The Role of Inequality in Poverty Measurement

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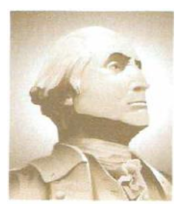

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Introduction

Two forms of technologies for evaluating poverty

Unidimensional

- **Single** welfare variable – eg, calories
- Variables **can** be meaningfully **combined** – eg, expenditure

Multidimensional

- Variables **cannot** – eg, sanitation conditions and years of education
- Want variables **disaggregated** for policy – eg food and nonfood consumption

Introduction

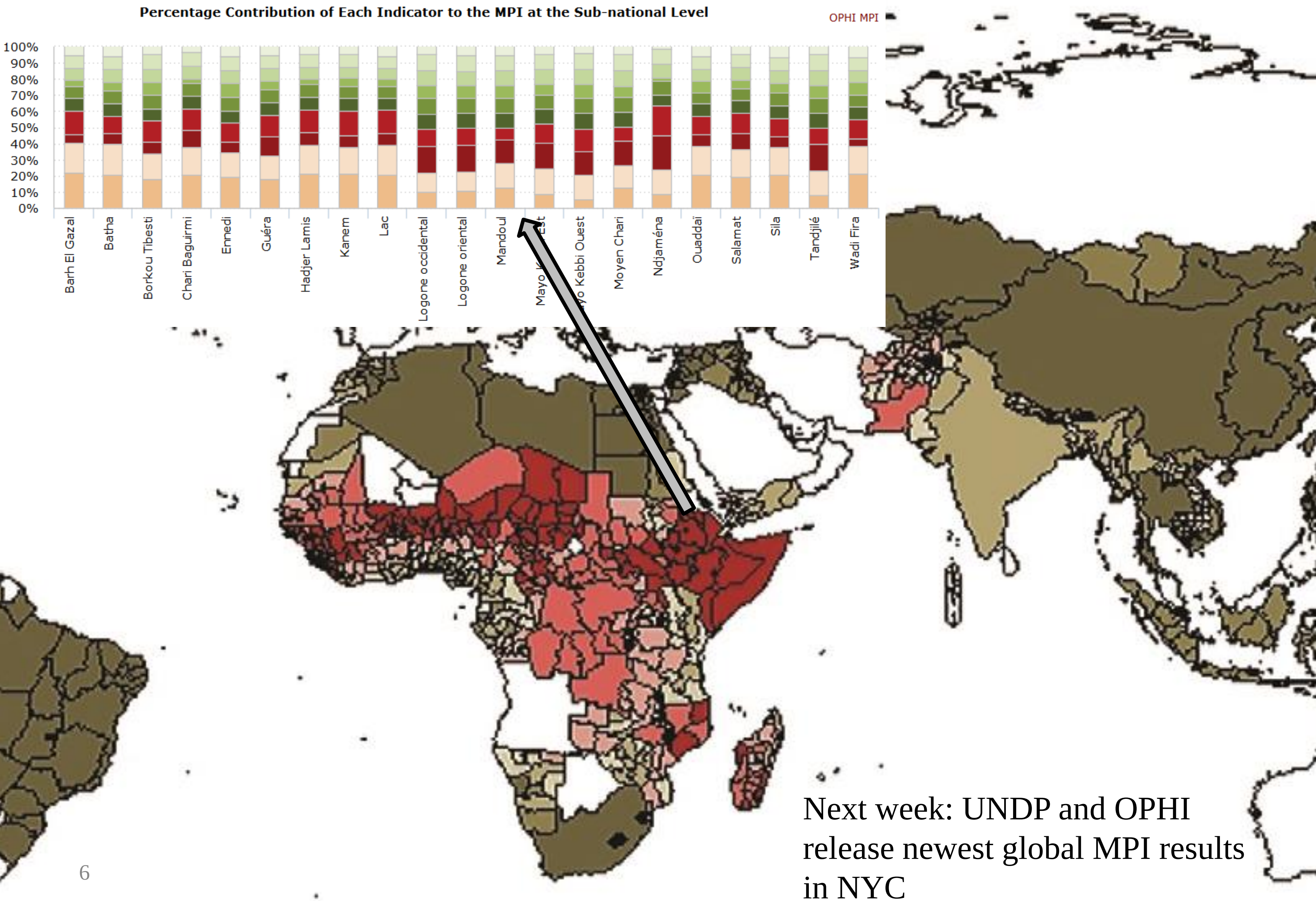
- **Demand** for multidimensional tools ⇧
  - International organizations, countries
- Literature has many measures
  - Anand and Sen (1997), Tsui (2002), Atkinson (2003), Bourguignon and Chakravarty (2003), Deutsch and Silber (2005), Chakravarty and Silber (2008), Maasoumi and Lugo (2008)
- Problems
  - Inapplicable to **ordinal** variables
    - Found in multidimensional poverty
  - Or methods **extreme**
    - Union identification
    - Violates basic axioms

Introduction

- New methodology Alkire-Foster (2011)
  - Adjusted headcount ratio $\mathbf{M_0}$ or **MPI**
- Designed for **ordinal** variables
  - Floor material
- Has **intermediate** identification
  - Dual cutoff approach
- Satisfies key **axioms**

Introduction

- Key axioms
  - **Ordinality**
    - Can use with ordinal data
  - **Dimensional Monotonicity**
    - Reflects deprivations of poor
  - **Subgroup Decomposability**
    - Gauge contributions of population subgroups
  - **Dimensional Breakdown**
    - Gauge contribution of dimensions
- See example
  - Chad



Next week: UNDP and OPHI release newest global MPI results in NYC

Introduction

Critique

- M_0 **not sensitive to distribution among the poor**

Axioms?

- Some only for **cardinal**
- Others **weak**: $\leq$ and not $<$. M_0 satisfies!

Questions addressed here

- Formulate **strict** axiom?
- **Construct** measures satisfying this and other key properties?
- Work in **practice**?

Paper Summary

1. **Axioms**

   Ordinality, Dimensional Breakdown and Dimensional Transfer

2. **Class**

   M-Gamma M_0^γ for $\gamma \geq 0$

   $M_0^0 = H$ headcount ratio

   $M_0^1 = M_0$ adjusted headcount ratio

   M_0^2 squared count measure

3. **Impossibility**
4. **Resolution**

   Shapley Breakdown

   Use M-Gamma like P-alpha

5. **Application** Cameroon

Review: Poverty Measurement

- Traditional two step framework of **Sen** (1976)
  - **Identification** Step "Who is poor?"
    - Targeting
  - **Aggregation** Step "How much poverty?"
    - Evaluation and monitoring

Unidimensional Poverty Measurement

- **Identification** step
  - Typically uses **poverty line**
    - Poor if strictly below cutoff
  - Example: Distribution x = (7,3,4,8) poverty line $\pi = 5$
    - Who is poor?
- **Aggregation** Step:
  - Typically uses **poverty measure**
  - Formula aggregates data into poverty level

Unidimensional Poverty Measurement

FGT or P-alpha class

- **Incomes** $x = (7,\underline{1},\underline{4},8)$
- **Poverty line** $\pi = 5$

Deprivation vector $g^0 = (0,1,1,0)$

- **Headcount ratio** $P_0(x;\pi) = H = \mu(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 4/5, 1/5, 0)$

- **Poverty gap** $P_1(x;\pi) = HI = \mu(g^1) = 5/20$

Squared gap vector $g^2 = (0, 16/25, 1/25, 0)$

- **FGT Measure** $P_2(x;\pi) = \mu(g^2) = 17/100$

Note: All based on normalized gap $\frac{\pi - x_i}{\pi}$ raised to power $\alpha \geq 0$

Our Methodology

- Alkire and Foster (2011)
  - Generalized **FGT** to multidimensional case
  - **Dual cutoff** identification
    - Deprivation cutoffs $z_1, \ldots, z_d$ within dimensions
    - Poverty cutoff k across dimensions
- **Concept** of poverty
  - A person is poor if **multiply deprived** enough
- **Consistent** with
  - Cardinal and **ordinal** data
  - Union, Intersection, and **intermediate** identification
- Example will clarify

Our Methodology

Achievement matrix with equally valued dimensions

Dimensions

$$y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix}$$

Persons

$$\boldsymbol{z = (13 \quad 12 \quad 3 \quad 1)}$$

Cutoffs

Our Methodology

Deprivation Matrix

Deprivation Score

c_i

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0/4 \\ 2/4 \\ 4/4 \\ 1/4 \end{matrix}$$

Our Methodology

Deprivation Matrix

Deprivation Score

c_i

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

0/4

<u>2/4</u>

<u>4/4</u>

1/4

Identification: Who is poor?

If poverty cutoff is **k = 2/4**, middle two persons are poor

Our Methodology

Censored Deprivation Matrix

Censored Deprivation Score $c_i(k)$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0/4 \\ \underline{2/4} \\ \underline{4/4} \\ 0/4 \end{matrix}$$

Why censor? To focus on the **poor**, must ignore the deprivations of **nonpoor**

Our Methodology

Aggregation: Adjusted Headcount Measure

$\mathbf{M_0 = \mu(g^0(k)) = \mu(c(k)) = 3/8}$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} c_i(k) \\ 0/4 \\ \underline{2/4} \\ \underline{4/4} \\ 0/4 \end{matrix}$$

$\mathbf{M_0 = HA}$ where

H = multidimensional **headcount** ratio = ½

"incidence"

A = average deprivation share among poor = 3/4

"intensity"

Note: Easily generalized to different weights summing to 1

Adjusted Headcount Ratio

Properties

- Invariance Properties: **Ordinality**, Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus
- Dominance Properties: Weak Monotonicity, **Dimensional Monotonicity**, Weak Rearrangement, Weak Transfer
- Subgroup Properties: Subgroup Consistency, Subgroup Decomposability, **Dimensional Breakdown**

Digression

- **Definitions** of Ordinality and Dimensional Breakdown

Ordinality

Definition An *equivalent representation* rescales all variables and deprivation cutoffs.

Ordinality An equivalent representation leaves poverty unchanged.

Eg Change scale on self reported health from 1,2,3,4,5 to 2,3,5,7,9, and poverty level should be unchanged

Note

- Measure violates if relies on scale or normalized gaps
- M_0 satisfies

Dimensional Breakdown

Dimensional Breakdown after identification has taken place and the poverty status of each person has been fixed, multidimensional poverty can be expressed as a weighted sum of dimensional components.

Note

Component function for j depends only on dimension j data

Breakdown formula for M_0

$M_0 = \Sigma_j w_j H_j$ or weighted average of censored headcount ratios

Example

Dimensional Breakdown – Cameroon MPI

Indicator	Censored Headcount Ratio H_j	Dimensional Breakdown $w_j H_j$	Relative Contribution $w_j H_j / M_0$
Years of Schooling	16.7	2.8	11.2%
School Attendance	18.4	3.1	12.4%
Child Mortality	27.4	4.6	18.4%
Nutrition	18.3	3.1	12.3%
Electricity	37.3	2.1	8.4%
Sanitation	34.7	1.9	7.8%
Water	28.9	1.6	6.5%
Flooring	34.5	1.9	7.7%
Fuel	45.5	2.5	10.2%
Assets	23	1.3	5.2%
		24.8	100.0%

New Property

Recall property in Alkire-Foster (2011)

- **Dimensional Monotonicity** Multidimensional poverty should rise whenever a poor person becomes deprived in an additional dimension

New property

- **Dimensional Transfer** Multidimensional poverty should fall as a result of a dimensional rearrangement among the poor
- A dimensional rearrangement among the poor An association-decreasing rearrangement among the poor (in achievements) that is simultaneously an association-decreasing rearrangement in deprivations.

New Property

Example with $z = (13,12,3,1)$

Achievements

$$\begin{bmatrix}12 & \mathbf{13} & 2 & 1\\ 10 & \mathbf{7} & 1 & 0\end{bmatrix} \to \begin{bmatrix}12 & \mathbf{7} & 2 & 1\\ 10 & \mathbf{13} & 1 & 0\end{bmatrix}$$

Dominance   No dominance

Deprivations

$$\begin{bmatrix}1 & \mathbf{0} & 1 & 0\\ 1 & \mathbf{1} & 1 & 1\end{bmatrix} \to \begin{bmatrix}1 & \mathbf{1} & 1 & 0\\ 1 & \mathbf{0} & 1 & 1\end{bmatrix}$$

Dominance   No dominance

Dimensional Transfer implies poverty must **fall**

Note: Adjusted Headcount M_0

- Just *violates* Dimensional Transfer
- Same average deprivation score

Question: Are there measures satisfying **DT**?

M-Gamma Class M_0^γ

Identification: Dual cutoff

Aggregation:

$$M_0^\gamma = \mu(c^\gamma(k)) \text{ for } \gamma \geq 0$$

where $c_i^\gamma(k)$ is the censored deprivation score for person i raised to the γ power

Note: Based on "normalized attainment gap"

$c_i^\gamma(\mathrm{k}) = (\frac{d-a_i}{d})^\gamma$ for poor i

$c_i^\gamma(k) = 0$ for nonpoor i

where a_i is person i's attainment score

M-Gamma Class M_0^{γ}

Main measures

$\gamma = 0$ headcount ratio $M_0^0 = H$

$\gamma = 1$ adjusted headcount ratio $M_0^1 = M_0$

$\gamma = 2$ squared count measure M_0^2

Note: Multidimensional analog to P-alpha

Dimensional Transfer satisfied for $\gamma > 1$ ✔

But Dimensional Breakdown **violated** for $\gamma > 1$ ✖

Impossibility

- Recall
  - Dimensional Breakdown: M can be expressed as a weighted average of component functions (after identification)
- **Why** does M_0^2 **violate**?
  - Marginal impact of each dimension depends on **all** dimensions
- Question: Any other measures satisfy **both**?
- **Proposition** There is **no** symmetric multidimensional measure satisfying both Dimensional Breakdown and Dimensional Transfer
- Proof
  - Follows Pattanaik et al (2012)
  - Idea: DT requires fall in poverty; DB requires unchanged

Impossibility

- Importance of Dimensional Breakdown
  - **Coordination** of Ministries
    - Coordinated dashboard of censored headcount ratios
  - **Governance**
    - Stay the course in bad financial times
  - **Policy Analysis**
    - Composition of poverty across groups, space, and time
- Conclusion
  - **Easy to construct** measure satisfying Dimensional Transfer
  - But at a **cost**: lose Dimensional Breakdown

Resolution?

1. Use multiple measures?

   M-gamma class analogous to P-alpha class ✔

2. Relax Dimensional Transfer?

   Already weak ✖

3. Relax Dimensional Breakdown?

   Already weak ✖

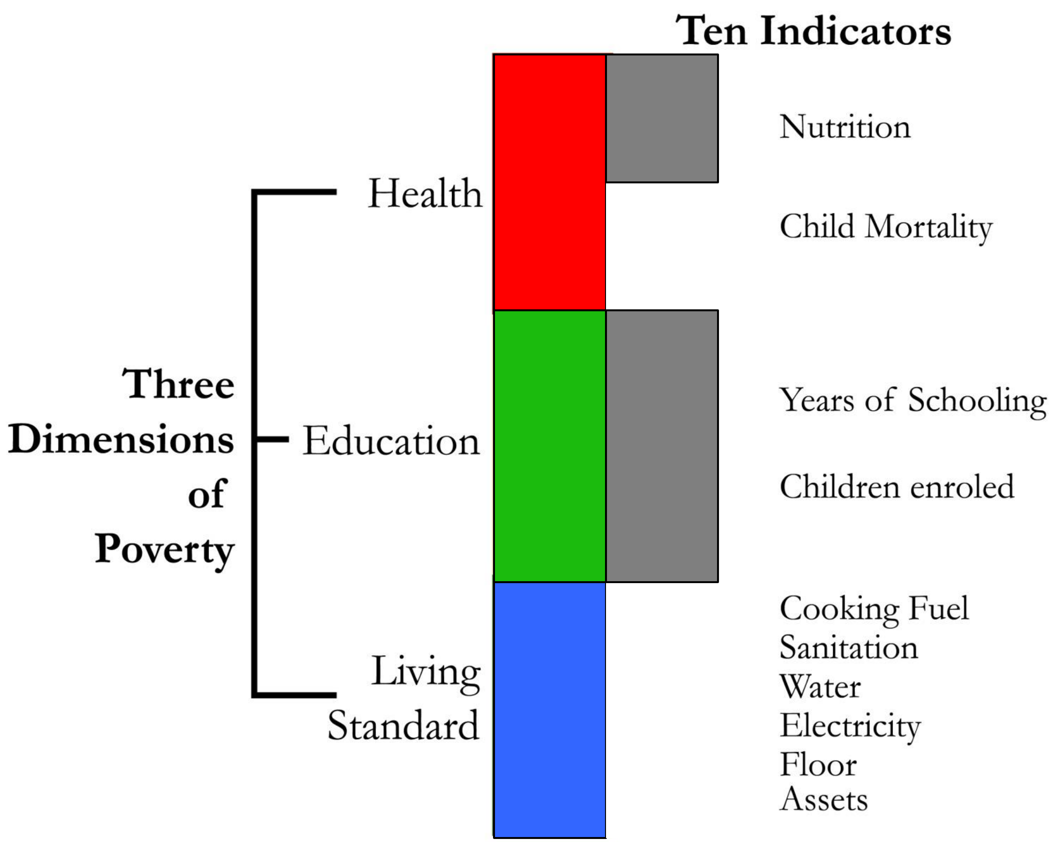
   Datt (2017) suggests Shapley methods

Shapley Breakdown

- Shapley Value
  - Finds **contributions** of parts to whole
  - Especially useful for nonlinear functions: M_0^γ for $\gamma \neq 1$
- **Example**: One person, 10 indicators and union ident.
  - Poverty is censored deprivation score to γ power: $(c_i(k))^\gamma$
    - If not poor, then total and parts are zero
    - If poor and not deprived in j, then j has zero contribution
    - If poor and deprived in j, then the marginal impact of j depends on which dimensional indicator goes first, second, third…
- Shapley: average marginal product across all permutations
  - Tedious to calculate
  - No intuitive link for policy
  - Makes no sense for hierarchical indicators

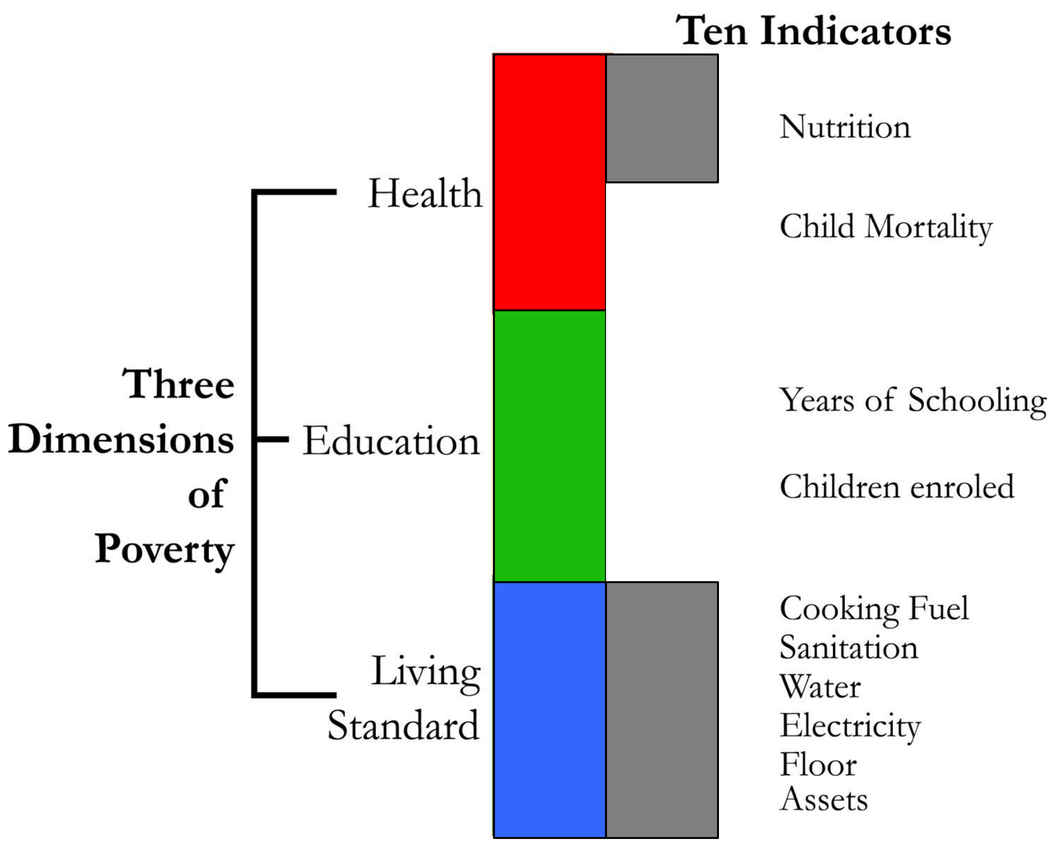
Shapley Breakdown

Example: Pat



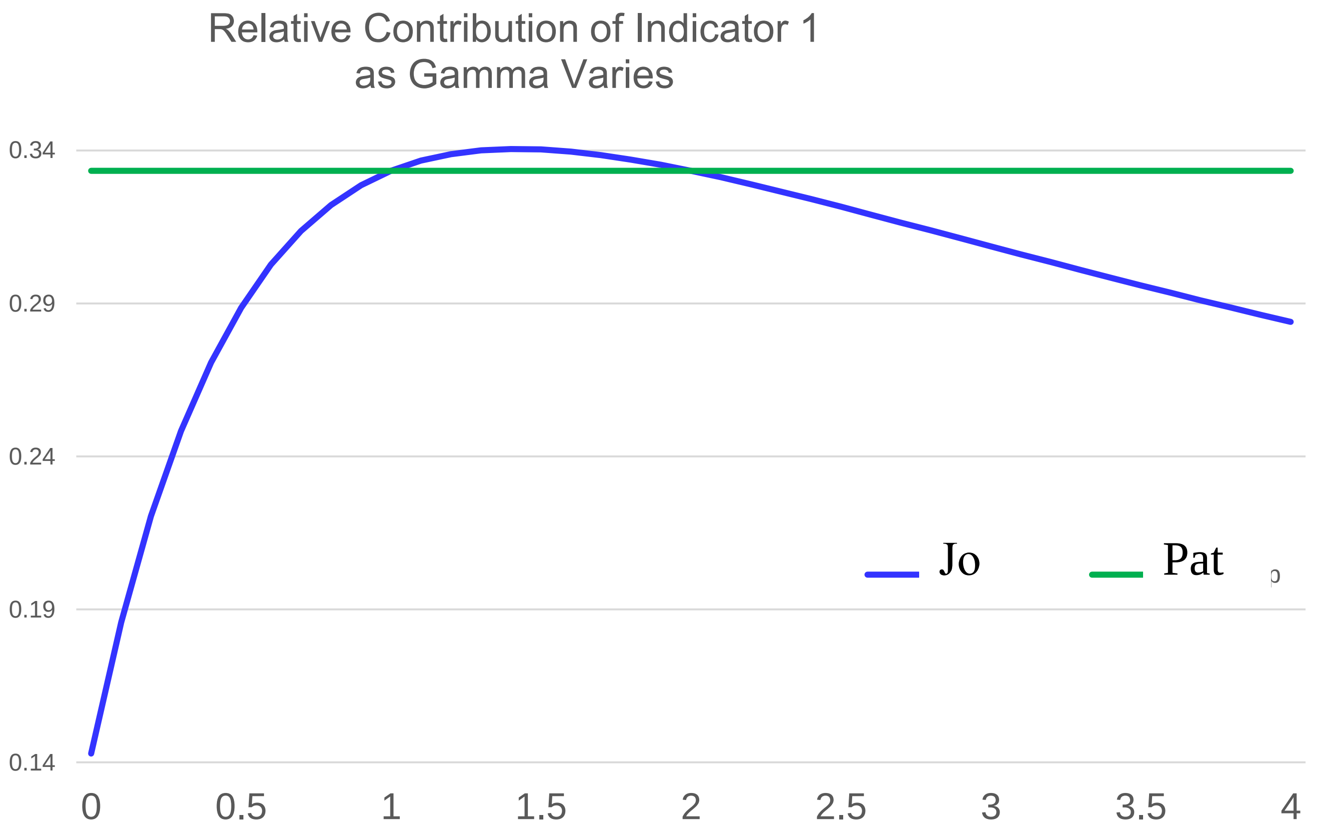
Shapley Breakdown

Example: Jo



Shapley Breakdown Breaks Down

Inconsistency due to hierarchical variables



But ok for $\gamma = 2$

Results

Definition

- Consider set of people **poor** and **deprived** in j
- Censored intensity A_j = average intensity or breadth of poverty in this group

Recall

- Censored headcount ratio H_j = incidence of this group in overall population

Theorem The Shapley breakdown for M_0^2 has a closed form solution. Each component is obtained by multiplying each component of the dimensional breakdown of M_0 by A_j.

Dimensional and Shapley Breakdown – Cameroon

Table V: Breakdowns of M_0^1 and M_0^2

Indicator	Censored Headcount Ratio H_j	Dim Breakdown $w_j H_j$	Censored Intensity A_j	Censored Adjusted Headcount M_{0j}	Shapley Breakdown $w_j M_{0j}$	Relative Contribution $w_j H_j / M_0^1$	Relative Contribution $w_j M_{0j} / M_0^2$	Percentage Point Diff Δ
Schooling	16.7	2.8	66.2	11.1	1.8	11.2%	12.5%	1.3
Attendance	18.4	3.1	66.1	12.2	2.0	12.4%	13.8%	1.4
Child Mortality	27.4	4.6	57.4	15.7	2.6	18.4%	17.8%	-0.6
Nutrition	18.3	3.1	63.9	11.7	1.9	12.3%	13.3%	1.0
Electricity	37.3	2.1	56.4	21.0	1.2	8.4%	7.9%	-0.4
Sanitation	34.7	1.9	53.9	18.7	1.0	7.8%	7.1%	-0.7
Water	28.9	1.6	56.1	16.2	0.9	6.5%	6.1%	-0.4
Flooring	34.5	1.9	56.4	19.5	1.1	7.7%	7.4%	-0.4
Fuel	45.5	2.5	54	24.6	1.4	10.2%	9.3%	-0.9
Assets	23	1.3	57	13.1	0.7	5.2%	5.0%	-0.2
		$M_0^1 = 24.8$			$M_0^2 = 14.7$			

Note similarity of relative contributions

Conclusion

- Derived closed form solution for Shapley breakdown of squared count measure
- Should use the three main M-gamma measures in tandem analogous to P-alpha measures
  - M_0 as the central measures for analysis satisfying Dimensional Breakdown and just violating Dimensional Transfer
  - H as a key partial measure of incidence of poverty
  - M_0^2 (and its Shapley breakdown) to evaluate the effects of inequality among the poor

Thank you!