Poverty Measurement and the Distribution of Deprivations among the Poor

Sabina Alkire

OPHI, Oxford

James E. Foster

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- Two forms of technologies for evaluating poverty identification and aggregation of Sen (1976)
- 1 Unidimensional methods apply when:
 - **Single** welfare variable eg, calories
 - Variables can be combined into **one** aggregate variable eg, expenditure
- 2 Multidimensional methods apply when:
 - Variables **cannot** be meaningfully aggregated eg, sanitation conditions and years of education
 - Desirable to leave variables **disaggregated** because subaggregates are policy relevant – eg food and nonfood consumption

Recently, strong **demand** for tools for measuring poverty multidimensionally

Governments, international organizations, NGOs

Literature has responded with new measures

Anand and Sen (1997)

Tsui (2002)

Atkinson (2003)

Bourguignon and Chakravarty (2003)

Deutsch and Silber (2005)

Chakravarty and Silber (2008)

Maasoumi and Lugo (2008)

Problems

Most inapplicable to **ordinal** variables Encountered in poverty measurement Or yield methods that are far too **crude** Violate Dimensional Monotonicity Non-discerning identification: Very few poor or very few nonpoor

Methodology introduced in Alkire-Foster (2011) Identification: Dual cutoff z and k Measure: Adjusted headcount ratio M_0 Addressed these problems Applies to **ordinal** And even categorical variables Not so **crude** Satisfies Dimensional Monotonicity Discerning identification: not all poor or all nonpoor Satisfies key properties for policy and analysis **Decomposable** by population **Breakdown** by dimension after identification

Specific implementations include:
Multidimensional Poverty Index (UNDP) Cross country implementation of M₀ by OPHI and HDRO
Official poverty index of Colombia Country implementation of M₀ by Government of Colombia
Gross National Happiness index (Bhutan) Country implementation of (1-M₀) by Center for Bhutan Studies
Women's Empowerment in Agriculture Index (USAID) Cross country implementation of (1-M₀) by USAID, IFPRI, OPHI

One possible critique

 M_0 is not sensitive enough to distribution among the poor

Two forms of distribution sensitivity among poor

To inequality within dimensions

Kolm (1976)

To positive association **across dimensions** Atkinson and Bourguignon (1982)

Many existing measures satisfy one or both

Adjusted FGT of Alkire-Foster (2011)

However, adjusted FGT not applicable to ordinal variables

This Paper

Asks

Can M_0 be altered to obtain a method that is both

- **sensitive to distribution** among the poor
- and applicable to **ordinal data**?

Answer

Yes. In fact, as easy as constructing unidimensional measures satisfying the transfer principle

Key

Intuitive transformation from unidimensional to multidimensional measures

Offers insight on the structure of M_0 and related measures

This Paper

However we lose

Breakdown by dimension after identification

Question

Is there any multidimensional measure that is sensitive to the distribution of deprivations and also can be broken down by dimension?

Answer

Classical impossibility result

Can have one **or** the other but **not both**!

Bottom line

Recommend using M_0 with an associated inequality measure

Outline

Poverty Measurement Unidimensional Multidimensional Transformations Measures Axioms Impossibilities and Tradeoffs Conclusions

Poverty Measurement

Traditional framework of Sen (1976) Two steps Identification: "Who is poor?" Targeting Aggregation "How much poverty?" Evaluation and monitoring

Typically uses **poverty line** for identification Early definition: Poor if income below or equal to cutoff Later definition: Poor if income strictly below cutoff Example: Income distribution x = (7,3,4,8) poverty line $\pi = 5$ Who is poor?

Typically uses poverty measure for aggregation

- Formula aggregates data to poverty level
 - Examples: Watts, Sen
 - Example: FGT

Where: g_i^{α} is $[(\pi - x_i)/\pi]^{\alpha}$ if *i* is poor and 0 if not, and $\alpha \ge 0$ so that

 $\alpha = 0$ headcount ratio

 $\alpha = 1$ per capita poverty gap

 $\alpha = 2$ squared gap, often called FGT measure $P_{\alpha}(\mathbf{X}; \pi) = \mu(\mathbf{g}_{1}^{\alpha}, ..., \mathbf{g}_{n}^{\alpha}) = \mu(\mathbf{g}^{\alpha})$

Example **Incomes** x = (7, 1, 4, 8)**Poverty line** $\pi = 5$ **Deprivation vector** $g^0 = (0,1,1,0)$ Headcount ratio $P_0(x; \pi) = \mu(g^0) = 2/4$ Normalized gap vector $g^1 = (0, 4/5, 1/5, 0)$ **Poverty gap** = $HI = P_1(x; \pi) = \mu(g^1) = 5/20$ Squared gap vector $g^2 = (0, 16/25, 1/25, 0)$ **FGT Measure =** $P_2(x; \pi) = \mu(g^2) = 17/100$

FGT Properties

For $\alpha = 0$ (headcount ratio)

Invariance Properties: Symmetry, Replication Invariance, Focus Composition Properties: Subgroup Consistency, Decomposability

For $\alpha = 1$ (poverty gap)

+Dominance Property: Monotonicity

For $\alpha = 2$ (FGT)

+Dominance Property: Transfer

Poverty line actually has two roles

- In identification step, as the separating **cutoff** between the target group and the remaining population.
- In aggregation step, as the **standard** against which shortfalls are measured

In some applications, it may make sense to **separate** roles A **poverty standard** π_A for constructing gap and aggregating A **poverty cutoff** $\pi_I \leq \pi_A$ for targeted identification Example 1: Measuring ultra-poverty Foster-Smith (2011) Forcing standard π_A down to cutoff π_I distorts the evaluation of ultrapoverty Example 2: Measuring hybrid poverty Foster (1998)

Broader class of poverty measures $P(x; \pi_A, \pi_I)$

Example: FGT $P_{\alpha}(x; \pi_A, \pi_I)$ **Incomes** x = (7, 1, 4, 8)**Poverty standard** $\pi_{\Delta} = 5$ **Poverty cutoff** $\pi_{I} = 3$ **Deprivation vector** $g^0 = (0,1,0,0)$ (use π_1 for identification) Headcount ratio $P_0(x; \pi_A, \pi_I) = \mu(g^0) = 1/4$ Normalized gap vector $g^1 = (0, 4/5, 0, 0)$ (use π_A for gap) **Poverty gap** = $HI = P_1(x; \pi_A, \pi_I) = \mu(g^1) = 4/20$ Squared gap vector $g^2 = (0, 16/25, 0, 0)$ **FGT Measure** = $P_2(x; \pi_A, \pi_I) = \mu(g^2) = 16/100$

All properties are easily generalized to this environment FGT Properties

For $\alpha = 0$ (headcount ratio)

Invariance Properties: Symmetry, Replication Invariance, and Focus Composition Properties: Subgroup Consistency, Decomposability,

For $\alpha = 1$ (poverty gap)

+Dominance Property: Monotonicity

For $\alpha = 2$ (FGT)

+Dominance Property: Transfer

Idea of poverty measure $P(x;\pi_A,\pi_I)$

Allows flexibility of targeting group below poverty cutoff π_{I} while maintaining the poverty standard at π_{A} Particularly helpful when different groups of poor have

different characteristics and hence need different policies

How to evaluate poverty with many dimensions? Previous work mainly focused on **aggregation**

While for the **identification** step it:

- First set cutoffs to identify deprivations
- Then identified poor in one of three ways
 - Poor if have any deprivation
 - Poor if have all deprivations
 - Poor according to some function left unspecified

Problem

First two are **impractical** when there are many dimensions Need intermediate approach

Last is indeterminate, and likely inapplicable to ordinal data

Alkire and Foster (2011) methodology addresses these problems

- It specifies an **intermediate** identification method that is consistent with **ordinal** data
- Dual cutoff identification
- Deprivation cutoffs z₁...z_j one per each of j deprivations
 Poverty cutoff k across aggregate weighted deprivations
 Idea

A person is poor if multiply deprived enough Example

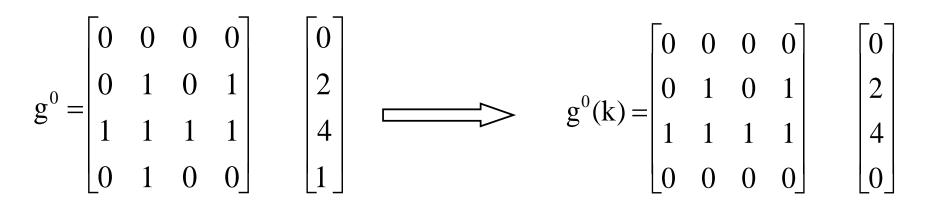
Achievement Matrix (say equally valued dimensions)

Dimensions $Y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix}$ $z = (13 \quad 12 \quad 3 \quad 1)$ Cutoffs

AF Methodology

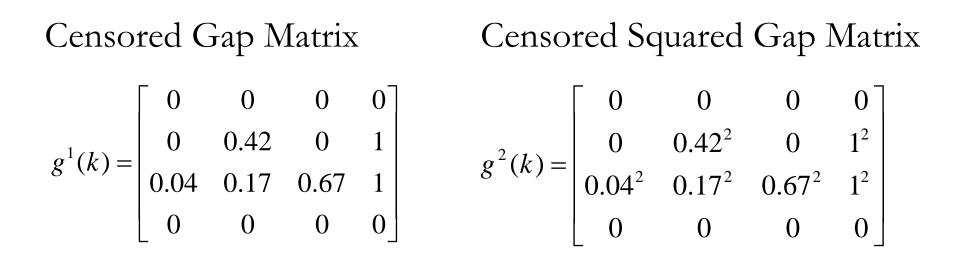
Deprivation Matrix

Censored Deprivation Matrix, k=2



Identification Who is poor? If poverty cutoff is k = 2 Then the two middle persons are poor Now censor the deprivation matrix Ignore deprivations of nonpoor

If data cardinal, construct two additional censored matrices



Aggregation

$$M_{\alpha} = \mu(g^{\alpha}(k)) \text{ for } \alpha \ge 0$$

Adjusted FGT M_{α} is the mean of the respective censored matrix

Properties

For $\alpha = 0$ (Adjusted headcount ratio)

- Invariance Properties: Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus
- Dominance Properties: Weak Monotonicity, Dimensional Monotonicity, Weak Rearrangement
- Composition Properties: Subgroup Consistency, Decomposability, Dimensional Breakdown
- For $\alpha = 1$ (Adjusted poverty gap)

+Dominance Property: Monotonicity, Weak Transfer

For $\alpha = 2$ (Adjusted FGT)

+Dominance Property: Transfer

Note

The poverty measures with α > 0 use gaps, hence require cardinal data

Impractical given data quality

Focus here on measure with $\alpha = 0$ that handles **ordinal** data

Adjusted Headcount Ratio M₀

Practical and applicable

Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mu(g^0(k)) = 3/8$

Domains
$$c(k) c(k)/d$$

$$g^{0}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 2/4 & Persons \\ 0 & 4/4 & 0 & 0 \end{bmatrix}$$

H = multidimensional headcount ratio = 1/2
A = average deprivation share among poor = ³/₄
Note: Easily generalized to where deprivations have different values v₁, v₂, v₃, v₄ summing to d = 4

Adjusted Headcount Ratio

Properties

Invariance Properties: Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus

- Dominance Properties: Weak Monotonicity, Dimensional Monotonicity, Weak Rearrangement, a form of Weak Transfer
- Composition Properties: Subgroup Consistency,
 - Decomposability, Dimensional Breakdown

Note

- No transfer property within dimensions
 - Requires cardinal variables!
- No transfer property across dimensions
 - Here there is some scope

New Property

- **Recall: Dimensional Monotonicity** Multidimensional poverty should rise whenever a poor person becomes deprived in an additional dimension (*cet par*) (AF, 2011)
- **New: Dimensional Transfer** Multidimensional poverty should fall as a result of an association decreasing rearrangement among the poor that leaves the total deprivations in each dimension unchanged, but changes their allocation among the poor.
- Adjusted Headcount Satisfies Dimensional Monotonicity, but just violates Dimensional Transfer.
- Q/ Are there other related measures satisfying **DT**?

New Measures

Idea

- Construct attainment matrix
- Aggregate attainment values to create attainment count vector
- Apply a unidimensional poverty measure P to obtain a multidimensional poverty measure M
- The properties of P are directly linked to the properties of M Perhaps M satisfying dimensional transfer can be found

Attainments

Recall Achievement matrix

Dimensions $Y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix}$ $z = (13 \quad 12 \quad 3 \quad 1)$ Cutoffs

Attainments

Construct **attainment** matrix (recall equal value case) 1 if person attains deprivation cutoff in a given domain 0 if not

Domains

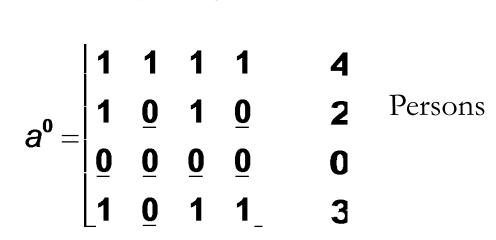
$$a^{0} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \underline{0} & 1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ 1 & \underline{0} & 1 & 1 \end{bmatrix}$$
 Persons

Note

Opposite of the deprivation matrix

Attainments

Counting Attainments (equal value case) 1 if person attains cutoff in a given domain 0 if not



a

Attainment vector

$$a = (4, 2, 0, 3)$$

Domains

Now apply unidimensional poverty measure

Define $M_P(x;z) = P(a; \pi_A, \pi_I)$

where a is the attainment vector associated with x

P is a unidimensional poverty measure

M_p called *attainment count measure*

Process of obtaining M_p from P is called *attainment count transformation*

Example 1

 $P = P_0$ unidimensional headcount ratio, $0 < \pi_I < \pi_A = d$ Poor identified using \leq

Then M_P is multidimensional headcount ratio H with dual cutoff identification having poverty cutoff $k = d - \pi_I$

Define $M_P(x;z) = P(a; \pi_A, \pi_I)$

where a is the attainment vector associated with x

P is a unidimensional poverty measure

M_p called *attainment count measure*

Process of obtaining M_p from P is called *attainment count transformation*

Example 2

 $P = P_1$ unidimensional poverty gap ratio, 0 < π_I < π_A= d Poor identified using ≤

Then M_P is adjusted headcount ratio M_0 with dual cutoff identification having poverty cutoff k = d - π_I

Note: This is the standard AF methodology

Define $M_P(x;z) = P(a; \pi_A, \pi_I)$

where a is the attainment vector associated with x

P is a unidimensional poverty measure

M_p called *attainment count measure*

Process of obtaining M_p from P is called *attainment count transformation*

Example 3

 $P = P_0$ unidimensional headcount ratio, $0 < \pi_I = \pi_A < d$ Poor identified using <

Then M_P is multidimensional headcount ratio H with alternate dual cutoff identification having poverty cutoff k = d - π_I

Alternate: a person is poor if attainment count exceeds k

Define $M_P(x;z) = P(a; \pi_A, \pi_I)$

where a is the attainment vector associated with x

P is a unidimensional poverty measure

M_p called *attainment count measure*

Process of obtaining M_p from P is called *attainment count transformation*

Example 4

 $P = P_1$ unidimensional poverty gap ratio, $0 < \pi_I = \pi_A < d$ Poor identified using <

Then M_P is adjusted headcount ratio M_0 with alternate dual cutoff identification having poverty cutoff k = d - π_I

Note: The Mexican version of the AF methodology

Example 2 Recall a = (4, 2, 0, 3)Identification using $\pi_{I} = 3$ and \leq Who is poor? (4, 2, 0, 3)Aggregation using $\pi_A = 4$ and poverty gap ratio P_1 Gap vector $g^1 = (0, 2/4, 4/4, 0)$ Then

$$P_1 = \int (g^1) = 6/16 = M_0 \text{ AF Methodology}$$

Example 4 Recall a = (4, 2, 0, 3)Identification using $\pi_{I} = 3$ and <Who is poor? (4, 2, 0, 3)Aggregation using $\pi_A = 3$ and poverty gap ratio P_1 Gap vector $g^1 = (0, 1/3, 3/3, 0)$ Then $P_1 = \int (g^1) = 4/12 = Mexican version$

Note: Properties of M_P depend on properties of P In particular:

If P satisfies monotonicity, then M_{P} satisfies dimensional monotonicity.

If P satisfies transfer, then M_P satisfies dimensional transfer.

Lesson

Trivial to construct multidimensional measures sensitive to inequality across deprivations – just use distribution sensitive unidimensional measure and transform

Question

But at what cost?

Impossibility

Crucial property

Dimensional Breakdown: M can be expressed as an average of dimensional functions (after identification)

Note

The measure associated with P₂ does not satisfy dimensional breakdown

Theorem There is no symmetric multidimensional measure M satisfying both dimensional breakdown and dimensional transfer Proof

Follows impossibility result in literature.

Impossibility

Importance of Dimensional Breakdown Policy

- Composition of poverty
- Changes over time by indicator
- Analysis
 - Composition of poverty across groups, time Interconnections across deprivations Efficient allocations

Conclusion

Easy to construct measure satisfying dimensional transfer But at a cost: lose this key element of the toolkit

Concluding Remarks

Alternative way forward:

Apply M_0 class of measures for ordinal data

Satisfies dimensional breakdown

Construct associated measure of inequality among the poor

Note

 P_0 headcount ratio, P_1 poverty gap and FGT P_2 have long been used in concert to analyze the incidence, depth, and distribution of (income) deprivations

Analogously, can use H headcount ratio, adjusted headcount ratio M_0 and inequality measure to analyze the incidence, breadth and distributions of deprivations

With a focus on the measure M_0 and its useful breakdown

Thank you