On Inequality and the Poverty Line.
Making the poverty line dependent on reference groups: implications for the extent of poverty in some Asian countries

Satya R. Chakravarty, Indian Statistical Institute, Kolkata, India

Nachiketa Chattopadhyay, Indian Statistical Institute, Kolkata, India

Zoya Nissanov, Ariel University, Israel.

Jacques Silber, Bar-Ilan University, Israel.

Motivation and Introduction

- Several indices suggested to measure poverty

- They all depend explicitly on the poverty line, which is often taken as given.

- The choice of a poverty line has hence clearly a high impact on anti-poverty policies.

- Based on a report documented in Ravallion, Datt and van de Walle(1991), the World Bank used a $1 per day poverty line for the developing world.

- Following work by Ravallion, Chen and Sangraula (2009), this international poverty line was updated and this new international line became $1.25 a day at 2005 purchasing power parity (PPP)
- This poverty line is obtained by regressing national poverty lines on the (log) of per capita consumption expenditures.
- Deaton (2010) argued that many problems are involved in the calculation of a global poverty.
- He discussed the issue of making inter-country comparisons where relative prices and consumption patterns are different.

Our approach

- As is well known, a   distinction is made in the literature between an 'absolute poverty line' and a 'relative poverty line'.

- The former is a fixed real value over time and is given exogenously.

- The latter is made responsive to the income distribution (e.g. a poverty line equal to sixty percent of the median income).

- Our approach follows a long tradition of identifying welfare with utility.

- We assume that utility depends on the absolute income *and* the relative income, that is, income relative to some reference standard.

- There is in fact a growing economic literature that stresses the importance of incorporating relative position in decision making analysis.

- Of particular interest in this literature is the analysis of the impact of so-called reference groups on life satisfaction or satisfaction with income.

- This question is in fact related to a much older hypothesis, which assumes that utility depends not only on one's own income but also on that of others (Duesenberry, 1949)).

- The income of other individuals is also central in the literature dealing with the notion of relative deprivation.

- See the pathbreaking work of Runciman (1966), and later on the papers, among others, of Yitzhaki (1979), Berrebi and Silber (1985), Chakravarty and Moyes (2003), Bossert and D'Ambrosio (2007) and Zheng (2007).

- Selecting the reference group:

       - *First possibility*: the reference group is made of colleagues in which case the emphasis is on "the relation between income gaps in the professional sphere and various notions of satisfaction ranging from job to life satisfaction" (Senik, 2009).

Clark and Oswald (1996) defined the reference group of a worker as the income of employees who had the same age and level of qualification as the worker and were doing the same kind of job.

- *Other studies: a*ssumed that the reference group was composed of people with the same characteristics as the individual, with, for example, the same age, level of education and region of residence (see, Ferrer-i-Carbonell, 2005).

- *Some authors*: used space-based reference incomes such as the average income of individuals of the same race in the cluster and district where the individuals surveyed live (see, Kingdon and Knight, 2007).

- Another issue: the direction of the impact of the reference income on subjective well-being.

There are two possible impacts:

The role of status:

- More common in the literature: ceteris paribus a higher reference income affects negatively satisfaction from life or income (see, for example, the studies of Senik, 2009, and Clark and Senik, 2009).

The signalling effect:

- Idea originally introduced by Hirschman and Rothschild (1973) "Suppose that the individual has very little information about his future income, but at some point a few of his relatives, neighbours, or acquaintances improve their economic or social position. Now he has something to go on: expecting that his turn will come in due course, he will draw gratification from the advances of others – for a while. It will be helpful to refer to this initial gratification as the 'tunnel effect'."

- Evidence confirming the existence of such signalling effects was provided by Senik (2004; 2008).

- <u>Empirical studies</u>:

- Only few papers estimated the impact on happiness, ceteris paribus, of an increase in one's own income, on one hand, of a rise in the reference group's income on the other hand. Moreover the effect of a change in the reference income, when estimated, was generally derived indirectly.

- Knight et al. (2007) looked at subjective well-being in China, introduced in their regression a dummy variable indicating whether the household income was much above, above, below or much below the village average.

- Clark et al. (2013) report the results of a regression where the dependent variable refers to satisfaction with income. It then appears that the coefficient of own income is about three times as high as that of self-reported reference income, and of opposite sign, even when a variable measuring the "comparison intensity" of the individual (how important it is for the respondent to compare her income with that of others) is introduced.

Formal Framework

- The individual utility is assumed to be:

    - increasing, concave in absolute income

    - but decreasing, convex in the reference standard (Clark & Oswald, 1998).

- Our analysis relies on a general reference income level, of which some proportions of mean or median income can be special cases.

- An additive form and a multiplicative form of the utility function are characterized using two different sets of axioms.

The general idea:

- Given a reference income and a person with income equal to an arbitrarily set poverty line, we determine the level of the corresponding utility.

- We then consider an alternative setting where a person's income is at a given poverty line. Further, his utility is assumed not to be affected by the reference income (this is obtained by setting his own income to be his reference income).

- We then assume that the person is equally satisfied in both positions. This assumption of equal satisfaction is quite plausible because in each case the individual is at the existing poverty line income.

- Consequently, we refer to it as utility-consistent poverty line.

- An innovative feature of our paper is that for either form of the utility function the new poverty line becomes a weighted average of the given poverty line and the reference income.

- The weights assigned to the two components have in fact a nice policy interpretation.

- A second attractive feature of our framework is that some of the suggestions that exist in the literature (e.g. Atkinson- Bourguignon, 2001, and EU standard) for basing the poverty line directly on some location parameter, such as the mean or median, become particular cases of our formulation.

- Let x and m respectively be the absolute income and reference income of an individual in the society.

- The reference income m can be treated as a positional good and it is assumed that m is not less than x. Examples of m can be the mean and the median incomes in the population or some positive scalar transformations of them.

- As mentioned previously, the utility function $U(x, m)$ is assumed to be increasing, concave in x and decreasing, convex in m .

- Increasingness and concavity assumptions in absolute income are quite standard.

- Suppose a person with a low income regards the income level m as his/her targeted income. An increase in m might increase his/her difficulty to fulfil the objective of receiving the higher targeted income. This means that his/her additional utility from an increase in m will be negative, that is, U is decreasing in m. Convexity means that his dissatisfaction from an increase in m increases at a non-decreasing rate.

First case:

- The difference form comparison demands that the utility function should be of the form $U(x, (x - m))$. The argument $(x - m)$ can be thought of as capturing dis-utility from comparison. That is, in this case the determinant of relative status depends on the difference $(x - m)$.

- We propose the following axioms for the utility function involving a difference form comparison.

Linear Translatability (LIT):
$$U\big(x + c, \big((x + c) - (m + c)\big)\big)$$
$$= U\big(x, (x - m)\big) + kc$$

where $k > 0$ is a scalar.

The intuition: Since under equal increase of the absolute and reference incomes the relative status remains unchanged but the absolute income increases, individual utility should increase.

LIT demands that if the absolute and reference incomes are changed by a given amount, then utility changes by a constant multiplied by the given amount.

Linear Homogeneity (LIH): For any $c > 0$,

$$U\big(cx, (cx - cm)\big) = cU(x, (x - m))$$

The intuition: An equi-proportionate change in the absolute and the reference incomes changes utility equi-proportionately.

We then prove the following proposition.

Proposition 1: The only utility function that satisfies LIT and LIH is of the form

$$U\big(x, (x - m)\big) = (k - a)x + am$$

where $k > 0$ is same as in LIT and $a < 0$ is a constant.

- This utility function is in fact a particular form of the '*additive comparisons model*' suggested by Clark and Oswald (1998). However, no characterization has been developed by them.

- Let us now consider a situation in which an individual does not compare his/her absolute income with the reference income because the reference income itself is identical to the absolute income. If we denote this absolute income by z_0, then we have $U(z_0, 0) = kz_0$. This absolute income can be taken as the current poverty line.

- Next, we state that the utility corresponding to some arbitrary poverty line z_1 and the reference income m is given by

$$U(z_1, (z_1 - m)) = (k - a)z_1 + am$$

.

- We now want to find out which income z_1 would guarantee the individual a level of utility identical to the utility level $U(z_0, 0)$, that is, the level of happiness that the person had in the earlier scenario when he was enjoying the poverty line income z_0. In other words we want to solve

$$U(z_1, (z_1 - m)) = U(z_0, 0)$$

- By equating the two expressions $U(z_0, 0)$ and $U(z_1, (z_1 - m))$,

it is easy to derive that the revised poverty line z_1 is a weighted average of the existing poverty line and of the specified reference income, that is,

$$z_1 = qz_0 + (1 - q)m$$

with $q = (k/(k - a))$.

- As the weight q increases from 0 to 1 , more and more importance is assigned to the current poverty line in the averaging derived above. Therefore, q may be interpreted as a policy parameter in the sense that it reflects the relative importance of the current poverty line in getting its revised estimate. Since generally $m > z_0$, we conclude that $z_1 > z_0$.

Second case:

- As Clark and Oswald (1998) argued, an alternative specification can be a '*ratio comparisons model*'. In this case the individual's utility depends directly on the absolute income x and also on the relative factor. Thus, in this case the determinant of the status is the ratio $\left(\dfrac{x}{m}\right)$.

- We consider a general form of the utility function $U\left(x, f\left(\dfrac{x}{m}\right)\right)$, where f is a positive valued and increasing transformation of the ratio (x/m).

- As before, we keep the assumptions that U is increasing, concave in x and decreasing, convex in m. So by our formulation, U is increasing in $f(x/m)$.

The axioms:

Linear Homogeneity (LIH):

$U(cx, f(cx/cm)) = cU(x, f(x/m))$ where $c > 0$.   ,

Normalization (NOM): If $x = 1$, then $U(x, f(x/m)) = f(x/m)$

Continuity (CON): U is continuous in its arguments

Intuition:
- Since $f(x/m)$ remains unaltered under positive scale transformation of the absolute income x and the reference income m, LIH shows how utility should be adjusted under such transformation of the variables.
- Continuity assures that minor observational errors in incomes will not change utility abruptly.

We then derive the following proposition.

Proposition 2: The only utility function that satisfies LIH, NOM and CON is of the form

$$U[x, f(x/m)] = xf(x/m)$$

- Clark and Oswald (1998) specified, without characterization, a utility function which is additively separable in the absolute income and the relative income . However, the functional form we have characterized is of product type in its arguments.

- Further, our form becomes additively separable under the logarithmic transformation.

- As in the additive case, we now wish to determine the value of z_1 such that

$$U(z_0, f(z_0/z_0)) = U(z_1, f(z_1/m))$$

- It is easy to show that we finally end up with

$$z_1 = wz_0 + (1 - w)m$$

with $0 < w < 1$.

Thus, here also the revised poverty line becomes a weighted average of the existing poverty line and the reference income. The parameter w has the same policy interpretation as q.

Thus, irrespective of the form of the utility function, we have the same procedure generating a relative poverty line from an existing poverty line and a reference income.

It is easy to show that

- If we take $(1 - w) = [(0.6m - z_0)/(m - z_0)]$,
     where m is the median, we get the poverty line set by the EU.

- Likewise, for $(1 - w) = [(0.37m - z_0)/(m - z_0)]$,
     where m now stands for the mean,
     we get the Atkinson- Bourguignon (2001) relative poverty line.

Our dataset

- It does not provide any information on the reference income of individuals.

- We know only the shares in total income of the various deciles as well as the mean and median of the income distribution (or rather the distribution of expenditures) in the various countries for which data are available.

- We chose for the reference income either the mean or the median.

- If the mean is selected, one implicitly assumes that the extent of poverty should also be a function of the income of those who are not poor, or more generally of the standards of living of all the individuals in the population.

- If the reference chosen is the median income, then, since the latter does not depend on the incomes of those who are not poor, one really assumes that the extent of poverty depends on the standards of living of those individuals who belong to the middle class, and are in the middle of the income distribution.

"amalgam poverty line": a weighted average of an absolute poverty line and of the median income

absolute poverty line:

    - we assume a monthly income of \$38 (at 2005 PPP) which corresponds to \$1.25 per day (Ravallion et al., 2009).

weights: we present results where the weight w given to the absolute poverty line (the weight of the median being then $1 - w$) is 1, 0.9 , 0.66 or 0.5. The results are derived on the basis of two approaches:

- Either the parametrization of the Lorenz curve proposed by Kakwani and Podder.

- Or the Shorrocks and Wan approach

Empirical Illustrations

Table 1: Headcount ratios under various scenarios.

Weighting scheme (weight given to the absolute poverty line)	Bangladesh (2010) Kakwani and Podder approach	Bangladesh (2010) Shorrocks and Wan approach	Cambodia (2009) Kakwani and Podder approach	Cambodia (2009) Shorrocks and Wan approach
Absolute poverty line: $38. It is weighted with the median.				
100%	0.43	0.43	0.28	0.19
90%	0.43	0.44	0.31	0.22
66%	0.45	0.46	0.37	0.30
50%	0.46	0.47	0.40	0.35
Absolute poverty line: $38. It is weighted with the mean.				
90%	0.44	0.46	0.32	0.25
66%	0.48	0.52	0.41	0.39
50%	0.50	0.56	0.46	0.48

Table 1 (cont.): Headcount ratios under various scenarios.

Weighting scheme (weight given to the absolute poverty line)	China rural (2009) Kakwani and Podder approach	China rural (2009) Shorrocks and Wan approach	China urban (2009) Kakwani and Podder approach	China urban (2009) Shorrocks and Wan approach
Absolute poverty line: $38. It is weighted with the median.				
100%	0.29	0.21	0.00	0.00
90%	0.31	0.24	0.00	0.02
66%	0.37	0.31	0.16	0.11
50%	0.40	0.36	0.26	0.21
Absolute poverty line: $38. It is weighted with the mean.				
90%	0.33	0.27	0.00	0.02
66%	0.42	0.41	0.22	0.18
50%	0.47	0.49	0.33	0.31

Table 1 (cont.): Headcount ratios under various scenarios.

Weighting scheme (weight given to the absolute poverty line)	India rural (2010) Kakwani and Podder approach	India rural (2010) Shorrocks and Wan approach	India urban (2010) Kakwani and Podder approach	India urban (2010) Shorrocks and Wan approach
Absolute poverty line: $38. It is weighted with the median.				
100%	0.36	0.34	0.34	0.29
90%	0.37	0.36	0.36	0.31
66%	0.40	0.40	0.40	0.37
50%	0.42	0.42	0.43	0.40
Absolute poverty line: $38. It is weighted with the mean.				
90%	0.38	0.38	0.38	0.34
66%	0.43	0.47	0.45	0.46
50%	0.47	0.52	0.49	0.52

Table 1 (cont.): Headcount ratios under various scenarios.

Weighting scheme (weight given to the absolute poverty line)	Indonesia rural (2011) Kakwani and Podder approach	Indonesia rural (2011) Shorrocks and Wan approach	Indonesia urban (2011) Kakwani and Podder approach	Indonesia urban (2011) Shorrocks and Wan approach
Absolute poverty line: $38. It is weighted with the median.				
100%	0.24	0.15	0.28	0.18
90%	0.27	0.19	0.31	0.21
66%	0.34	0.28	0.37	0.30
50%	0.39	0.34	0.41	0.36
Absolute poverty line: $38. It is weighted with the mean.				
90%	0.29	0.21	0.33	0.25
66%	0.38	0.36	0.42	0.41
50%	0.44	0.45	0.47	0.49

Table 1 (cont.): Headcount ratios under various scenarios.

Weighting scheme (weight given to the absolute poverty line)	Pakistan (2008) Kakwani and Podder approach	Pakistan (2008) Shorrocks and Wan approach	Philippines (2009) Kakwani and Podder approach	Philippines (2009) Shorrocks and Wan approach
Absolute poverty line: $38. It is weighted with the median.				
100%	0.29	0.21	0.28	0.19
90%	0.32	0.24	0.31	0.22
66%	0.37	0.31	0.37	0.31
50%	0.40	0.36	0.41	0.36
Absolute poverty line: $38. It is weighted with the mean.				
90%	0.33	0.27	0.33	0.26
66%	0.40	0.39	0.42	0.41
50%	0.45	0.47	0.48	0.50

Table 1 (end): Headcount ratios under various scenarios.

Weighting scheme (weight given to the absolute poverty line)	Thailand (2010) Kakwani/ Podder approach	Thailand (2010) Shorrocks and Wan approach	Vietnam (2008) Kakwani and Podder approach	Vietnam (2008) Shorrocks and Wan approach
Absolute poverty line: $38. It is weighted with the median.				
100%	0.00	0.00	0.23	0.17
90%	0.06	0.02	0.26	0.20
66%	0.22	0.11	0.33	0.29
50%	0.30	0.21	0.37	0.34
Absolute poverty line: $38. It is weighted with the mean.				
90%	0.09	0.03	0.28	0.22
66%	0.29	0.22	0.38	0.36
50%	0.38	0.37	0.43	0.44

Table 2: Number of poor (in million) in each country, depending on the weighting scheme.

Country	$38;median;100%	$38;median;90%	$38;median;50%	$38;mean;90%	$38;mean;50%
Bangladesh (2010)	65.43	66.49	70.63	69.65	84.95
Cambodia (2009)	2.72	3.13	4.99	3.55	6.75
China Rural (2009)	142.84	165.95	251.45	187.37	342.89
China Urban (2009)	2.05	9.69	131.31	13.97	194.79
India Rural (2009)	285.87	299.92	353.72	318.04	434.58
India Urban (2009)	108.67	116.49	149.60	126.85	194.15
Indonesia Rural (2011)	18.21	23.08	40.49	25.43	53.99
Indonesia Urban (2011)	21.81	26.00	43.89	30.62	60.97
Pakistan (2007)	35.59	40.61	60.52	44.49	78.61
Philippines (2009)	17.20	20.27	33.15	23.68	45.54
Thailand (2010)	0.32	1.20	14.04	1.90	24.61
Vietnam (2008)	14.79	17.30	28.99	19.12	37.70