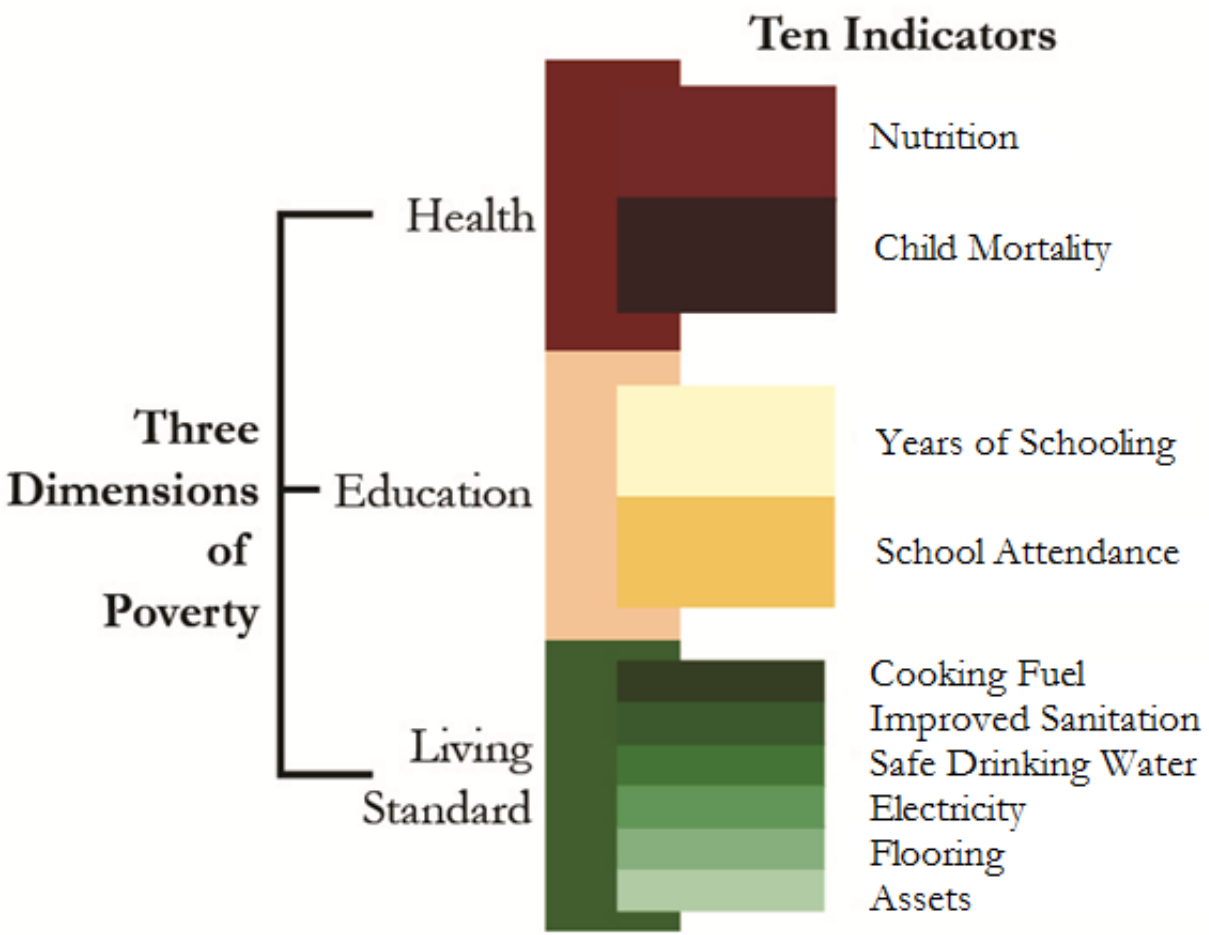
Dimensional Inclusive Growth

For poverty, several new technologies are available.

- Here I use adjusted headcount ratio: Alkire and Foster (2011)
- OPHI is working on a book on multidimensional poverty
- Also presenting event in UNGA
  - "Multidimensional poverty measurement in the post-2015 development context"
  - live webcast of side-event at the UN General Assembly
  - 1.15-2.30 pm (EST), 24 September 2013, United Nations, New York
  - Live and on-demand webcast coverage will be available on UN Web TV:
  - http://webtv.un.org

- Results are from "How Multidimensional Poverty Went Down: Dynamics and Comparisons," Sabina Alkire and José Manuel Roche, March 2013, OPHI, Oxford

MPI Indicators



Published in *Human Development Reports* since 2010 for over 100 countries
Uses DHS data – as in the MDGs

Dimensional Inclusive Growth

Countries	M_0			Per Capita GDP			Growth Elasticity of M_0
	First Year	Second Year	Percentage decrease p.a.	First Year	Second Year	Growth p.a	
Armenia 2005-2010	0.003	0.001	-12.90%	$4,096.44	$4,900.47	3.93%	-3.286
Bangladesh 2004-2007	0.365	0.289	-7.00%	$1,114.63	$1,290.69	5.27%	-1.330
Bolivia 2003-2008	0.175	0.089	-9.80%	$3,597.70	$4,172.33	3.19%	-3.068
Cambodia 2005-2010	0.298	0.212	-5.80%	$1,508.01	$1,968.13	3.22%	-1.803
Colombia 2005-2010	0.040	0.023	-8.40%	$7,304.56	$8,479.35	3.22%	-2.611
Ethiopia 2000-2005	0.677	0.605	-2.10%	$527.30	$636.07	4.13%	-0.509
Ethiopia 2005-2011	0.605	0.523	-2.20%	$636.07	$979.21	8.99%	-0.245
Ghana 2003-2008	0.309	0.202	-6.90%	$1,134.15	$1,380.12	4.34%	-1.591
Guyana 2005-2009	0.053	0.041	-5.40%	$2,536.38	$2,979.60	4.37%	-1.236
India 1998/9-2005/6	0.300	0.251	-2.40%	$1,632.30	$2,293.16	5.78%	-0.415
Jordan 2007-2009	0.011	0.011	-3.60%	$4,844.75	$5,245.63	4.14%	-0.870
Kenya 2003-2008/9	0.296	0.244	-3.20%	$1,274.30	$1,441.08	2.38%	-1.345
Lesotho 2004-2009	0.239	0.182	-4.80%	$1,185.99	$1,383.86	3.34%	-1.438

Dimensional Inclusive Growth

Countries	M_0			Per Capita GDP			Growth Elasticity of M_0
	First Year	Second Year	Percentage decrease p.a.	First Year	Second Year	Growth p.a	
Madagascar 2004-2008/9	0.383	0.400	1.00%	$855.71	$915.36	1.55%	0.646
Malawi 2004-2010	0.381	0.334	-2.00%	$644.62	$780.37	3.51%	-0.570
Nepal 2006-2011	0.350	0.217	-7.60%	$969.65	$1,105.72	2.81%	-2.708
Nigeria 2003-2008	0.368	0.313	-3.00%	$1,577.12	$1,945.47	4.67%	-0.642
Peru 2005-2008	0.085	0.066	-7.30%	$6,386.96	$7,967.33	8.25%	-0.885
Rwanda 2005-2010	0.460	0.330	-5.60%	$840.47	$1,077.01	5.63%	-0.995
Senegal 2005-2010/11	0.440	0.423	-0.70%	$1,677.00	$1,737.55	0.66%	-1.066
Tanzania 2008-2010	0.367	0.326	-5.70%	$1,208.45	$1,293.08	3.50%	-1.628
Uganda 2006-2011	0.417	0.343	-3.50%	$977.07	$1,187.65	4.31%	-0.812
Zimbabwe 2006-2010/11	0.180	0.145	-4.20%	-	-	-	-

Summary

Framework for measuring inclusive growth

- Based on “ends” and “means”
- Three forms of measure: absolute, relative, benchmarked
- Three types of inclusivity: vertical, horizontal, dimensional
- Examples of “ends”: income standards, multidimensional poverty

Q/

- What is your conception of inclusivity?
- What does this framework miss?