Quantifying the contribution of a subpopulation to inequality

An application to Mozambique

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Motivation

- The analysis of inequality by **subpopulations**: key element for understanding **inequality levels and trends** across countries.
  - To identify **sources of inequality** and dynamics.
  - However, only **aggregate decompositions** (between-group and within-group), or **group inequality** analyses.
  - In general, **no explicit contribution of each group** to total inequality or each component.

Aim

- Proposing a **detailed decomposition** of inequality by subpopulations (contribution of each subpopulation to overall inequality).
    - + to **between-group and within-group** inequality (**additively decomposable indices**)
  - The **sum of the contributions of its members**
    - The impact that a marginal increase in the proportion of people with a specific income would have on total inequality using the **Recentered Influence Function** (RIF).
      - Consistently with **RIF regressions**.
      - Various **good properties**.

Aim (cont.)

- **Alternative approaches** adapted from the factor inequality decomposition literature (esp. marginal and Shapley factor decompositions)
  - **Mean Log Deviation (M)**, index with best additive decomposability properties: approaches are almost equivalent.
- **Empirical illustration: Mozambique**
  - Low-income sub-Saharan African country, increase in inequality in recent years.
  - Disproportional contributions of **affluent groups** to inequality and its increase over time:
    - top percentiles, urban areas, especially Maputo, and households with heads having higher education.

The RIF detailed decomposition of inequality by subpopulations: general case

- Exhaustive partition, $K \geq 1$ disjoint groups
  - **Population:** $\boldsymbol{y} = (\boldsymbol{y^1}, \dots, \boldsymbol{y^K})$, size n, mean income μ
  - **Group k:** $\boldsymbol{y^k} = (y_1^k, .., y_{n_k}^k)$, size n^k, mean income μ^k.

Contribution to inequality

- Contribution of the **individual** j in group k:

$$S_j^k = \frac{1}{n} RIF\left(y_j^k; I(\boldsymbol{y})\right).$$

- Contribution of **group** k:

$$S^k = \sum_{j=1}^{n^k} S_j^k.$$

Impact on $I(\boldsymbol{y})$ of marginally increasing the population mass at y_j^k.

$\boldsymbol{y}_{\boldsymbol{\varepsilon}}$ = mixture distribution with probabilities: $1 - \varepsilon$ to $\boldsymbol{y}$, ε to x :

$IF(x; I(\boldsymbol{y})) = \frac{\partial}{\partial \varepsilon} I(\boldsymbol{y}_{\boldsymbol{\varepsilon}})|_{\varepsilon=0}$; $E\left(IF\left(x; I(\boldsymbol{y})\right)\right) = 0$ (Hampel, 1974)

$RIF(x; \mathrm{I}(\boldsymbol{y})) = I(\boldsymbol{y}) + IF(x; \mathrm{I}(\boldsymbol{y}))$; $E\left(RIF\left(x; I(\boldsymbol{y})\right)\right) = I(\boldsymbol{y})$ (Firpo, Fortin & Lemieux, 2007,09)

Properties

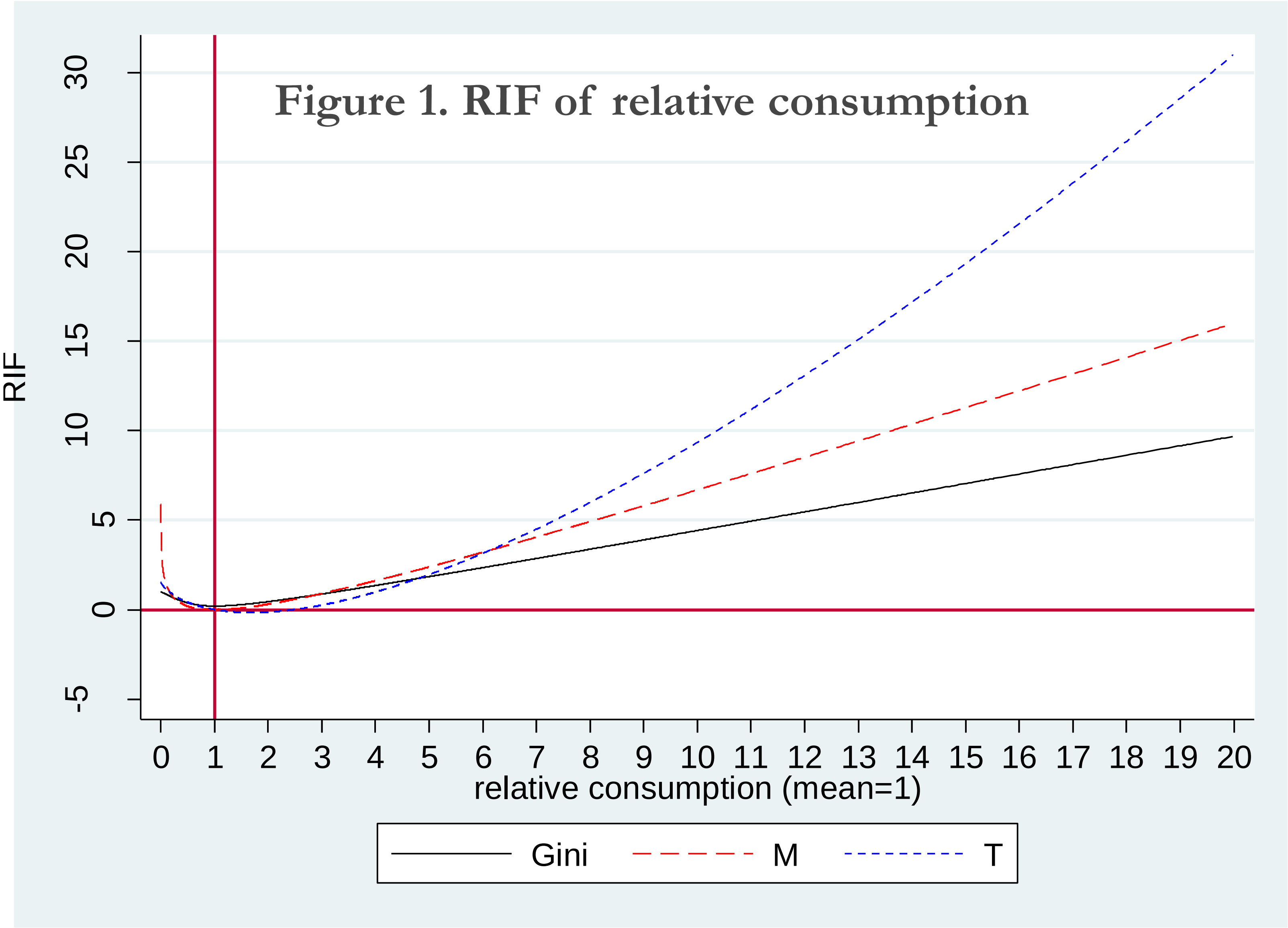
- Invariant to replications of the entire population (**population principle**):

$$S^k(I(\boldsymbol{y}))= S^k\big(I(\boldsymbol{y}')\big) \text{ for any replication } \boldsymbol{y}' = (\boldsymbol{y}, \dots, \boldsymbol{y}).$$

- Invariant to the multiplication of all incomes in the population by the same factor (**scale invariance**):

$$S^k(I(\boldsymbol{y}))= S^k(I(\lambda\boldsymbol{y})) \text{ for any } \lambda > 0.$$

- Asymmetric **U-pattern** with respect to income, reflecting the specific degree of **sensitivity to income transfers** that occur at different points of the distribution.



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Properties (cont.)

- **Consistency**: $I(\boldsymbol{y}) = \sum_{k=1}^{K} S^k = \sum_{k=1}^{K} \sum_{j=1}^{n^k} S_j^k$.

  → $s^k = S^k / I(\boldsymbol{y})$ (relative contribution)

- **Path independence** (order of groups)
- Invariant to the **level of aggregation** of groups.
- **Normalization** property (Gen. Entropy family):
  - $S^k = 0$ if $y_j^k = \mu, \forall\, j = 1, \dots, n^k$;
  - $S^1 = I(\boldsymbol{y})$ if K=1.
- **Range** property (M): S^k will always fall between 0 and $I(\boldsymbol{y})$.

The case of additively decomposable indices

- $I(\boldsymbol{y}) = I_B + I_W;$
- $I_W = I(\boldsymbol{y}) - I(\boldsymbol{\mu^k}) = \sum_{k=1}^{K} I\left(\boldsymbol{y^k}\right) w_I^k;$
- $I_B = I\left(\boldsymbol{\mu^k}\right);$ with $\boldsymbol{\mu^k} = (\mu^1 \mathbf{1}_{\boldsymbol{n}^1}, \dots, \mu^K \mathbf{1}_{\boldsymbol{n}^K})$
- This (+ scale and replication invariance) defines the Generalized Entropy class (Shorrocks, 1984), including limit cases $\alpha = 0{,}1$:

$$I_\alpha(\boldsymbol{y}) = \frac{1}{\alpha(\alpha-1)} \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{\mu}\right)^\alpha - 1\right];$$

$$\text{with } w_{I_\alpha}^k = \frac{n^k}{n} \left(\frac{\mu^k}{\mu}\right)^\alpha.$$

Mimicking aggregate decomposition

- $S^k = S_B^k + S_W^k.$

- $S_W^k = S^k(I(\boldsymbol{y})) - S^k\left(I(\boldsymbol{\mu}^{\boldsymbol{k}})\right),$

  with $I_W = \sum_{k=1}^{K} S_W^k$

- $S_B^k = S^k\left(I(\boldsymbol{\mu}^{\boldsymbol{k}})\right) = \frac{n_k}{n} RIF\left(\mu^k; I(\boldsymbol{\mu}^{\boldsymbol{k}})\right),$

  with $I_B = \sum_{k=1}^{K} S_B^k.$

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For limit cases, M and T

	$M \equiv I_0$	$T \equiv I_1$
I	$\frac{1}{n}\sum_{i=1}^{n} ln\frac{\mu}{y_i}$	$\frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\mu} ln\frac{y_i}{\mu}$
S^k	$\frac{n^k}{n}\left[M^k + \frac{\mu^k - \mu}{\mu} + ln\frac{\mu}{\mu^k}\right]$	$\frac{n^k}{n}\left[\left(\frac{\mu - \mu^k}{\mu}\right)(T+1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu} + \frac{\mu^k}{\mu} T^k\right]$
S_B^k	$\frac{n^k}{n}\left(\frac{\mu^k - \mu}{\mu} + ln\frac{\mu}{\mu^k}\right)$	$\frac{n^k}{n}\left[\left(\frac{\mu - \mu^k}{\mu}\right)(T_B+1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu}\right]$
S_W^k	$\frac{n^k}{n} M^k$	$\frac{n^k}{n}\left[\frac{\mu^k}{\mu} T^k + T_W\left(\frac{\mu - \mu^k}{\mu}\right)\right]$

M: + sensitivity to transfers at the **bottom** and better decomposability properties (**independent of the path** for defining BG and WG terms).

Other approaches: factor decomposition

- **Marginal and Shapley factor decomposition (zero or equalizing subpopulation)**
  - **Marginal:** change after removing a factor (e.g. Kakwani, 1977)
    - Inconsistent decomposition + not invariant with the level of aggregation of the target group
  - **Shapley:** average marginal contribution over all possible sequences (Chantreuil and Trannoy, 2013; Shorrocks, 2013)
    - Consistent decomposition + not invariant with the level of aggregation of groups, cumbersome to compute.
- **Natural decomposition rules of some inequality indices** (Shorrocks, 1982, Morduch and Sicular, 2002)
  - Index-specific (CV, Gini, Theil) and does not fully account for the contribution of a factor.

Equalizing subpopulations

- **Marginal:** $\delta^k = I(\boldsymbol{y}) - I\left(\left(\boldsymbol{y}^{-\boldsymbol{k}}, \mu\mathbf{1}_{\boldsymbol{n}^k}\right)\right)$. $\widetilde{\boldsymbol{\delta}}^{\boldsymbol{k}}$ if normalized to add up to I
- **Shapley:** $\delta'^k = \frac{1}{2}\left(I(\boldsymbol{y}) + I\left((\mu\mathbf{1}_{\boldsymbol{n}^{-k}}, \boldsymbol{y}^{\boldsymbol{k}})\right) - I\left((\boldsymbol{y}^{-\boldsymbol{k}}, \mu\mathbf{1}_{\boldsymbol{n}^k})\right)\right)$.

$$\delta_W^k(M) = \delta'^k_W(M) = S_W^k(M),$$

$$\delta_B^k(M) = \frac{n^k}{n} ln\frac{\mu}{\mu^k} - ln\left(1 + \theta^k\right) \approx S_B^k(M)$$

$$\delta'^k_B(M) = \frac{n^k}{n} ln\frac{\mu}{\mu^k} + \frac{1}{2} ln\left(\frac{1+\theta^k}{1-\theta^k}\right) \approx S_B^k(M)$$

If small

$$\theta^k = \frac{n^k}{n}\frac{\mu^k - \mu}{\mu}$$

Empirically similar

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Empirical analysis: Mozambique

- **Data**: 2 most recent Household Budget Surveys.
  - *Inquéritos ao Orçamento Familiar* (IOF 2008/09 and 2014/15, INE)
- **Wellbeing**: Daily **real per capita consumption** (MEF/DEEF, 2016)
- **Sample:** about 11,000 households (>50,000 ind.) interviewed once in 2008/2009; similar but interviewed 1-3 times in 2014/15 (pool).
- **Subpopulations**:
  - consumption percentile groups,
  - area of residence (rural or urban),
  - province,
  - head's attained education.

Table 2: Consumption inequality

Lorenz dominance

Index	2008/09	20014/15
Gini	0.415	0.468
I_{-1}	0.409	0.532
I_0=M	0.303	0.381
I_1=T	0.367	0.520
I_2	0.887	2.242

C. Gradín and F. Tarp (2017), "Investigating growing inequality in Mozambique", UNU-WIDER WP 208/2017

Table 3: Relative RIF contributions to inequality by percentile

Range	%pop	%y	Gini	I_0=M	I_1=T
Bottom 5	5	0.8	8.4	14.7	9.7
6-25	20	6.5	23.7	24.5	25.6
26-75	50	34.3	31.1	12.5	23.2
76-95	20	30	12.9	6.5	-4.1
Top 5	5	28.5	24.0	41.9	45.6
Total	**100**	**100**	**100**	**100**	**100**

… to inequality increase between 2008/09 and 2014/15

Range	%pop	%y	Gini	I_0=M	I_1=T
Bottom 5	5	0.7	-0.1	0.8	4.5
6-25	20	5.6	15.1	21.5	21.6
26-75	50	30.6	39.9	19.5	32.0
76-95	20	29.1	7.1	-1.8	-8.7
Top 5	5	34.0	38.1	60.1	50.7
Total	**100**	**100**	**100**	**100**	**100**

Table 5a: RIF decomposition of M by province and area in 2014/15

Province	%pop.	μ_k/μ	M^k	s^k%	s_B^k%	s_W^k%
Niassa	6.4	66.1	0.267	5.7	1.3	4.5
Cabo Delgado	7.4	87.8	0.243	4.8	0.2	4.7
Nampula	19.5	77.7	0.304	17.0	1.5	15.5
Zambezia	18.8	76.0	0.291	16.0	1.7	14.3
Tete	9.8	97.6	0.247	6.3	0.0	6.3
Manica	7.5	93.2	0.259	5.1	0.0	5.1
Sofala	7.9	102.7	0.382	7.9	0.0	7.9
Inhambane	5.8	95.0	0.340	5.2	0.0	5.2
Gaza	5.5	89.8	0.345	5.1	0.1	5.0
Maputo province	6.6	169.4	0.376	9.3	2.9	6.5
Maputo City	4.9	280.1	0.583	17.3	9.8	7.5
All	**100**	**100**	**0.381**	**100**	**17.5**	**82.5**
Area						
Rural	68.3	78.8	0.243	48.3	4.7	43.6
Urban	31.7	145.7	0.541	51.7	6.7	44.9
All	**100**	**100**	**0.381**	**100**	**11.4**	**88.6**

Table 6a: RIF decomposition of ΔM by province and area, 2008/09-2014/15

Province	Δ%pop	$\Delta\mu_k/\mu$	ΔM^k	$\%\Delta S^k/\Delta M$	$\%\Delta S_B^k/\Delta M$	$\%\Delta S_W^k/\Delta M$
Niassa	0.5	-68.9	-0.078	-2.0	2.3	-4.3
Cabo Delgado	-0.5	-20.6	0.046	3.6	0.4	3.2
Nampula	0.3	-22.9	0.001	8.8	7.4	1.5
Zambezia	-0.2	-2.3	0.060	15.2	1.6	13.6
Tete	0.8	0.3	0.039	7.0	0.0	7.0
Manica	0.5	7.9	0.049	5.3	-0.8	6.1
Sofala	-0.2	8.3	-0.038	-5.2	-0.1	-5.1
Inhambane	-0.3	-3.5	0.082	5.2	0.1	5.1
Gaza	-0.8	5.7	0.013	-3.0	-0.7	-2.3
Maputo P.	0.3	74.6	0.125	25.3	14.0	11.3
Maputo C.	-0.4	95.2	0.148	39.8	32.5	7.3
All	**0.0**	**0.0**	**0.078**	**100**	**56.5**	**43.5**
Area						
Rural	-1.3	-9.6	0.003	15.7	16.5	-0.9
Urban	1.3	19.3	0.139	84.3	21.2	63.1
All	**0.0**	**0.0**	**0.078**	**100**	**37.7**	**62.3**

Table 5b: RIF decomposition of M by education in 2014/15

Education	%pop.	μ_k/μ	M^k	s^k%	s_B^k%	s_W^k%
Less than primary	30.5	72.4	0.285	26.6	3.8	22.8
Lower Primary	43.9	82.1	0.247	30.5	2.1	28.4
Upper Primary	13.9	105.9	0.300	11.0	0.1	11.0
Lower Secondary	4.1	139.8	0.338	4.3	0.7	3.6
Upper Secondary	3.3	207.1	0.432	6.8	3.0	3.8
Technical	0.7	250.9	0.470	2.0	1.1	0.9
Some college	2.5	469.1	0.574	17.8	14.0	3.8
Unknown	1.1	94.7	0.334	0.9	0.0	0.9
All	**100**	**100**	**0.381**	**100**	**24.8**	**75.2**

Table 6b: RIF decomposition of ΔM by education, 2008/09-2014/15

Education	Δ%pop	$\Delta\mu_k/\mu$	ΔM^k	$\%\Delta S^k/\Delta M$	$\%\Delta S_B^k/\Delta M$	$\%\Delta S_W^k/\Delta M$
Less than primary	5.4	-10.2	0.032	43.1	12.9	30.1
Lower Primary	-11.4	-5.7	0.018	-18.9	4.5	-23.4
Upper Primary	1.3	-6.9	0.026	8.5	-0.9	9.4
Lower Secondary	1.1	-21.6	0.015	3.2	-2.0	5.1
Upper Secondary	1.8	-24.5	0.057	16.2	5.2	11.0
Technical	-0.1	12.3	0.138	0.8	-0.1	0.8
Some college	1.3	-8.5	0.023	44.2	34.4	9.8
Unknown	0.6	32.0	0.152	3.1	-0.5	3.6
All	**0.0**	**0.0**	**0.078**	**100**	**53.5**	**46.5**

Table 7a: Relative Decomposition of M and T by percentile range, 2014/15

	M			T		
	RIF	Marginal	Shapley	RIF	Marginal	Shapley
Range	S^k	$\tilde{\delta}^k$	δ'^k	S^k	$\tilde{\delta}^k$	δ'^k
Bottom 5%	14.7	12.9	14.5	9.7	8.0	7.3
6-25	24.5	23.1	23.8	25.6	20.2	17.5
26-75	12.5	13.3	11.7	23.2	18.5	13.5
76-95	6.5	6.8	7.0	-4.1	-2.9	1.8
Top 5%	41.9	43.7	43.0	45.6	56.3	59.9
All	**100**	**100**	**100**	**100**	**100**	**100**

Note: Marginal, normalized to add up to 100

Table 7b: Relative Decomposition of M and T by province, 2014/15

	M			T		
	RIF	Marginal	Shapley	RIF	Marginal	Shapley
Province	S^k	$\tilde{\delta}^k$	δ'^k	S^k	$\tilde{\delta}^k$	δ'^k
Niassa	5.7	5.7	5.7	5.3	5.1	4.1
Cabo D.	4.8	4.8	4.8	4.7	4.5	4.2
Nampula	17.0	17.0	17.0	20.0	18.9	17.8
Zambezia	16.0	16.0	16.0	14.8	14.0	12.5
Tete	6.3	6.2	6.3	5.2	5.0	5.1
Manica	5.1	5.0	5.1	4.5	4.4	4.3
Sofala	7.9	7.8	7.9	7.3	7.2	7.4
Inhambane	5.2	5.1	5.2	4.7	4.6	4.5
Gaza	5.1	5.0	5.1	4.6	4.5	4.3
Maputo P.	9.3	9.4	9.4	7.8	8.2	10.1
Maputo C.	17.3	18.0	17.4	21.2	23.5	25.6
All	**100**	**100**	**100**	**100**	**100**	**100**

Table 7c: Relative Decomposition of M and T by area, 2014/15

	M			T		
	RIF	Marginal	Shapley	RIF	Marginal	Shapley
Area	S^k	$\tilde{\delta}^k$	δ'^k	S^k	$\tilde{\delta}^k$	δ'^k
Rural	48.3	48.2	48.1	46.6	39.5	38.7
Urban	51.7	51.8	51.9	53.4	60.5	61.3
All	**100**	**100**	**100**	**100**	**100**	**100**

Table 7d: Relative Decomposition of M and T by education, 2014/15

	M			T		
	RIF	Marg.	Shapley	RIF	Marg.	Shapley
Education	s^k	$\tilde{\delta}^k$	δ'^k	s^k	$\tilde{\delta}^k$	δ'^k
Less than primary	26.6	26.7	26.5	27.5	25.8	23.1
Lower Primary	30.5	30.4	30.5	28.7	26.9	24.6
Upper Primary	11.0	10.7	11.0	9.1	9.1	9.6
Lower Secondary	4.3	4.2	4.3	3.2	3.2	4.0
Upper Secondary	6.8	6.8	6.8	6.8	7.2	8.7
Technical	2.0	2.0	2.0	1.9	1.9	2.4
Some college	17.8	18.4	17.9	22.1	25.1	26.8
Unknown	0.9	0.9	0.9	0.8	0.8	0.8
All	**100**	**100**	**100**	**100**	**100**	**100**

Conclusions

- A **detailed decomposition of inequality indices by subpopulations** based on RIF.
  - **Overall inequality** can be decomposed into the contribution of the distinct groups making up the population.
  - Additively decomposable indices: further decomposed into their **between-group and within-group components**.
  - Consistent with **RIF regressions**.
  - Verifies several **appealing properties** (e.g. consistency, path independence, and independence on the level of aggregation) and easy to compute.

Conclusions (cont.)

- Other natural **alternatives**,
  - Especially, **marginal and Shapley decomposition** using the equalizing subpopulation approach,
    - more appropriate for attributing the contribution of each group, especially with additive decomposable indices.
  - All three approaches are **approximately equal** in the case of the Mean Log Deviation (best decomposable index).

Conclusions (cont.)

- **Empirical analysis** of consumption inequality in Mozambique
  - Choice of approach is not empirically relevant (Mean Log Deviation)
    - Non-negligible differences with very extreme groups
  - The **richest groups**, such as people living in Maputo or in other urban areas, with higher educational level, or in the top of the consumption distribution are responsible for the largest shares of inequality and for its increasing trend over time.
    - **Even higher** contributions with Shapley decomposition of the **Theil index**, qualitative results are very similar.