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Nonlinearity and cross-country dependence of income inequality *

Leena Kalliovirta Helsinki Center of Economic Research, University of Helsinki, Helsinki, Finland

Tuomas Malinen[†] Helsinki Center of Economic Research, University of Helsinki, Helsinki, Finland

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Abstract

We use top income data and the newly developed regime switching Gaussian mixture vector autoregressive model to explain the dynamics of income inequality in developed economies during the last 100 years. Our results indicate that the process of income inequality consists of two equilibriums identifiable by high inequality, high variance and low inequality, low variance. Our results also show that income inequality in the US is the driver of changes in income inequality in other developed economies.

JEL classification: C32, C33, D30

Keywords: top 1% income share, GMAR model, developed economies

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[†] Corresponding author. Address: Department of Political and Economic Studies, University of Helsinki, P.O.Box 17 (Arkadiankatu 7), FIN–00014 University of Helsinki, Finland, Tel: +358 50 3182261, E-mail: tuomas.malinen@helsinki.fi.

1 Introduction

Income inequality has, once again, become a global topic. Estimates on the level of global income inequality vary,¹ but the share of the total income going to the top income earners has not been this high in many developed economies since the 1920's (Alvaredo *et al.* 2013). The history of the distribution of product is embodied by large fluctuations in the share of income massing at the top. According to Piketty (2014, p. 274) in the history of inequality "there have been many twists and turns and certainly no irrepressible, regular tendency toward a natural equilibrium". In a similar vein, Roine and Waldenström (2011) have found global break points from the top 1% income share series that could be changes between different phases of income inequality. In this study we show that this is indeed the case: income inequality follows a regime switching process where higher inequality leads to higher variance in income shares and *vice versa*. We also show that changes in the income inequality in the US have driven the level of inequality of other developed economies during the last 100 years.

The structure of income has varied quite heavily throughout the last century. In the beginning of the 20th century, high incomes consisted mostly on capital (Piketty 2014; Piketty and Saez 2003). Concentrated capital was the primary reason for high income inequality in developed economies before the Second World War. The period after the mid-1970's was marked by liberalization of financial markets, which raised the share of private capital to same levels as in the beginning of the 20th century (Bolt and Van Zanded 2013; Piketty 2014). However, biggest driver of the resurgence of income inequality after 1970's was the increasing share of high wages. According to Piketty (2014), two-thirds of the increase in inequality after the 1970's is attributable to raise in wages of the top 1% income earners. It seems that the structure of inequality has changed, but are the characteristics of inequality the same now as they were in the beginning of the 20th century?

Recent studies have uncovered that the variance of earnings has been increasing in developed economies during the last few decades. Gottschalk and Moffitt (2009) found that the transitory variance of male annual earnings in the U.S. have almost doubled

¹See Anand and Segal (2008); Chotikapanich et al. (2012); Sala-i-Martin (2002), among others.

from the 1970's. Beach *et al.* (2010) find that the total earnings variance in Canada has increased since the year 1982. Daly and Valletta (2008) show that the transitory earnings inequality in the United States, Germany and Great Britain has converged substantially during the 1990s. Although these developments have occurred during a period marked by increasing income inequality (Alvaredo *et al.* 2013), research on their relationship has been almost nonexistent.² Moreover, there are no empirical studies looking at the historical relation between the level of income inequality and the fluctuation of income shares. To our knowledge, there are also no studies looking at the possible dependence of income inequality of an individual country on that of others. In this study we are set to fill these gaps.

As argued by Piketty (2014), income inequality seems not to have been following any kind of mean-reversing process (see above). This has been confirmed in many econometric studies, which have been unable to reject the unit root hypothesis in the autoregressive models for different measures of income inequality (e.g., Herzer and Vollmer 2013; Jäntti and Jenkins 2010; Malinen 2012; Mocan 1999; Parker 2000).³ The breaks in the top 1% income share series identified by Roine and Waldenström (2011) could be one reason for these finding. If breaks are actually shifts between different phases of income inequality identified by, e.g., different levels of variance, there would be no tendency towards a single equilibrium but shifts between multiple equilibria. A linear autoregressive model will be misspecified due to the observed jumps, whereas the so called trend-break models ignore the strong autocorrelation in the series.

We employ the newly developed Gaussian mixture autoregressive (GMAR) model studied in Kalliovirta, Meitz, and Saikkonen (2012) and its multivariate generalization, the Gaussian mixture vector autoregressive (GMVAR) model of Kalliovirta, Meitz, and Saikkonen (2014) to estimate the dynamic properties of income inequality during the last 100 years. We use the GMAR and GMVAR models to identify the different regimes and autoregressive dynamics in the top income series, because they are able to model

 $^{^{2}}$ In the only study we could find Beach *et al.* (2010) shows that rise in the total earnings variance in Canada after 1982 is mostly attributable to increase in overall inequality.

³This is a problematic result in empirical literature as series of commonly used measures of income inequality, like the Gini index and the top income share are bounded between 0 and 1, whereas unit root series has a time-increasing variance.

multiple equilibria. We analyze an updated version of the top 1% income share data ranging from the end of the 19th century to the beginning of the 21st century for six countries: Australia, Canada, France, Finland, Japan, and the USA.

We find that in all analyzed countries, the process of income inequality has consisted on two or three different regimes. Two of these regimes are also found to be common to all the aforementioned countries. We find that the regimes are characterized by different means, or levels, and also with different variances, or scales of variation. Moreover, our GMVAR results show that not only is the variance of income inequality highly dependent across countries, but that income inequality in the United States drives the changes in levels of income inequality in other developed economies.

Rest of the paper is organized as follows. Section 2 presents the data and the GMAR and GMVAR models. Section 3 presents the univariate and panel estimations of GMAR and GMVAR models. Section 4 discusses the economic implications of the estimation results and section 5 concludes.

2 Data and methods

The top 1% income share of population is used to proxy the income inequality. These shares are the only aggregate measures of income inequality that currently contain enough observations for meaningful testing of the time series properties. Leigh (2007) has also demonstrated that the top 1% income share series have a high correlation with other measures of income inequality, like the Gini index. Our data on top income share is obtained from the World Top Income Database (WTID, Alvaredo *et al.* 2013). During the time of writing, WTID had long, continuous time series on six developing countries: Australia, Canada, Finland, France, Japan, and the US.⁴ For these countries, the data on the top 1% income shares starts at the end of the 19th or the beginning of the 20th century. For other countries, the data either starts only after the Second World War and/or it has gaps extending to several years.

⁴For Japan, the observation from the year 1946 is missing, and it has been replaced with the average of the top 1% income share from years 1945 and 1947. For Canada the top 1% income share data is continued with the top 1% income share-LAD data after the year 2000. For Finland, data on top 1% income share-tax data is continued with top 1% income share-IDS data after the year 1992.

We assume that in each country the observed top 1% income share series follows a regime switching GMAR process. This assumption is reasonable, because regime switches are a natural way to adequately model jointly both the dynamic structure of these series and the breaks found in them. Especially, unlike linear AR models they allow for multiple equilibria. Similar regime switching approach has been successfully used for example in Hamilton (1989) to model the U.S. business cycle. The Markov switching AR model of Hamilton (1989) and the constrained version of the GMAR model are closely connected; the latter is a special case of the generalizations of the former. In Hamilton model the probability of a regime switch is constant, whereas in the GMAR model the change in regime is varying in time; thus allowing for more flexibility. However, the general flexibility of these regime switching models comes with a price: one has to be careful how to interpret them. Instead, an estimate of the probability of the series being in a certain regime is available for each time point. These estimated probabilities are henceforth referred as time-varying mixing weights.

The GMAR model has several advantageous properties compared to the more general Markov switching AR model or other nonlinear models. First, the GMAR model is more parsimonious, a considerable advantage when only yearly data for less than hundred years are available. Second, the GMAR model is known to be stationary: It suffices that the usual stationarity condition of the conventional linear AR model is fulfilled in the regimes. Third, the stationary distribution of the GMAR model is known exactly. Thus, we are able to make direct comparisons to the unconditional moments of the original observations (as in Table 1) which can be interpreted as different equilibrium points. This would be unavailable if any other nonlinear model had been used, because the conditions for making the transition from the conditional to the unconditional distribution are then unknown. To learn more about the GMAR model and its competing nonlinear alternatives, see Kalliovirta, Meitz, and Saikkonen (2012).

To understand the joint behavior of the 1% income share series in all six countries, we employ the GMVAR model of Kalliovirta, Meitz, and Saikkonen (2014). In particular, this multivariate model is able to depict regime switches and dynamic structures common in all these six countries.

2.1 The univariate GMAR model

We assume that the top 1% income share series y_t is generated by

$$y_t = \sum_{m=1}^M s_{t,m}(\varphi_{m,0} + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \sigma_m \varepsilon_t),$$

where unobservable random variables $s_{t,m}$ indicate the regimes m = 1, ..., M (M = 2 or 3). Parameters $\varphi_{m,0}$, φ_1 , φ_2 , and σ_m fulfill restrictions: $\varphi(z) = 1 - \varphi_1 z - \varphi_2 z^2 \neq 0$ for $|z| \leq 1$ and $\sigma_m > 0$. For each *t*, exactly one of $s_{t,m}$ random variables takes the value one and others are equal to zero and random variables ε_t are i.i.d. N(0,1). Further, variables ε_t and $s_{t,m}$ are independent given the history of the observed series y_t , $\{y_{t-j}, j > 0\}$. The conditional probabilities $P(s_{t,m} = 1|y_{t-j}, j > 0) = \alpha_{m,t}$ are time-dependent mixing weights. So $\alpha_{m,t}$ yields the probability of the series being in regime *m* at time point *t*. Or, the probability of the observation y_t being generated by the AR(2) model of the *m*:th regime, $\varphi_{m,0} + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \sigma_m \varepsilon_t$, is equal to $\alpha_{m,t}$. Thus, these weights have to satisfy $\sum_{m=1}^{M} \alpha_{m,t} = 1$ for all *t*. In the GMAR model, the mixing weights depend on the past observations, the parameters $\varphi_{m,0}$, φ_1 , φ_2 , and σ_m , and additional weight parameters $\alpha_m > 0$, $\sum_{m=1}^{M} \alpha_m = 1$, according to

$$\alpha_{m,t} = \frac{\alpha_m \mathsf{n}_2(\mathbf{y}_{t-1}; \boldsymbol{\mu}_m, \boldsymbol{\Gamma}_m)}{\sum_{n=1}^M \alpha_n \mathsf{n}_2(\mathbf{y}_{t-1}; \boldsymbol{\mu}_n, \boldsymbol{\Gamma}_n)},$$

where $\mathbf{y}_{t-1} = (y_{t-1}, y_{t-2}), \mu_m = \varphi(1)^{-1} \varphi_{m,0}, \mathbf{1}_2 = (1, 1),$ and

$$\mathsf{n}_{2}(\mathbf{y}_{t-1};\mu_{m},\mathbf{\Gamma}_{m}) = \{2\pi\}^{-1} det(\mathbf{\Gamma}_{m})^{-1/2} \exp\left\{-\frac{1}{2}(\mathbf{y}_{t-1}-\mu_{m}\mathbf{1}_{2})'\mathbf{\Gamma}_{m}^{-1}(\mathbf{y}_{t-1}-\mu_{m}\mathbf{1}_{2})\right\}.$$

The symmetric, 2x2 Toeplitz matrix Γ_m is a function of parameters φ_1 , φ_2 and σ_m^2 according to

$$\operatorname{vec}\left(\mathbf{\Gamma}_{m}\right) = \left(I_{2^{2}} - \left[\begin{array}{cc}\varphi_{1} & \varphi_{2}\\ 1 & 0\end{array}\right] \otimes \left[\begin{array}{cc}\varphi_{1} & \varphi_{2}\\ 1 & 0\end{array}\right]\right)^{-1} \left(\left[\begin{array}{cc}1\\ 0\end{array}\right] \otimes \left[\begin{array}{cc}1\\ 0\end{array}\right]\right) \sigma_{m},$$

where \otimes denotes the Kronecker product. Notice that if $\varphi_2 = 0$, then matrix Γ_m simplifies to $\frac{\sigma_m^2}{1-\varphi_1^2}$ and the normal distributions above are univariate. Also, the restriction $\sum_{m=1}^{M} \alpha_m = 1$ reduces the number of free weight parameters α_m into M - 1. Thus, if M = 2 we only have to estimate one weight parameter α_1 .

The stationary distribution of the GMAR model, $\sum_{m=1}^{M} \alpha_m n_2(\mathbf{y}_{t-1}; \mu_m, \Gamma_m)$, yields an alternative parameterization that employs μ_m , $\boldsymbol{\varphi}$, $\gamma_{m,0}$, and α_m , m = 1, ..., M. We used this alternative parameterization in the univariate analysis and estimate the model parameters using maximum likelihood as suggested in Kalliovirta, Meitz, and Saikkonen (2012).

2.2 The multivariate GMVAR model

The GMAR model generalizes easily into the multivariate GMVAR model. We thus assume that the 6 dimensional 1% income share series y_t is generated by

$$\mathbf{y}_{t} = \sum_{m=1}^{3} s_{t,m} \Big(\phi_{m,0} + A_1 \mathbf{y}_{t-1} + A_2 \mathbf{y}_{t-2} + \Omega_m^{1/2} \boldsymbol{\varepsilon}_t \Big),$$

where unobservable random variables $s_{t,m}$ indicate the regimes m = 1, ..., 3 and ε_t are i.i.d. N(**0**, I_6) random vectors. Parameters $\phi_{m,0}$, A_1 , A_2 , and Ω_m fulfill the following conditions: det $A(z) \neq 0$ for $|z| \leq 1$ with $A(z) = I_6 - A_1 z - A_2 z^2$ and covariance matrix Ω_m is positive definite. The random variables ε_t and $s_{t,m}$ are independent given $\{y_{t-j}, j > 0\}$. For each *t*, the variables $s_{t,m}$ are defined as in the univariate case so exactly one of them takes the value one and others are equal to zero. The time-dependent mixing weights $\alpha_{m,t}$ are the conditional probabilities $P(s_{t,m} = 1 | y_{t-j}, j > 0)$. Also, the probability of the observation y_t being generated by the VAR(2) model of the *m*:th regime, $\phi_{m,0} + A_1 y_{t-1} + A_2 y_{t-2} + \Omega_m^{1/2} \varepsilon_t$, is equal to $\alpha_{m,t}$. Thus, similar to GMAR model these weights satisfy $\sum_{m=1}^{3} \alpha_{m,t} = 1$ for all *t*. In the GMVAR model, the mixing weights depend on the past observations, the parameters $\phi_{m,0}$, A_1 , A_2 , and Ω_m , and additional weight parameters $\alpha_m > 0$, $\sum_{m=1}^{3} \alpha_m = 1$, according to

$$\alpha_{m,t} = \frac{\alpha_m \mathsf{n}_{12} \left(\boldsymbol{Y}_{t-1}; \boldsymbol{\mu}_m, \boldsymbol{\Gamma}_m \right)}{\sum_{n=1}^3 \alpha_n \mathsf{n}_{12} \left(\boldsymbol{Y}_{t-1}; \boldsymbol{\mu}_n, \boldsymbol{\Gamma}_n \right)},$$

where $Y_{t-1} = (y'_{t-1}, y'_{t-2})', \mu_m = A(1)^{-1}\phi_{m,0}, \mathbf{1}_2 = (1, 1)$, and

$$\mathsf{n}_{12}(\boldsymbol{Y}_{t-1};\boldsymbol{\mu}_m,\boldsymbol{\Gamma}_m) = \{2\pi\}^{-6} det(\boldsymbol{\Gamma}_m)^{-1/2} \exp\left\{-\frac{1}{2}(\boldsymbol{Y}_{t-1}-\boldsymbol{1}_2\otimes\boldsymbol{\mu}_m)'\boldsymbol{\Gamma}_m^{-1}(\boldsymbol{Y}_{t-1}-\boldsymbol{1}_2\otimes\boldsymbol{\mu}_m)\right\}.$$

The symmetric, 12x12 Toeplitz matrix Γ_m is a function of matrices A_1 , A_2 , and Ω_m , according to

$$\operatorname{vec}\left(\mathbf{\Gamma}_{m}\right) = \left(I_{12^{2}} - \left[\begin{array}{cc}A_{1} & A_{2}\\I_{6} & \mathbf{0}\end{array}\right] \otimes \left[\begin{array}{cc}A_{1} & A_{2}\\I_{6} & \mathbf{0}\end{array}\right]\right)^{-1} \left(\left[\begin{array}{cc}I_{6}\\\mathbf{0}\end{array}\right] \otimes \left[\begin{array}{cc}I_{6}\\\mathbf{0}\end{array}\right]\right) \operatorname{vec}\left(\Omega_{m}\right),$$

where \otimes denotes the Kronecker product. The stationary distribution of the GMVAR model, $\sum_{m=1}^{3} \alpha_m n_{12}(Y_{t-1}; \mu_m, \Gamma_m)$, yields an alternative parameterization that employs μ_m , A_1 , A_2 , Γ_m , and α_m , m = 1, 2, 3. We report these alternative parameterizations also in the multivariate analysis and estimate the model parameters using maximum likelihood as suggested in Kalliovirta, Meitz, and Saikkonen (2014).

3 Results

3.1 Univariate model

As a starting point for the analysis of each series, we estimated linear Gaussian AR models. However, residual diagnostics (not reported) rejected these models due to nonnormality and conditional heteroskedasticity, which is a clear indication of nonlinearity in the modeled series. Table 1 presents properties of the original series and the estimation results for GMAR models that pass the quantile residual diagnostics of Kalliovirta (2012).⁵ Clearly, the original series are persistent in all six countries, and the variances are also highly fluctuating from around 24 in Japan to around 5 in Australia. In the GMAR models, there are two regimes in top 1% income series in all countries except Australia, where three regimes are found. The series of France, Japan, and USA require two lags in the GMAR model, whereas one lag is enough for the other three countries. The autocorrelation in the top 1% income series diminishes quite clearly in all countries, when the effect of regime-wise constants and variances are taken into account.

The regimes of GMAR models seem to be marked with quite clear and similar char-

⁵The accuracy of the mean, variance, and weight parameter estimates suffer from the lack of data. Testing the significance of the mixing weights is a theoretically highly demanding nonstandard testing problem common to all regime switching models like the STAR and Markov switching models (see Kalliovirta, Meitz, and Saikkonen (2012) for more explanation), and it has not been solved yet for GMAR models. For the same reason one cannot test the equality of the means or variances simply by comparing their estimates and standard errors, because these parameters are closely connected to the time varying mixing weights. Further, testing the equality of means and variances jointly would again lead to the nonstandard testing problem. However, we can test them separately. For example, in the income series of Canada the LR tests for equality of means has p-value 0.31 and equality of variances has p-value < 10^{-12} . The quantile residual diagnostics indicate that the model with equal means describes inadequately the autocorrelation of the series. Thus, the model reported in the table is chosen.

For this reason, we base the model specification on the theoretically appropriate quantile residual diagnostics that supports nonlinearity over linearity in all six models. Further, information criteria like AIC and BIC (not reported) clearly indicate that the nonlinear models are superior. More details on the estimated models and residual diagnostics are available upon request.

	Australia	Canada	Finland	France	Japan	USA
Original data						
First autocorrelation	0.94	0.96	0.95	0.96	0.98	0.95
mean	8.2	11.4	8.8	10.8	12.2	12.8
variance	4.5	8.8	9.5	14.9	24.4	14.7
years	1921-	1920-	1920-	1915-	1886-	1913-
	2010	2010	2009	2009	2010	2012
GMAR model						
autocorrelation (φ_1)	0.90	0.95	0.94	1.11	1.33	1.14
	(0.04)	(0.03)	(0.03)	(0.11)	(0.08)	(0.10)
autocorrelation (φ_2)				-0.16	-0.42	-0.24
				(0.12)	(0.09)	(0.10)
mean 1 (μ_1)	4.8	9.6	4.9	8.4	8.1	8.2
	(0.1)	(0.8)	(0.5)	(0.4)	(0.3)	(0.3)
mean 2 (μ_2)	6.0	14.3	7.9	15.5	16.7	15.1
	(0.4)	(4.4)	(1.6)	(2.2)	(1.5)	(1.2)
mean 3 (μ_3)	9.1					
	(1.0)					
variance 1 (γ_1)	0.01	1.4	0.5	0.6	0.6	0.2
	(0.01)	(0.7)	(0.3)	(0.3)	(0.2)	(0.1)
variance 2 (γ_2)	0.3	15.2	4.5	11.3	12.7	6.6
	(0.1)	(9.6)	(2.1)	(5.8)	(5.2)	(2.5)
variance 3 (γ_3)	2.9					
	(1.3)					
α_1	0.08	0.95	0.22	0.42	0.62	0.17
	(0.1)	(0.08)	(0.2)	(0.3)	(1.4)	(0.3)
α_2	0.47					
	(0.2)					

Table 1: Estimation results on the top 1% income share

Standard errors (in parentheses) are calculated using the estimated Hessian.

acteristics in all countries. In one regime, the mean and variance of the top 1% income series are clearly higher, whereas in the other regime both are considerably lower. So at least in these countries, income inequality has consisted on two notably different regimes. First one is a low income inequality, low income fluctuations regime and the second is a high income inequality, high income fluctuations regime. Even though the Australian series has three regimes, the same characteristics are found in them.

Further, our analysis points out that the evolution of the top 1% income series cannot be modeled adequately using a linear model. The nonlinear structure of the series with different constants and variances between regimes increases the autocorrelation observed in the original series. This indicates that, although the dynamics of the process can be *approximated* with a stochastic trend, i.e. unit root process, it might not be its true form.

Figure 1 presents the top 1% income shares and the estimated time-dependent mixing weights for the above mentioned six countries. In all six subfigures the mixing weights $\hat{\alpha}_{1,t}$ (dashed line) (or $\hat{\alpha}_{2,t}$ (dotted line) in the subfigure for Australia) are given on the right axis while the share of total income earned by the top 1% income earners (solid line) is given on the left axis.

In Australia, the probability that income inequality is in the third regime is above 90% until 1955. In 1955, the probability of the second regime begins to rise. Transition from the second regime into the first regime happens around 1975 and back into the second regime in 1987. In 1999, the series moves back into the third regime. In Canada, France and Japan, income inequality switches the regime right after the Second World War. The probability that the income inequality series is in the first regime increases into 99% in Canada in 1944, into 98% in France in 1948, and into 98% in Japan in 1948. In Finland, the probability of income inequality being in the first regime increases into 33% in 1976 and decreases below 2% in 1998. In the USA, the probability that income inequality is in the first regime increases into 61% in 1955. After 1988, the probability of the second regime is 100%.

The results based on GMAR models imply that many of the structural breaks found by Roine and Waldenström (2011) are points, where the series of income inequality change regime and the characteristics of the series change in terms of means and variances. We find the following correspondences between the breaks of Roine and Waldenström and the regime switches: 1) in Australia, the regime change in 1987 corresponds the structural break in the country-specific series in 1985; 2) in Canada, the countryspecific break point in 1994 corresponds the probability of second regime beginning to increase in 1998; 3) in Finland, the probability of income inequality being in the first regime increases into 73% in 1981 which corresponds to the break in post-war data on Nordic countries, and the probability of second regime rices over 68% in 1997, which corresponds to the country-specific break; 4) in Canada, France, and Japan, the changes



Top 1% income shares and the time-dependent mixing weights for Australia, Canada, Finland, France, Japan and the USA based on the univariate GMAR models.

from the second regime into the first regime correspond to the global trend break point of 1946; and 5) in Australia and USA, the changes in regime around 1955 and 1987 correspond to the common structural break in 1953 and the common post-war break in Anglo-Saxon countries in 1987.

3.2 Multivariate, panel data model

Next we combine the six individual series into a panel data over the years 1921 and 2009 to find out whether the regime switches and other dynamics in these series move in tandem. The GMVAR model that passes quantile residual diagnostics has three regimes and the VAR structure is common to all regimes and has maximum of two lags. ⁶

We report estimated GMVAR model component by component to make comparisons easy with the estimated univariate models and report the estimated Hessian based standard errors in parentheses below. The estimated weight parameters for the first and second regimes in the GMVAR model are $\hat{\alpha}_1 = 0.14$ and $\hat{\alpha}_2 = 0.85$. Note that these estimates also yield the unconditional probabilities $P(s_{t,1} = 1) = 0.14$, $P(s_{t,2} = 1) = 0.85$, and $P(s_{t,3} = 1) = 0.01$. We denote the *i*th element of vector $\hat{\Omega}_2^{1/2} \varepsilon_t$ with $u_{t,i}$ and report separately the estimated covariance matrix $\hat{\Omega}_2$, because it is not diagonal like $\hat{\Omega}_1$. The third regime is added to allow the constants of France and Japan to change within the second regime so there is no need for the third covariance matrix. The series of Australia, Canada, Finland, France, Japan, and USA follow:

$$y_{t,Aus} = \underbrace{0.93}_{(0.04)} y_{t-1,Aus} + \underbrace{0.02}_{(0.04)} y_{t-1,USA} \\ + s_{t,1} \left(\underbrace{0.23}_{(0.27)} + \underbrace{\sqrt{0.08}}_{(0.02)} \varepsilon_{t,Aus} \right) + (1 - s_{t,1}) \left(\underbrace{0.28}_{(0.44)} + u_{t,1} \right),$$

$$y_{t,Can} = \underbrace{1.00}_{(0.11)} y_{t-1,Can} + \underbrace{0.11}_{(0.05)} y_{t-1,USA} - \underbrace{0.15}_{(0.10)} y_{t-2,Can} \\ + s_{t,1} \left(\underbrace{0.35}_{(0.28)} + \underbrace{\sqrt{0.07}}_{(0.02)} \varepsilon_{t,Can} \right) + (1 - s_{t,1}) \left(\underbrace{0.17}_{(0.46)} + u_{t,2} \right),$$

$$\begin{aligned} y_{t,Fin} &= \begin{array}{l} 0.91y_{t-1,Fin} + 0.07y_{t-1,USA} \\ &+ s_{t,1} \left(\begin{array}{c} 0.13 + \sqrt{0.47} \varepsilon_{t,Fin} \\ (0.27) & (0.07) \end{array} \right) + (1 - s_{t,1}) \left(\begin{array}{c} -0.23 + u_{t,3} \\ (0.41) \end{array} \right), \end{aligned}$$

⁶Similar to the univariate models, the accuracy of mean, variance, and mixing weights estimates suffer from the lack of data. One may suspect that there are several redundant mean and variance parameters based on their standard errors. However, the testing of their equivalence has to be based on LR tests (e.g. a LR test for equality of means in regimes 1 and 2 for France has p-value 0.002), and hypotheses that contain unidentified nuisance parameters lead again to the nonstandard testing problems common to all regime switching models. Thus, we base model selection on the information criteria and theoretically appropriate quantile residual diagnostics, which strongly support the GMVAR model. Note also that compared to the univariate case the joint modeling has led to more efficient parameter estimates for the regime variances.

$$y_{t,Fra} = \frac{0.88}{(0.03)} y_{t-1,Fra} + \frac{0.06}{(0.02)} y_{t-1,USA} + s_{t,1} \left(\frac{0.54}{(0.23)} + \sqrt{0.07} \varepsilon_{t,Fra} \right) + s_{t,2} \left(\frac{0.19}{(0.30)} + u_{t,4} \right) + s_{t,3} \left(\frac{0.83}{(0.49)} + u_{t,4} \right), y_{t,Jpn} = \frac{1.22}{(0.09)} y_{t-1,Jpn} + \frac{0.13}{(0.03)} y_{t-1,USA} - \frac{0.33}{(0.08)} y_{t-2,Jpn} + s_{t,1} \left(-\frac{0.21}{(0.28)} + \sqrt{0.04} \varepsilon_{t,Jpn} \right) + s_{t,2} \left(-\frac{0.75}{(0.46)} + u_{t,5} \right) + s_{t,3} \left(-\frac{0.16}{(0.62)} + u_{t,5} \right). y_{t,USA} = \frac{1.21}{(0.10)} y_{t-1,USA} - \frac{0.28}{(0.10)} y_{t-2,USA}$$

$$+s_{t,1}\left(\underset{(0.37)}{0.55}+\sqrt{0.03}\varepsilon_{t,USA}\right)+(1-s_{t,1})\left(\underset{(0.67)}{0.90}+u_{t,6}\right)$$

The autoregressive dynamics within countries remain very similar to what is found in the univariate models. However, the first lag of the income inequality in USA affects the autoregressive dynamics for all countries. The positive coefficients indicate that an increase (decrease) in the income inequality in USA will cause an increase (decrease) in the income inequality in these other countries. Thus, the changes in the income inequality in USA are exported to other countries.

The mean vectors of the stationary distribution, solved using $\mu_m = A(1)^{-1}\phi_{m,0}$, are:

$$\boldsymbol{\mu}_{1} = \begin{bmatrix} \mu_{1,Aus} \\ \mu_{1,Can} \\ \mu_{1,Fin} \\ \mu_{1,Fra} \\ \mu_{1,Jpn} \\ \mu_{1,USA} \end{bmatrix} = \begin{bmatrix} 6.0 \\ (0.6) \\ 9.0 \\ (0.5) \\ 7.5 \\ (1.2) \\ 8.6 \\ (0.4) \\ 7.8 \\ (0.6) \\ 8.5 \\ (0.5) \end{bmatrix}, \quad \boldsymbol{\mu}_{2} = \begin{bmatrix} 8.6 \\ (1.8) \\ 12.0 \\ (1.9) \\ 7.6 \\ (1.8) \\ 8.5 \\ (1.2) \\ 9.2 \\ (2.5) \\ 14.0 \\ (2.1) \end{bmatrix}, \quad and \quad \boldsymbol{\mu}_{3} = \begin{bmatrix} \mu_{2,Aus} \\ \mu_{2,Can} \\ \mu_{2,Fin} \\ 13.7 \\ (2.0) \\ 14.3 \\ (3.2) \\ \mu_{2,USA} \end{bmatrix}$$

The mean vectors of the stationary distribution of the GMVAR model have roughly the same values as what is found in the univariate GMAR models. The differences are found in Australia, where the third, lowest mean regime of the univariate GMAR model becomes redundant, and in Finland, where the low mean regime has increased significantly.

3.2.1 Time-dependent mixing weights

Figure 2 depicts the top 1% income shares and the estimated time-dependent mixing weights for the above mentioned six countries. In all subfigures the mixing weights $\hat{\alpha}_{1,t}$



Top 1% income shares and the time-dependent mixing weights for Australia, Canada, Finland, France, Japan and the USA based on the GMVAR model.

(dashed line) (or $\hat{\alpha}_{2,t}$ (dotted line) in the subfigures for France and Japan) are given on the right axis while the share of total income earned by the top 1% income earners (solid line) is given on the left axis. In the beginning of the period the series of France and Japan are in the third regime and the other series in the second regime with probability of 100%. Both these regimes have high mean and high variance. France and Japan change into the second regime around 1940 and thereby the means become much lower. Between 1955 and 1987, all the series are in the first regime with probability of 99 % or higher.⁷ This first regime has low mean and low variance. Since 1988 the probability of the second regime is above 82% for all countries. Thus, the top 1% income share has returned to the much higher level similar to the time before the Second World War. The regime changes common to all six countries in the multivariate model are the same ones observable in the univariate model for USA, the common structural breaks in 1953 and 1987. Thus, this further illustrates the significant effect the US series has on the dynamics for all the series in the multivariate model.

3.2.2 Regime specific covariances

The different behavior of the series within regimes is also visible in the covariance matrices. In the first regime, where the means and variances are low, the covariance matrix $\hat{\Omega}_1$ is diagonal. So the shocks of the components do not affect each other and in each country the variation is country-specific. The estimated covariance matrix of the second (and the third) regime

$$\hat{\Omega}_2 = \begin{bmatrix} 0.50 & 0.13 & 0 & 0.12 & 0.16 & 0.24 \\ (0.09) & (0.07) & (0.05) & (0.07) & (0.09) \\ 0.13 & 0.54 & 0 & 0 & 0.14 & 0.17 \\ (0.07) & (0.10) & (0.07) & (0.10) \\ 0 & 0 & 0.47 & 0 & 0 & 0 \\ (0.07) & & & & & \\ 0.12 & 0 & 0 & 0.30 & 0.22 & 0 \\ (0.05) & & (0.06) & (0.06) & \\ 0.16 & 0.14 & 0 & 0.22 & 0.57 & 0 \\ (0.07) & (0.07) & (0.06) & (0.11) & \\ 0.24 & 0.17 & 0 & 0 & 0 & 0.91 \\ (0.09) & (0.10) & & (0.18) \end{bmatrix}$$

shows that excepting Finland the components affect each other through shocks. In the second (and the third) regime, the means and variances are high and shocks in one country will affect the future values in the other countries. To make the strength of the dependence between countries easier to interpret, we also report the corresponding

⁷The common break points found by Roine and Waldenström (2011) were in 1945 and in 1980.

correlation matrix

[1	0.26 (0.13)	0	0.32 (0.12)	0.30 (0.12)	0.36 (0.12)
0.26 (0.13)	1	0	0	0.24 (0.12)	0.24 (0.13)
0	0	1	0	0	0
0.32 (0.12)	0	0	1	0.53 (0.10)	0
0.30 (0.12)	0.24 (0.12)	0	0.53 (0.10)	1	0
0.36 (0.12)	0.24 (0.13)	0	0	0	1

The effect the US series has on the series of Australia is weak in terms of the autoregressive dynamics, however the effect is significant in both directions (correlation 0.36) in the second regime through the shocks. The strongest dependence between shocks is observed for France and Japan, where correlation is 0.53. Further, the income inequality in these two and in the Anglo-Saxon countries Australia, Canada and USA are significantly connected through shocks.

3.2.3 Impulse response analysis

To gain better understanding of the dynamical system in the estimated GMVAR model, we compute the regime specific orthogonal impulse responses of all countries to a unit change in the U.S. series. We also include a linear VAR model in the analysis to obtain more comparison.⁸ We employ the orthogonal impulse responses, because the shocks in regime 2 of GMVAR model and in VAR model are contemporaneously correlated. These results are presented in Figure 3.

The different dynamics between the regimes is clearly visible in Figure 3. In the (low mean and low variance) regime 1, the impact of a shock in USA is negligible on other countries. For USA, the impact is positive and decays slowly to zero. In the (high mean and high variance) regime 2, the impact is much stronger on all countries, especially on the USA series itself but also on Canada and Japan. Note that the autocorrelation structure is the same in both regimes so the differences are explained by the different error covariance matrices in regimes.

⁸The details on this estimated VAR(2) model is available upon request. The log-likelihood in GMVAR model is larger than in the VAR even though the VAR(2) model has 114 parameters and the GMVAR model has only 48. Thus, it is evident that the information criteria support the GMVAR model. Further, the quantile residual diagnostics strongly support the GMVAR model over the VAR(2) model.



Figure 3. Orthogonal impulse responses of all countries to a unit change in the US top 1% income share series based on regime 1 in GMVAR (solid line), regime 2 in GMVAR (dashed line) and VAR(2) (dotted line) models.

In the VAR model, the impact on USA begins on level lower than in regime 2 but is more persistent. This might be explained by the fact that the largest root in the GMVAR model is 0.93 compared to 0.98 in the VAR model. Also, the impact on other countries is smaller than what is observed in regime 2. One may interpret that the VAR model represents a weighted average model over the two regimes, so its impact is also a weighted average. Therefore, the VAR model underestimates the present-day impact USA has on other countries.

To understand the overall effects of an impulse in U.S. series on the countries, we

compare the total accumulated impulse responses of the models. In regime 1, the total accumulated effect is between 1 (in Australia) and 3 (in Japan), in regime 2 it is between 4 (in Australia) and 15 (in Japan), and in VAR model it is between -2 (in France) and 9 (in USA). In the VAR model, another negative impact is also observed in Japan (-1). Thus, the VAR model implies that a increase in U.S. inequality will decrease inequality in France and Japan in the long run. This is in contrast to what the GMVAR model suggests. We consider the total accumulated impulse responses of the GMVAR model to be more reliable, because the GMVAR allows for multiple equilibria whereas the VAR model allows only a unique equilibrium. Thus, the GMVAR is in line with the observed nonlinear behavior and multiple equilibria found in the top 1% income shares (Piketty 2014; Roine and Waldenström 2011). Accordingly, the impulse response analysis supports the idea that the income inequality in USA affects income inequality in the other countries.

4 Discussion

The dynamics of income inequality seem to follow a joined path, and we can infer that income equality creates stability in the earned incomes of the top 1% (regime 1), while income inequality creates instability in the earned incomes of the same group (regime 2). Multivariate results confirm the findings of Roine and Waldenström (2011) on the "global" phases in income inequality. But, our results also show that the variance of income inequality is not only highly dependent across countries, but that income inequality in the United States is the driver of changes in levels of income inequality in other developed economies.

Results have three rather drastic implications. First, Leigh (2007) has shown that top income shares, especially the top 1% income share, have a high correlation with other measures of inequality (see also

Second, Herzer and Vollmer (2013) and Malinen (2012) have found that stochastic parts of income inequality and GDP per capita have a long-run *equilibrium* relation. Therefore, larger stochastic fluctuations in the top 1% income share in the second regime translate to larger stochastic fluctuations in the GDP per capita creating macroeconomic

instability. This finding is supported by Berg and Ostry (2011) who find that higher inequality is associated with shorter growth spells and *vice versa*.

Third, the level of inequality in US directly affects the future level of inequality in other developed countries. This level effect is also visible in how the regime changes occur: the regime change in the US is the regime change common to all six countries. In addition, in the high inequality high variance regime the changes in the level of inequality in the US is transmitted to all other countries through the covariance structure in that regime. This dynamic dependence on US level of inequality and its changes diminishes the control of individual countries on their distribution of income.

These empirical findings naturally raise two important questions: what are the driving forces of regime switches and, more importantly, what is the role of the US behind these forces? The high income inequality in developed economies before the Second World War was mostly due to the larger share of income coming from highly concentrated capital (Piketty 2014). Fluctuations in dividends and stocks added volatility in the share of income going to the top of income earners. Before the Great Depression in the 1930's, global capital markets were already highly 5integrated (Obstfeld and Taylor 1997), and by 1920's US had accumulated the largest pool of private and national capital (Bolt and Van Zanded 2013; Piketty 2014). In other words, US became the dominant power in capital markets after the First World War. The effect of the US on the global capital markets was multiplied during Great Depression, which started from the US and spread through the developed world. In the 1980's US began to liberalize its financial sector, which led to a wave of financial liberalization in other developed economies (Stiglitz 2004). This increased the share of private capital to income, but the renewed raise in income inequality in developed economies was mostly caused by the rise of high wages. Two-thirds of the increase of income inequality that occurred in the US after mid-1970's is attributable to raise in wages of the top 1% earners (Piketty 2014). This also drives income inequality in other developed economies, because the wages of top managers in Europe (and elsewhere) need to keep up with the wages in the US (Petit 2010). Salaries of top managers started to increase in the US and they were exported to other developed economies. The high volatility of incentives, bonuses and option prices (mostly through stock market fluctuations) of the top managers also explains the fluctuations in top income during recent years (Gottschalk and Moffitt 2009; Piketty 2014).

Through the last 100 years high income inequality has led to higher variance in the share of income of the top earners through two interlinked channels. First, periods of high income inequality have been associated with periods of concentrated capital (Piketty 2014). Because financial capital has been an integral part of concentrated capital accumulation, higher share of volatile income from capital has increased the volatility of income of the top 1%. Second, during the latest era of globalization, price fluctuation of incentives, bonuses and options received by top managers have caused additional fluctuations in the top 1% income share series (Piketty 2014). Most importantly, history of capital accumulation and salaries reveals that during the last 100 years developments in the US have driven changes in capital markets and in salaries of top managers. Our results indicate that these developments have had a direct effect on the dynamics of income distribution in other developed economies, meaning that future developments on income inequality have been driven by changes of income inequality in the US.

5 Conclusions

In his recent path-breaking book, Piketty (2014) shows that income inequality in many developed economies has followed an U-shaped path instead of the inverted-U shaped path hypothesized by Kuznets (1955). Results presented in this article add to this by showing that the level of inequality determines the characteristics of income distribution similarly as with inflation: it can be either equal and stable or unequal and volatile. Moreover, results indicate that changes in the dynamics of income inequality of developed economies are driven by changes in the inequality in the US.

Results have some drastic policy implications. Because increase in the mean share of top 1% income in the high inequality, high variance regime is higher than any conceivable short to medium term growth of GDP, shift to higher inequality, high variance regime is more harmful for the bottom 99% income earners. Larger fluctuations in the top 1% income share in this regime also translate to larger stochastic fluctuation in the GDP per capita, because stochastic parts of income inequality and GDP per capita have been found to have an *equilibrium* relation (Herzer and Vollmer 2013; Malinen 2012). This combination makes poor and middle-income households bearers of the costs of income inequality in more ways than one: increasing income inequality lowers their share of the total income disproportionately and it increases the uncertainty of their future income. In addition, stronger business cycle fluctuations can exacerbate income inequality meaning that inequality may enforce itself in the high inequality high variance regime (Ashley 2007; Fawaz *et al.* 2012). The responses of sovereign nations on the costs associated to income inequality are diminished by the dependence of it on the level of inequality in the US.

US has dominated the capitalist word since the beginning of the 20th century. This holds also for the dynamics of income distribution that seem to have been more integrated across developed economies than previously thought. Due to the continuing integration of the global economy, it is likely that dynamics of income inequality become even more bounded between developed economies, or maybe even globally, in the future.

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