WIDER Working Paper 2020/162

Identifying structural changes in the exchange rates of South Africa as a regime-switching process

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December 2020

Abstract: Exchange rate volatility is said to exemplify the economic health of a country. Exchange rate break points (known as structural breaks) have a momentous impact on the macroeconomy of a country. Nonetheless, this country study makes use of both unsupervised and supervised machine learning algorithms to classify structural changes as regime shifts in real exchange rates in South Africa. Weekly data for the period January 2003–June 2020 are used. To these data we apply both non-linear principal component analysis and Markov-switching generalized autoregressive conditional heteroscedasticity. The former approach is used to reduce the dimensionality of the data using an orthogonal linear transformation by preserving the statistical variance of the data, with the proviso that a new trait is non-linearly independent, and it identifies the number of regime switches that are to be used in the Markov-switching model. The latter is used to partition the variance in each regime by allowing an estimation of multiple break transitions. The transition breakpoints estimates derived from this machine learning approach produce results that are comparable to other methods on similar system sizes. Application of these methods shows that the machine learning approach can also be employed to identify structural changes as a regime-switching process. During times of financial crisis, the growing concern over exchange rate volatility, including its adverse effects on employment and growth, broadens the debates on exchange rate policies. Our results should help the South African monetary policy committee to anticipate when exchange rates will pick up and be prepared for the effects of periods of high exchange rates.

Keywords: machine learning, Markov-switching process, non-linear principal component, South Africa

JEL classification: B23, C01, C24, C41

Acknowledgements: Very special gratitude to the South African Reserve Bank for providing us with weekly data that have realized volatility. Many thanks to our anonymous reviewers for their constructive comments and suggestions. We thank you all.

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This study has been prepared within the UNU-WIDER project Southern Africa—Towards Inclusive Economic Development (SA-TIED).

Information and requests: publications@wider.unu.edu

ISSN 1798-7237 ISBN 978-92-9256-919-8

https://doi.org/10.35188/UNU-WIDER/2020/919-8

Typescript prepared by Joseph Laredo.

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The Institute is funded through income from an endowment fund with additional contributions to its work programme from Finland, Sweden, and the United Kingdomas well as earmarked contributions for specific projects from a variety of donors.

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1 Introduction

Exchange rate movements have significant implications for the domestic macro-economy, including the worldwide transmission of business cycles and inflation, and changes between current and financial accounts. In addition, exchange rate volatility can initiate insecurity and increment vulnerability in every one of these areas, so that settling on an economic choice turns out to be more difficult for firms, buyers, and policy-makers (Nyawo and van Wyk 2018). As economies become more globally incorporated, these linkages conceivably become more crucial (van Wyk et al. 2020). The key transmission component in these connections is price changes. Consequently, the relation between exchange rates and prices is crucial to macro-economic research and financial policy.

The South African rand (ZAR) has depreciated substantially since the end of apartheid. At the end of 2003, the ZAR lost approximately 50 per cent of its value due to South Africa's (SA) high inflation rates compared with its trading partners. Makatjane and Xaba (2016) stated that this high inflation episode was caused by (i) sudden reversals of capital, (ii) the country's difficulties in attracting foreign direct investment (FDI), (iii) the replication of macro-economic policy for the maintenance of stability and improvement of growth rates, and (iv) attempts to avoid the over-heating and appreciation of the real exchange rate by the South African Reserve Bank (SARB).

Decisions that consumers formulate about their living standards are warped by unstable inflation rates, which have further slowed South Africa's dawdling economic growth. Because of elevated inflationary factors, circulation of money in the economy declines. Bonga-Bonga and Kabundi (2015) report the importance of stabilizing prices, and van Wyk and Dlamini (2018) reveal that in the long-run such distortions harm households that have low incomes. Edwards and Hlatshwayo (2020) emphasize that there has been an abrupt downgrading, followed by a recuperation due to substantial unpredictability during this time as some crisis patterns were urbanized in 1997 and in 2001. Oscillations in the real exchange rate may create considerable apprehension among policy-makers and businesses, as they may have a disruptive impact on trade flows if hedging is costly or incomplete. They may also deter investment decisions associated with such trade flows. It is, therefore, appropriate to study whether these movements in the real exchange rate are in equilibrium or not (Ricci 2005).

The area that requires attention is the implication of changes in the real exchange rate (RER) to exporters and importers. Hlatshwayo and Saxegaard (2016) discovered that the RER has previously been an essential driving factor of intensification in SA exports and imports. In recent years, there has been a feeble relationship between the real effective exchange rate (REER) and exports. This has been reflected by the weak response of exports to the considerable real reduction of ZAR from 2010 to 2016 (Hlatshwayo and Saxegaard 2016). Are inflationary pressures in SA provoked by exchange rate dynamics? If yes, are these the prevailing cause of inflationary pressure? These questions are the focus of this study.

Exchange rate pass-through to prices has offered some diverse findings regarding whether the pass-through is high or low (Nxazonke and van Wyk 2020; Devereux and Yetman 2002). Notable among them are those of Asafo (2019) and Chifurira et al. (2016), which showed that the pass-through of high exchange rate dynamics to prices is attributable to the depreciation of the currency as the ZAR is subject to a bendy exchange rate regime.

Given these facts, the novelty of our study is its identification of structural changes in the South African real exchange rate as a regime-switching process. A two-fold empirical analysis is set up.

We first perform a non-linear principal component Markov-switching generalized autoregressive conditional heteroscedasticity (NPC-MS-GARCH) analysis. A non-linear principal component is used to identify the number of regimes in the RER and significantly reduce the dimensionality of the featured space across the sample period by the use of orthogonal linear alterations so as to conserve the statistical discrepancy of the data under the condition that new features are non-linearly independent.

The Markov-switching model (MSM) is based on Cruz and Mapa (2013), who indicated that the MSM model does not distinguish ex-ante between high and low volatility episodes and that the Markov process can identify periods of high and low exchange rates through a latent volatility state in each regime. Proposed machine learning (ML) methods are able to partition structural changes according to the similarity principle in dissimilar classes. The distribution of the partitions with reference to time series is linked with structural changes that predict the transition point. Therefore, this is an experimental process for detecting structural changes.

In applying the proposed ML methods, the dimensionality of time-varying features across regimes using an orthogonal non-linear transformation conserves the statistical inconsistency of each regime and shows that the identified regimes are non-linearly independent. This study contributes to the existing literature by showing how unsupervised ML methods, in conjunction with Markov-switching models, identify structural changes in a volatile series such as real exchange rates. To our knowledge, this is the first study to identify structural changes as a process of regime shifts using the proposed statistical methods.

2 Literature review

The status of a country's global competitiveness is vital when determining key macro-economic objectives. The relative competitiveness of a country in global trade is often determined by the RER of an economy (Walters and De Beer 1999). Movements or changes in the exchange rate influence international relationships significantly. Therefore, shocks or structural breaks in exchange rates have a significant impact on the macro-economy of a country. The debate on the effects of structural breaks in exchange rates is ongoing. Kočenda (2001) argued that globalization has reconfigured the way changes in exchange rates affect economies, and this is true because structural breaks in exchange rates depend largely on the economic conditions of a country. Furthermore, Kołodziejczyk (2020) and Nomsobo and van Wyk (2018) found that periods of low and high volatility in exchange rates have significantly impacted currencies of Southern countries, as experienced in the United States (US) during 2008. After this global financial crisis, growing concerns over exchange rate volatility, including its adverse effects on employment and growth, have broadened exchange rate policies.

According to Jeelani et al. (2019), the health of a country's economy is demonstrated by the stability of its exchange rate. However, Dropsy (1996) has shown that structural shocks in the economy affect the long-run real exchange rate. In 2007, the exchange rate in Kenya experienced momentous structural breaks and the shilling drastically depreciated as compared with global currencies. Studies have shown that the characteristics of a country are imperative, which include policies related to monetary response and structural characteristics that influence the sensitivity of an economy's currency movements (Hafner et al. 2019). Volatility in forecasting variables may lead to uncertainty in policy-making. According to Asghar and Urooj (2012), data forecasts are often different from the outcome, which causes uncertainty in forecasting. Policy uncertainty may be harmful to the macroeconomy of a country (Nxazonke and van Wyk 2020). In addition, Kočenda (2001) asserted that a one-time shock produced by structural changes and/or measures that are imposed by policy-makers significantly hinders exchange rates. It is therefore vital to identify structural breaks effectively. Machine learning algorithms can be utilized to determine and classify the degree to which structural breaks occur. This methodology is particularly effective in providing instinctive measures of predictive accuracy such as sensitivity and specificity, as well as cross-validation to confirm the generalizability of the model and conduct model-free permutation tests.

It is difficult to model behavioural aspects of financial time series utilizing linear models. This is especially challenging when modelling phenomena such as volatility and structural breaks in time series (Ismail and Isa 2007). Therefore, this study uses a Markov-switching GARCH and non-linear principal component analysis methodology like Ismail and Isa (2007) to determine the structural breaks in exchange rates in South Africa. This enables the capturing of regime shifts caused in times of financial crisis or vulnerability. Nonetheless, numerous economists have questioned whether structural breaks in exchange rate volatility really matter. Su et al. (2011) argue that researchers should consider structural breaks when forecasting and estimating macro-economic variables. Yet, the results of estimation could be meaningfully different from developing to developed economies.

Empirical evidence shows that structural breaks in exchange rates are prevalent among developing countries. Kočenda (2001) measured structural breaks in exchange rates in Balkan countries. These countries are economically volatile and monetary policies implemented by policy-makers have resulted in structural breaks in the behaviour of exchange rates. Kočenda (2001) utilized a Vogelsang (1997) testing procedure, which produced strong evidence for the existence of structural changes in the exchange rates of these countries. A study by Makatjane et al. (2018) measured the structural changes caused by the 2008 US financial crisis on South Africa's real exchange rate between 2000 and 2017. These authors utilized a Seasonal Autoregressive Integrated Moving Average (SARIMA) intervention to identify significant impacts of this financial crisis. Their results revealed that the financial crisis in SA occurred in March 2008 and significantly affected the exchange rate. Hence, the impact pattern was abrupt. But the current study uses latent Markov chains (MC) to identify multiple interventions in the sampled period. Chifurira et al. (2016) studied structural changes in the US/ZAR exchange rate and corresponding cointegration with the SA gold mining index. These authors made use of a Granger causality test and Zivot-Andrews unit root test. They recognized that there were structural breaks and that the gold mining index is not key to determining trends in exchange rates and vice versa.

After China relinquished its fixed exchange rate to the US$, Zeileis et al. (2010) conducted a study on the exchange rate regime in China in order to track the development of exchange rate regimes in India. They used a linear regression model to classify the exchange rate regime. This was done to complement an inferential methodology for determining regime stability. These authors found long periods of increased flexibility and marginal appreciation of the Chinese currency between 2006 and 2008. The Indian currency was largely linked to the US$ before 2004, becoming more flexible thereafter. Jeelani et al. (2019) tested structural changes between the correlation of macro-economic variables, such as gross domestic product (GDP), FDI, interest rates, and exchange rates using an ordinary least squares estimation. This study revealed that the correlation between variables is significantly impacted by major economic events such as the 2008 US financial crisis. Su et al. (2011) conducted a study using GARCH models to determine structural breaks and exchange rate volatility in Asia-Pacific currencies and found a significant difference in the exchange rates of Canada, Switzerland, and the United Kingdom (UK) as compared with Asia-Pacific exchange rates.

Kołodziejczyk (2020) investigated the exchange rate returns of the euro against three Central European currencies using Hamilton's regime-switching model for the period January 2014 to December 2018. Results indicated that periods of low and high volatility are dependent on each other between the countries. On the other hand, the empirical analysis of Dropsy (1996) investigated long-run exchange rate shifts in the euro relative to the US and German currencies in order to determine the causes of exchange rate deviation from an equilibrium state. Hafner et al. (2019) estimated the structural factor-augmented VAR models of 47 economies and found that independence of the central bank can facilitate inflation stabilization after vast currency fluctuation and allow the exchange rate to be a buffer against external structural shocks.

3 Data and materials

This study uses a weekly exchange rate retrieved from the South African Reserve Bank database. The series covers the period from January 2003 to June 2020. Various routines are done in R 3.6.3 with included packages such as MS-GARCH of Ardia et al. (2019) and the 'pcadapt' package of Luu et al. (2017).

3.1 Non-linearity test

Recent research has uncovered that macro-economic and financial time arrangements such as stock exchanges, are portrayed by the presence of shocks. Douc et al. (2014) found that the underlying linear model of such data may give ambiguous market movement specifications. If the dynamic spread of shocks diverges from the usual behaviour of the time series, then a model that relies on a linear propagation mechanism will necessarily be incorrect (Bradley and Jansen 2004). Scholars like Xaba et al. (2017) and Ismail and Isa (2007) used the Ramsey RESET test. However, in the current study, we make use of the Lagrange Multiplier (LM) test to check for the presence of non-linearities.

Denoting autocorrelations of raw data by $\rho_1, \dots, \rho_m$ with $m = \frac{n}{4}$, Engle (1982) established the hypothesis of dependence on Y_t as follows: $\begin{matrix} H_0: \rho_{ij} = 0 \\ H_a: \rho_{ij} \neq 0 \end{matrix}$. This hypothesis is tested by a Q-statistic given by

$$Q_m = n(n+2) \sum_{k=1}^{m} \frac{\rho_k^2}{n-k} \sim \chi_{\alpha}^2, n - p. \quad (1)$$

Since the Q-statistic in model (1) is firmly related to the LM test for autoregressive conditional heteroscedasticity (ARCH), at that point Engle (1982) indicated that the test depends on the accompanying linear regression

$$\hat{e}_t^2 = \vartheta_0 + \sum_{i=0}^{m} \vartheta_i e_{t-i}^2 + \nu_t \quad (2)$$

where $\vartheta_0, \dots, \vartheta_m$ are parameter estimates and $\upsilon_t \sim \text{i. i. d}\ (0, \sigma_\varepsilon^2)$. Note that i.i.d is here referenced (independently and identically distributed). Therefore, the hypothesis to be tested is as follows:

$$\begin{matrix} H_0: \mathrm{Var}(\varepsilon_t) = \sigma_t^2 \\ H_1: \mathrm{Var}(\varepsilon_t) \neq \sigma_t^2 \end{matrix}.$$

The test statistic established in model (2) follows the usual F-statistic given by

$$F^* = \frac{R^2/_k}{1-R^2/_{l-k-1}} \sim F_{\alpha,(k,l-2k-1)}. \quad (3)$$

If the critical value of $F_{\alpha,(k,l-2k-1)}$ is less than the value of the F statistic in model (3), we reject the null hypothesis and conclude that the observed time series is non-linear.

3.2 Information criterion for model selection

Model selection is an enormous issue in practical data analysis. It is still believed that the reported results of a model with a high coefficient of determination (R^2) are spurious. The implication of this is that R^2 should not be the main archetype for model selection. For model selection the current study makes use of the Akaike information criterion (AIC) by Akaike (1974), the Schwarz Bayesian information criterion (SBC) by Schwarz (1978), and the likelihood ratio test of Bevington and Robinson (2003). According to Bozdogan (2000), the requirement to introduce model evaluation has been recognized as one of the imperative technical areas, and the problem is posed on the choice of the best approximating model among a class of competing models by a suitable model evaluation criterion given a data set. All things considered, the current study recommends the utilization of the three information criteria for model determination to keep away from the issue of predispositions. The SBC measure is given by

$$\mathrm{SBC} = \mathrm{k}\ \ln(\mathrm{n}) - 2\ln(\hat{\mathrm{L}}) \tag{4}$$

where n is the sample size and k is the estimated number of parameters, while the likelihood function of the estimated model M is denoted by $\hat{\theta}$, which is $\hat{\theta} = \mathrm{p}(\mathrm{x}|\hat{\omega}, \mathrm{M})$, x is the observed data and $\boldsymbol{\omega}$ is the parameter of the estimated model. Therefore, the AIC is given by

$$\mathrm{AIC} = -2\mathrm{L}(\beta; \mathrm{D}) + 2\mathrm{p} \tag{5}$$

with L being the dimension of β and D the data series. We accomplish model selection by selecting from M models the one that minimizes the AIC. According to Ismail and Isa (2007), the likelihood test is represented in the following mathematical form:

$$\mathrm{LR} = |\ln \mathrm{L}_{\mathrm{MSARA}} - \mathrm{L}_{\mathrm{AR}}| \tag{6}$$

3.3 Unsupervised machine learning for identifying regime shifts

The ML approach used in this study follows a procedure similar to that used in the single-phase method. We use an 80:20 proportion for the training and test data. For this situation, we perform principal component analysis on the training data to reduce the dimensionality of the data and identify the number of regime shifts in the RER of SA. This method includes performing a symmetrical linear change to another feature space of equivalent or lesser dimension with the end goal that the principal components creating the projections onto the new feature space ensure the biggest clarified difference between the samples under the condition that the features are non-linearly independent. We ordered the principal components by their explained variance ratios. These smaller features are easier to analyse and allow a simple illustration of the data. Before performing NPC reduction, it is essential to scale the features in order to prevent inappropriate domination of a subset of the features over others. In this empirical analysis, we utilize scaling with the end goal that each of the features shares a common domain ranging from zero to one.

The mathematical procedure is as follows. Let $\mathrm{Y} = (\mathrm{y}_1, \cdots, \mathrm{y}_\mathrm{p})$ be a data matrix of n objects by p numerical variables and each column of Y be standardized, i.e. $y_i^T\ 1 = 0$ and $y_i^T y_i/n = 1$ for $i = 1, \cdots, p$, where 1_n is an $n-1$ vector of ones. Principal component analysis linearly transforms Y of p variables into a substantially smaller set of uncorrelated variables that contains much of the information of the original data set. Then PCA simplifies the description of Y and reveals the structure of Y and the variables. The principal component analysis postulates that Y is approximated by

$$\widehat{\mathrm{Y}} = \mathrm{ZA}^{\mathrm{T}} \tag{7}$$

where Z is an $\mathrm{n\,x\,r}$ matrix of n component scores on $\mathrm{r}(1 \leq \mathrm{r} \leq \mathrm{p})$ components and A is a $\mathrm{p\,x\,r}$ weight matrix that gives the coefficients of linear combination. Therefore, the following loss function formulates our PCA:

$$\sigma(\mathrm{Z}, \mathrm{A}) = \mathrm{tr}\left(\mathrm{Y} - \widehat{\mathrm{Y}}\right)^{\mathrm{T}}\left(\mathrm{Y} - \widehat{\mathrm{Y}}\right) = \mathrm{tr}(\mathrm{Y} - \mathrm{ZA}^{\mathrm{T}})^{\mathrm{T}}(\mathrm{Y} - \mathrm{ZA}^{\mathrm{T}}). \tag{8}$$

The minimum of a loss function (8) over Z and A is found by the eigen-decomposition of $\mathrm{Y}^{\mathrm{T}}\,\mathrm{Y}/\mathrm{n}$ or the singular value decomposition of Y.

3.4 Markov-switching generalized autoregressive conditional heteroscedasticity model

Using the residuals of the NPCA, we fit the MS-GARCH. To examine the behaviour of conditional volatility of exchange rates in SA, this study incorporates the dynamic features of the MS-GARCH and uses Gregoriou and Pascalau (2010)'s framework. As in the work of Klaassen (2002), we suppose the RERs to be X_t; and their logarithmic returns are computed by $\mathrm{R}_{\mathrm{t}} = 100(\ln(X_t) - \ln(X_{t-1})/\ln(X_{t-1}))$. Here, R_{t} constitutes the depreciation percentage of the RER between $t-1$ and t. The MS-GARCH model nevertheless involves four components: mean, regime process, variance, and distribution. The last is critical for decryption of the empirical results, since the differences between these models and the common one-regime GARCH model are directly associated with them. Ideally, a random walk with drift usually models the mean process of an econometric or financial time series. For example, in a short period, Mills and Markellos (2008) and Taylor (2007) showed that

$$\mathrm{X}_{\mathrm{t}} = \mu + \varepsilon_{\mathrm{t}} \tag{9}$$

where ε_{t} is the error term that has a mean of zero, and unit variance, while μ is the conditional mean of X_{t}. According to Billio et al. (2018), it is possible to include autoregressive terms in the conditional mean.

Based on Klaassen's argument, the goal of regimes is to clarify persistence of volatility, which means that regimes can remain persistent according to Haas et al. (2004). However, to model this persistence, the unobserved variance regime at time t should be $\mathrm{S}_{\mathrm{t}} \in \{1,2\}$ and the first regime is identified as a low-variance regime. Let $\mathrm{P}_{\mathrm{t-1}}\left(\mathrm{S}_{\mathrm{t}}\middle|\tilde{\mathrm{S}}_{\mathrm{t-1}}\right) = \mathrm{P}\left(\mathrm{S}_{\mathrm{t}}\middle|\mathrm{I}_{\mathrm{t-1}},\tilde{\mathrm{S}}_{\mathrm{t-1}}\right)$. Let $\mathrm{P}_{\mathrm{t-1}}\left(\mathrm{S}_{\mathrm{t}}\middle|\tilde{\mathrm{S}}_{\mathrm{t-1}}\right) = \mathrm{P}\left(\mathrm{S}_{\mathrm{t}}\middle|\mathrm{I}_{\mathrm{t-1}},\tilde{\mathrm{S}}_{\mathrm{t-1}}\right)$ be a likelihood of change to regime S_{t} at time t. Ardia et al. (2018) denoted observed information by $(\mathrm{X}_{\mathrm{t-1}}, \mathrm{X}_{\mathrm{t-2}}, \ldots)$ and Camacho et al. (2018) showed that S_{t} follows the first Markov process with the following transition probabilities:

$$\mathrm{p}_{\mathrm{ij}} = \Pr(\mathrm{S}_{\mathrm{t}} = \mathrm{j}|\mathrm{S}_{\mathrm{t-1}} = \mathrm{i}) = \begin{cases} \mathrm{P}_{11} & \text{if } \ \mathrm{S}_{\mathrm{t}} = \mathrm{S}_{\mathrm{t-1}} = 0 \\ \mathrm{P}_{22} & \text{if } \ \mathrm{S}_{\mathrm{t}} = \mathrm{S}_{\mathrm{t-1}} = 1 \end{cases}. \tag{10}$$

Using the law of iterated expectations and model (10), Hamilton and Raj (2013) derived the conditional variance $\sigma^2_{t-1}\{S_t\} = E_{t-1}\left[\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\}\right]$, so that researchers could concentrate on $\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\}$. For the sake of exposition, this study is confined to a model with only one ARCH (referenced herein as autoregressive conditional heteroscedasticity) and one GARCH term, as in Brunetti et al. (2008).

Nevertheless, there are three specifications followed when building the MS-GARCH model. The first specification is to apply the GARCH(1,1) model and this is specified in a regime-switching context as

$$\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\} = \omega_{Xt} + \alpha_{Xt}\varepsilon^2_{t-1} + \beta_{Xt}\sigma^2_{t-2}\{\varepsilon_{t-1}|\tilde{S}_{t-1}\} \quad (11)$$

where $\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\}$ is the conditional variance of ε_t with the following observable information I_{t-1}. This variance has the regime path on $\tilde{S}_t$. The parameters ω_{Xt}, α_{Xt}, and β_{Xt} represent an intercept, ARCH, and GARCH, and they are being determined by the current regime.

Salisu and Fasanya (2013) discovered that the specification in model (11) is practically infeasible because $\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\}$ is governed by the entire $\tilde{S}_t$ such that it depends on $\tilde{S}_t$ and $\sigma^2_{t-2}\{\varepsilon_{t-1}|\tilde{S}_{t-1}\}$. This led to several unlimited paths, which makes it difficult to estimate the model (Klaassen 2002).

According to Salisu and Fasanya (2013), the second specification is based on Hamilton and Susmel (1994). Specifically, Hamilton removed the GARCH term as a source of path dependence and therefore used only the ARCH term. An essential point is that the conditional variance depends on only the small number of regimes that can be included in the model.

Ardia et al. (2019) derived a third specification. These authors argue that the path dependency issue is often resolved by not neglecting the necessary persistence effects of the second GARCH term. It is therefore important to integrate a non-observed path of regime $\tilde{S}_{t-1}$ into the source of path dependency $\sigma^2_{t-2}\{\varepsilon_{t-1}|\tilde{S}_{t-1}\}$ in model (11). According to Ardia et al. (2018), this depends only on the current regime S_t and not on the regime path $\tilde{S}_t$. The likelihood is computed using a first-order recursive method that considerably speeds up the estimation process. As a result, the observable information at $t-2$ is used, and Billio et al. (2016) assumed that

$$\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\} = \omega_{Xt} + \alpha_{Xt}\varepsilon^2_{t-1} + \beta_{Xt}E_{t-2}\left[\sigma^2_{t-2}\{\varepsilon_{t-1}|\tilde{S}_{t-1}\}\right]. \quad (12)$$

An expectation can be across $\tilde{S}_{t-1}$, which is conditional on I_{t-1}. The main advantage of model (12) is that path dependence is not adverse; GARCH effects are still permissible. It is necessary to create a multi-period-ahead variance forecast such as $\sigma^2_{t-1}\{S_{t+1}\}$, particularly when it comes to volatility forecasting, because forecasts are very complex (Haas et al. 2004). This stimulates the present study to extend the work of Augustyniak et al. (2018) by pursuing one more design that incorporates one-step-ahead forecasting, which is more advantageous in preserving attractive features of the Gray (1996) model. Instead, the regime-switching GARCH (1,1) model is defined as

$$\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\} = \omega_{Xt} + \alpha_{Xt}\varepsilon^2_{t-1} + \beta_{Xt}E_{t-2}\left[\sigma^2_{t-2}\{\varepsilon_{t-1}|\tilde{S}_{t-1}\}|S_t\right] \quad (13)$$

where the expectation on the right-hand side is across the regime path $\tilde{S}_{t-1}$ and is conditional on information I_{t-1} and S_t. To ensure the positivity of $\sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\}$ at time t, Klaassen (2002) enforced $\omega_{Xt} > 0$ and $\alpha_{Xt}, \beta_{Xt} \geq 0$ just as for the single regime GARCH model. The conditional distribution is the last component of the regime-switching GARCH model. The assumption is that I_{t-1}, $\tilde{S}_{t-1}$and ε_t has a t-distribution with $ć$ degrees of freedom, in which $ć$ is presumed to remain unconditioned with mean zero and $\sigma^2_{t-1}\{\varepsilon_t|S_t\}$, i.e. with mean zero and variance

$$\varepsilon_t|I_{t-1},\tilde{S}_t \sim t\left(v, 0, \sigma^2_{t-1}\{\varepsilon_t|\tilde{S}_t\}\right). \quad (14)$$

The use of a t-distribution rather than a normal distribution is quite popular in the standard single-regime GARCH literature (see Bollerslev 1987). For regime-switching models, a t-distribution is further helpful since the regime-switching can account for large unconditional kurtosis and constancy of regimes. Note that a t-distribution includes the conventional normal distribution as a limiting case where the degrees of freedom go to infinity. We therefore use a skewed distribution. And a simple way to introduce skewness into any unimodal standardized distribution, via the additional parameter, is provided by $\xi > 0$, and if $x_i = 1$, the distribution turns out to be symmetric. For the current study, the skewed distributions are only fitted to the returns if the preliminary analysis reveals that the exchange rates are skewed. Furthermore, Chikobvu and Chifurira (2015), Chinhamu et al. (2017), and Huang et al. (2014) have shown that financial time series are skewed, and mostly to the left; hence the skewed distributions are considered for this study. The standardized Fernandez-Steel skewed distributions necessary for the estimation of the MS-GARCH model were derived by Trottier and Ardia (2016). However, Castillo et al. (2011) defined a skewed density distribution by using the approach of Fernández and Steel (1998) through re-parameterization to ensure that the distribution has zero mean and unit variance. Hence, as shown by Trottier and Ardia (2016), the resulting density is written as

$$f_\xi(z) \equiv \frac{2\sigma_\xi}{\xi+\xi^{-1}} f_1(z_\xi), \;\; z_\xi \equiv f(x) = \begin{cases} \xi^{-1}(\sigma_\xi z + \mu_\xi), \; if z \geq -\mu_\xi/\sigma_\xi \\ \xi(\sigma_\xi z + \mu_\xi), \; if z < -\mu_\xi/\sigma_\xi \end{cases}. \tag{15}$$

Here,

$$\mu_\xi \equiv M_1(\xi - \xi^{-1}), \;\; \sigma_\xi^2 \equiv (1 - M_1^2) + 2M_1^2 - 1, \;\; M_1 \equiv 2\int_0^\infty (u)du. \tag{16}$$

Note that $0 < \xi < \infty$, is the parameter describing the degree of asymmetry and $f_1(.)$ can be any symmetric unimodal density with $\mu_t = 0$ and$\sigma_t^2 = 1$. In general, the asymmetry of the central part of a distribution is controlled mainly by a skewness parameter. Consequently, a class that has one skewness parameter and two tail parameters for generalized asymmetric Student's t-distribution offers an opportunity to improve capacity fit and forecast empirical data in the tail regions, which are critical to risk management and other financial econometric applications.

3.5 In-sample goodness-of-fit for MS-GARCH model

In the literature, there are different tests of fitness. The chi-square goodness-of-fit (GoF) test is the most common and is generally used for large-sample data. The idea of the GoF test is to see whether the sample comes from the population with the claimed distribution. A likelihood test compares the model with a more complex one for detecting a lack of fit. Agresti (2018) suggests that a more complex model could have a non-linear impact such as a quadratic term, which would allow the impact of a predictor to change direction with an increase in value. This gives some assurance that a selected model is adequate when a more complex model does not fit better.

This section discusses an in-sample GoF test for the proposed asymmetric regime-switching models. The study follows Raihan (2017) and uses the Anderson-Darling (A-D) goodness-of-fit test, Shapiro-Wilk (S-W) test, and Jarque-Bera (J-B) test. The GoF tests that are proposed in this study are estimated to confirm that the empirical distribution of the exchange rates comes from a heavy-tailed distribution, as has been discovered in previous studies. Moreover, these tests are used to prove that the skewed distribution used to estimate the specified asymmetric regime-switching GARCH family models in this study is the correct distribution. Box et al. (2015) emphasized the shortcomings of wrong conclusions drawn from fitted models. Among the most relevant are heteroscedasticity or serial correlation of the error terms, fundamental changes in the backslide coefficients, non-linearities, utilitarian misspecification, and overlooked components. Some of these disadvantages are believed to be basic, especially in applied econometrics. They may even incite un-interpretable outcomes. Since the model with serially correlated residuals has some issue of least variance among the estimators, Maas and Hox (2004) contend that it is ideal to test for the relationships as they carry inclination into the standard variance estimates.

Jarque-Bera test

If a given sample X_s comes from a normal distribution and the empirical distribution of the estimated model is correctly specified, the current study uses the Jarque-Bera (J-B) test to perform a test of normality on the sample. With a sample of more than 50 observations the test is said to perform much better than with a sample of fewer than 50 observations. Therefore, the J-B test is calculated by

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \sim \chi^2, df \tag{17}$$

where S is the sample skewness, K is the sample kurtosis, n is the sample size, and df is the degree of freedom. For this study the null and alternative hypotheses are

$$H_0: E(X_s) = 0$$
$$H_a: E(X_s) \neq 0^{\cdot}$$

Makatjane and Moroke (2016) declared that the null hypothesis is rejected if the calculated probability of the J-B statistic is less than the observed probability or if the calculated J-B statistic is greater than the critical value obtained from chi-square distribution with two degrees of freedom.

Serial correlation

While the Durbin-Watson test is formulated with the $AR(1)$ alternative hypothesis, it should have some power in detecting other forms of serial correlation provided by $E[e_t e_{t-1}] \neq 0$ under the alternative hypothesis. Still, there are more powerful tests for high-order serial correlation that involve high-order autocorrelation estimators. Suppose the error terms are $AR(p)$ for $p > 1$. To prove that the estimated asymmetry regime-switching GARCH models used in this study have highly correlated residuals, implying some non-linear patterns, the Ljung-Box test is used. This is used to test whether there exists a group of significant autocorrelations in the residuals of the estimated model.

To test the remaining GARCH effects, Mokoena (2016) revealed that the Ljung-Box Q statistic is formed for squared residuals and the test proposed by Ljung-Box is

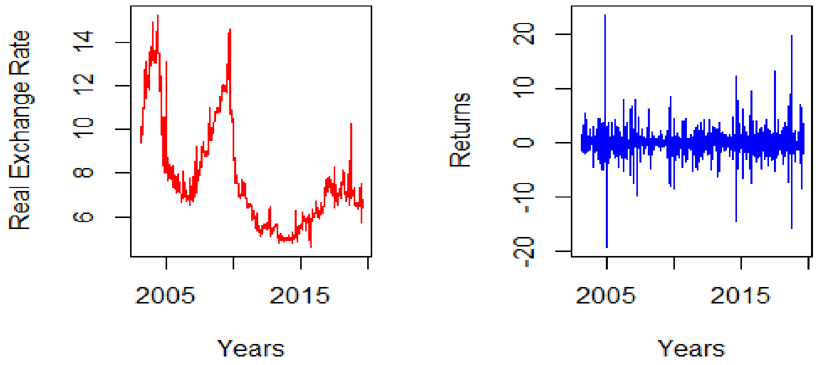
$$Q_m = n(n+2)\sum_{k=1}^{p}\frac{\rho_k^2}{n-k} \sim \chi^2_{\alpha,}, m-p. \tag{18}$$

Here, n is the sample size and k and p are the numbers of the estimated model parameters. The null hypothesis is not rejected if the calculated probability is greater than the observed probability value, implying that the estimated residuals are not correlated to each other over time and also there are no ARCH effects on the estimated model.

4 Empirical analysis

This empirical analysis is based on the real exchange rates of SA for the period January 2003 to June 2020. Figure 1 displays a time series plot and logarithmic exchange rate returns. As can be seen, the series in the left panel displays some upward and descending patterns related to seasonality. This is confirmed by the logarithmic returns shown in the right panel because the series shows volatile patterns. The descriptive statistics indicate that the series violates the normality assumption. This is evident in Table 1, as the reported kurtosis is greater than 3 and the skewness is greater than 0. The same result, of the kurtosis being greater than 3, is evidenced by Ray (2012). So the descriptive statistics show that all the values are not normally distributed about its mean and variance; in other words, we see no randomness in the data. So, the results of descriptive statistics raise the issue of the inefficiency of the RER market in SA.

Figure 1: Real exchange rate series



Source: authors' construction.

Table 1: Exploratory data analysis

Statistic	Exchange rates	p-values
Kurtosis	15.01045	0.0002
Skewness	0.6778014	0.0002

Source: authors' construction.

Having established that the exchange rate series is leptokurtic and asymmetric, it is equally important to investigate whether the series is non-linear. Referring to the results in Table 2, the null hypothesis of linearity is rejected at the 5 per cent level of significance, supporting the findings reported in Table 1. This proves that the distribution tails of the exchange rate are heavier than Gaussian. Bonga-Bonga and Makakabule (2010) in a similar study found that the all share index time series they used are non-linear. Similar studies by Xaba et al. (2015) and Brock et al. (2001) utilized a Brock-Dechert-Scheinkman (BDS) test and reported parallel results. The Lagrange Multiplier test in Table 2 further confirms the findings.

Table 2: LM test for non-linearity

LM test	F-Test	Sign.
Exchange rate	451.7496	***

Sign. Codes: *** 0.001 ** 0.01 * 0.05

Source: authors' construction.

4.1 Regime shifts in the real exchange rate

Since our study aims to identify the structural changes in the real exchange rates in a regime-switching atmosphere, we first verify whether the real exchange rates display regime-switching behaviour. For this purpose, we proceed to test the null hypothesis of no regime shift (i.e. the dynamics of the exchange rate are better reproduced via a linear AR model than by a regime-switching model thatcorresponds to an MS-GARCH model). A likelihood ratio test was used by Chkili and Nguyen (2014) to make the final choice of suitable modelling approaches. However, in this study, we introduce a new approach to test the null hypothesis of no regime shifts and use the non-linear principal competent of Mori et al. (2016). Using the Fronius norm, we obtain two principal components, with the total variability explained by the two principal components as 96.38 per cent (Table 3). With these results, the null hypothesis is rejected, and we conclude that the behaviour of the exchange rates in SA is better explained by a proposed supervised ML model (akin to the Markov-switching generalized autoregressive conditional heteroscedasticity model). Previous empirical studies, including Xaba et al. (2017) and Chkili and Nguyen (2014), found parallel results when utilizing the likelihood ratio test. From a theoretical perspective, this behaviour is normal and can be clarified by the changing economic structure in the exchange rate of SA, attributable to structural economic reform policies (financial liberalization, tax system adjustments, competition policy), as well as to progressive economical and financial crises at both local and worldwide levels.

Table 3: Principal component analysis

	PC1	**PC2**
Exchange rate	-0.7155	-0.6987
Time	-0.6987	0.7555
Standard deviation	0.9480	0.9748
Cumulative proportion	0.9528	0.9748

Source: authors' construction.

4.2 Markov-switching GARCH framework

Prior to estimation of the $MS(\mathrm{k}) - GARCH(\mathrm{p};\ \mathrm{q})$ model, the information criterion discussed in the previous section is used to identify the number of GARCH to be estimated in the MS-GARCH model. As reported in Table 3, the SBC and LR select the optimal lag length as 1. According to Tsay (2014), the final optimal lag length should be selected by the SBC. Xaba et al. (2017) acknowledged that with a large sample size, say $n \geq 30$, both the SBC and Hannan-Quinn perform much better in an optimal lag length selection. With this evidence, in this study the selected optimal lag length is 1 (Table 4).

Table 4: Model selection

Lag length	**AIC**	**SBC**	**LR**
1	-361.000	-461.090	335.5334
2	-340.657	-430.677	310.3736
3	-346.580	-436.180	273.0962
4	-335.521	-345.421	225.4341
5	-339.482	-309.082	160.4418

Source: authors' construction.

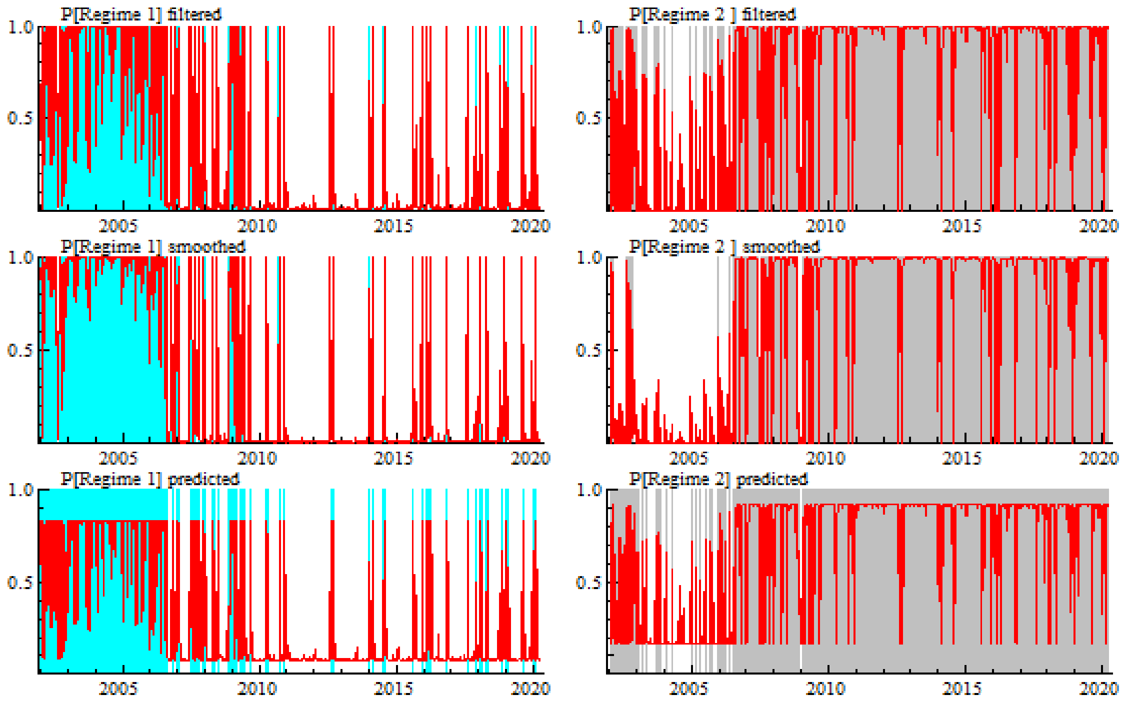
Because we have found that the RER returns are positively skewed, as reported in Table 1, we classify the two regimes as a Markov-switching by fitting a skewed Student's t-distribution with a Markov-switching GARC. Using the maximum likelihood estimator and Nelder-Mead parameter optimizer, we find that all the regimes are independent of each other, giving a classification rate of 75 per cent in regime 1 (high-volatility regime) and 86 per cent in regime 2. This finally produces a total number of 281 structural breaks in regime 1 and 666 structural breaks in regime 2, confirming the results of the principal components in Table 3, which indicated that the first PCA has lower variances than second principal component. The same results are confirmed in Figure 2. Furthermore, the regime-switching probabilities given by

$$P(S_t = 0|S_{t-1} = 0) = 0.9771$$
$$P(S_t = 1|S_{t-1} = 1) = 0.9932$$

suggesting that the probability of exchange rate in low regime is lower than that of high exchange regime by about 1.61 per cent. This indicates regime persistence (Raihan 2017).

The smoothed probability of staying in regime 1, which is displayed in the middle of Figure 2, shows several high-volatility periods that are common throughout the sample period.

Figure 2: Number of identified structural breaks



Source: authors' construction.

The first breakpoint occurred in June 2003. According to Makatjane and Molefe (2020), this was due to the invasion of Iraq, after which the price of oil weakened, causing a depreciation in exchange rates, especially in SA. However, Bonga-Bonga and Kabundi (2015) further indicate that the SARB had been going ahead with a contractionary policy. The monetary policy committee in SA had increased the repo rate by 4 percentage points, from 7 per cent to 11 per cent. In 2007, around the time of the filing of Chapter 11 bankruptcy by Lehman Brothers, the London Stock Exchange (LSE) leased yet another trading platform to the Johannesburg stock exchange (JSE), i.e. the JSE TradeElect system (Phiri 2020). Nevertheless, there are six factors that are believed might be the main cause of the 2007 crisis: (i) the repeal of Glass-Steagall act in 1999 by the Clinton administration[1]; (ii) the flood of sub-prime mortgages as a reaction to a significant level of housing speculation and the build-up of a bubble; (iii) the Depository Institutions Deregulation and Monetary Control Act of 1980; (iv) the establishment of new-fangled financial instruments that were difficult to weigh up, which shifted accountability between agents; (v) a decline in the RER

[1] One of the biggest post-depression pieces of legislation, separating commercial banks and investment banks (Wallison 2011).

combined with the Federal Reserve's expansionary monetary policy; and (vi) global financial imbalances.

According to Moroke et al. (2014), during the 2007–09 financial crisis assets were consigned and organizations conserved people, causing a rapid increase in the unemployment rate and the general debilitating of financial development. Moreover, intermediate exchange rate regimes promote flows of goods between countries and the results that depend on the anchor currency and indirect arrangements do not have any significant impact on international trade, although systemic banking crises negatively affect trade flows between countries; and the impact of the exchange rate regimes on trade during the crisis depends on the anchor currency and whether the crisis takes place in the exporting or the importing country. There is much evidence that the effects of this crisis are still being felt, and in many ways the crisis is still ongoing. For its part, the exchange rate of the ZAR was particularly high during the period covering the US sub-prime crisis and global financial crisis. This increase in exchange rate volatility can be explained by the relative importance of the United States, as the main trade partner, towards the South African economy, as found by Bildirici and Ersin (2018) and Chkili and Nguyen (2014).

Table 5: MS(2)-GARCH(1,1) for real exchange rate

Regime 1				
coefficient	**Estimate**	**Std error**	**t-value**	**p-value**
δ	0.0058	0.001	0.06	0.000
γ	0.0194	0.504	0.05	0.000
α	0.0676	0.003	0.55	0.000
β	0.9281	0.066	0.65	0.000
μ	13.1989	0.055	0.56	0.000
Regime 2				
δ	0.000	0.007	0.061	0.000
γ	0.0001	0.000	0.045	0.000
α	0.0001	0.096	0.087	0.000
β	0.9998	0.010	0.071	0.000
μ	7.1056	0.050	0.023	0.000
Transition matrix				
	$\mathbf{P_{11}}$	0.9771	$\mathbf{P_{12}}$	0.0229
	$\mathbf{P_{21}}$	0.0068	$\mathbf{P_{22}}$	0.9771

Source: authors' construction.

4.3 In-sample goodness-of-fit

Regardless of anything else, it must be stressed that testing the null hypothesis of a linear model or single regime model against a regime-switching model is a non-trivial endeavour. The intricacy generally develops on the grounds that standard likelihood-based acceptance is invalid, since transition probabilities linger as unidentified parameters under the null hypothesis (Gonzalez et al. 2017). This results in a probability proportion whose asymptotic dissemination is not a standard chi-square any longer and therefore may prompt misleading conclusions (Klaassen 2002). When estimating the asymmetric MS-GARCH model, a skewed *t*-distribution is used to capture the potential fat-tailed performance of the theoretical distribution of closing stock price returns. To confirm that skewed *t*-distribution is correctly specified in this study, GoF tests known as the J-B, A-D, and S-W are used. The study tests whether the null hypothesis of the estimated empirical distribution of the returns is a standard normal distribution against the alternative of the estimated empirical distribution of the returns being of other than normal distribution. The results of the study, tabulated in Table 6, suggest that the empirical distribution fitted to the MS-GARCH family model is a skewed *t*-distribution. The null hypothesis is rejected for the three GoF tests because all the calculated probability values are less than the observed probability of 5 per cent. Therefore, the conclusion is that the empirical distribution of a skewed *t*-distribution was correctly specified. This is confirmed by the results shown in Table 1, where the skewness coefficient indicated a skewed distribution. The GoF test also revealed non-normality in returns series.

Table 6: In-sample goodness-of-fit tests

Tests	A-D	S-W	J-B	LM	Q Test	BDS
Statistic	0.028	0.96	0.269	0.970	2.266	0.498
P-value	0.001	0.008	0.006	0.123	0.007	0.751

Source: authors' construction.

Furthermore, this study follows Mokoena's (2016) work and uses the LM test to see if there are still ARCH effects, and subsequently the Ljung-Q test. The results of the LM test for all the three models, as tabulated in Table 6, fail to reject the null hypothesis of no ARCH effects in the residuals of the estimated models. The conclusion made is that the estimated MS-GARCH is correctly specified and the ARCH and GARCH effects are perfectly captured. These results are also seen in Mokoena's work, where he used different single-regime GARCH models to test their ability to forecast stock volatility.

In contrast to the results obtained by Raihan (2017) and Mokoena (2016), however, in the current study, when testing for the presence of the serial correlation in the error term of the models, the null hypothesis of no serial correlation was rejected in favour of the alternative. Hence, the conclusion is that the residuals are correlated with each other over the sampled period. This gives more evidence that exchange rates are highly correlated with each other over time. It is worth noting that Mokoena and Raihan used standardized residuals in their tests. However, this study did not use standardized residuals because standardizing the residuals means that some transformation is made so that they give best linear unbiased estimates (BULE), and this contradicts non-linear modelling and its theories (see, for instance, Tsay 1986).

The BDS test was also used to test the null hypothesis of i.i.d. residuals of the three models. As the null hypothesis was not rejected, the BDS test confirmed that all three estimated models have residuals that are independent and are identically distributed. Moreover, seven statistical loss functions were used to select the best performing model in modelling the volatility of stock returns for the five banks. All the statistical loss functions led to inconclusive results. Because the focus here is to model volatility of stock returns with asymmetric regime-switching GARCH models and assess their prediction and forecasting ability, the study follows the work of Raihan (2017) and Mokoena (2016).

4.4 Evaluation of the classification experiment

Using the validation data, we were able to establish the NPC and MS (2)-GARCH (1,1) models where 2 is the number of identified regimes when estimating the non-linear principal component; and the first 1 represents the GARCH order while the second is the ARCH order of the model. To lessen variability, we further use a cross-validation approach of ten-fold. This approach partitions the training data into ten subsets and averages validation effects over ten rounds. The ten-fold cross-validation method was used for the MS (2)-GARCH (1,1). Table 7 presents the results of the predictive accuracy (ACC) and AUC. The NPCA has the highest predictive accuracy. This indicates that the model has accurately predicted the number of regimes. The MS(2)-GARCH

(1,1) accurately identified multiple breakpoints in each regime. We conclude that our proposed procedures have adequately identified the structural changes as regime processes.

Table 7: Performance of validation data

Parameters	NPCA	MS (2) – GARCH (1,1)
ACC	0.999	0.962
AUC	0.987	0.952

Source: authors' construction.

5 Conclusion and recommendations

Non-linear models have become a useful tool in modelling economic relationships given the experience gained and conclusions drawn from the 2007–09 financial crisis and the Great Recession. The main aim of this work lies in estimating and identifying structural changes in the real exchange rate of South Africa as a regime shift process. There are few studies that aim to identify structural changes as a regime shift process and, to our knowledge, this is the first use of such an approach. By performing the non-linear principal component and GARCH model subject to two regimes, we can better model the structural co-movement for this geographically diverse study area. The model also enables the capturing of regime shifts caused by structural breaks during times of financial crisis or vulnerability. This is useful when determining the economic conditions of the macro-economy, avoiding any policy uncertainties that may occur.

The results show that unsupervised ML methods can be used to both isolate the statistically relevant data in structural information and cluster those data into groups that represent transition points in exchange rates. The results of the Markov-switching model gave regime independence with a total number of 281 structural breaks in regime 1 and 666 structural breaks in regime 2, with a classification rate of 86 per cent and 75 per cent, respectively. The location of a sample along the first principal axis is also very well correlated with time series data and the partitions made by the second regime of the MSM. These features of the unsupervised ML approach to detect structural changes show that this method is promising for supervised classification of exchange rate structural change systems. The ranges constructed using the standard deviations of the exchange rates, though rather large, did indeed capture the true breaking points of the potentials. Various maximum volatilities beyond the expected breaking point did not affect the results with any statistical significance.

In this regard, policy-makers would benefit from the findings in both policy implementation and the economic planning process, as the estimated model can be used as an early warning system for fluctuations/volatility in the exchange rates in South Africa and the Monetary Policy Committee (MPC) can guard against the negative effects of highly volatile exchange rates.

This study also provides practical information to the SARB and decision-makers about the effects of high exchange rates and consumer behaviour policies. In this regard, decision-makers would benefit from the findings for their policy implementations and in the economic planning process. For instance, they may take the determinants of exchange rate volatility into account in policy designs. Furthermore, the MPC may implement policies aimed at protecting the economy by considering these structural changes, as they may depreciate the exchange rates and increase the inflation rate from the target margin of 3–6 per cent.

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