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The impact of the Employment Equity Act on female inter-industry labour mobility and the gender wage gap in South Africa

Mattie Susan Landman[1] and Neave O'Clery[1,2]

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Abstract: The Employment Equity Act No. 55 of 1998 was introduced by the South African government to address the legacy of apartheid and ensure equitable representation of black people and women in the South African labour market. Although the impacts of the Act are highly controversial, its widespread adoption among firms opens up questions on its impact on the structure of the South African labour market. This study primarily focuses on determining the impact of the Act on female inter-industry labour mobility and the gender wage gap. Using a regression discontinuity design, we show that as a firm becomes compliant with the Act, this increases the diversity of sectors from which the firm hires new female workers (the female inflow diversity), and increases the firm's average female wage. We also find that the more male-dominant an industry is, the higher its female inflow diversity and the smaller its gender wage gap. This relationship is significantly stronger among the group of firms that comply with the Act compared to those that are exempt. These results suggest that firms that comply with the Act, and particularly those in male-dominant industries, have adopted the following two recruitment strategies in order to feminize their workforce: they have diversified recruitment to a larger number of sectors, and they have increased the average female wage.

Key words: Employment Equity Act, gender wage gap, industry networks, labour dynamics

JEL classification: D04, J24

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[1] Mathematical Institute, University of Oxford, Oxford, UK; [2] Centre of Advanced Spatial Analysis, University College London, London, UK; corresponding author: landman@maths.ox.ac.uk

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Katajanokanlaituri 6 B, 00160 Helsinki, Finland

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1 Introduction

The South African government originally introduced the Employment Equity (EE) Act No. 55 of 1998 to rectify labour market inequalities caused by the apartheid regime. The Act's primary aim is to ensure fair representation of black people,[1] women, and people with disabilities in all sectors, occupations, and levels of the workforce through implementing affirmative action (Burger and Jafta 2010). The Act's impact is controversial among both the public and policy makers (Mzilikazi 2016). It has been criticized for creating a brain drain (Horwitz 2013), causing a greater skill mismatch (Dongwana 2016), and reducing the productivity of the workforce (Burger 2014; Kruger and Kleynhans 2014). Advocates of the policy, however, argue that the Act is vital to reverse the self-reinforcing inequality caused by the structure of the labour market (Visagie 1999). However, limited quantitative academic research has investigated the impact of the Act on the structure of the labour market (Horwitz and Jain 2011). Within this study, we are interested in determining how the Act has impacted female labour mobility and the gender wage gap.

The South African labour market has become feminized,[2] with an increase in the female share of employment from about 38 per cent in 1994 to about 44 per cent in 2018 (Trading Economics 2019). This follows the global trend of a greater female presence in the labour market, credited to lower marriage rates and changing household structures (Casale 2004). In South Africa, this increase has also been influenced by an increase in the average level of female education and a decrease in gender biases within the labour market. The EE Act has helped to reduce female discrimination and increased female labour market participation. In this study, we hypothesize that the Act has also caused firms to diversify their recruitment of female workers to a larger number of sectors and increase their average female wage as a recruitment strategy to increase female representation within their workforces.

Our approach is twofold. First, we investigate the impact of the Act on a firm's female labour inflow diversity and its gender wage gap. A firm's female labour inflow diversity is the diversity of sectors from which the firm hires new female workers. Second, we investigate whether male-dominant industries have been more heavily impacted by the Act as they require the largest workforce restructuring to attain the Act's required gender workforce representation. If so, they will display an even greater female labour inflow diversity and a smaller gender wage gap.

The analysis consists of constructing three networks: the inter-firm labour flow network, the inter-industry labour flow network, and the skill-relatedness network. The first two networks count the number of worker transitions between either firms or industries. They are used to investigate the structure of the labour market, first at a firm level and then at an industry level. The skill-relatedness network is a normalized inter-industry labour flow network that is constructed to measure the skill and knowledge overlap between industries and construct a new industry classification.

To measure the female inflow diversity of a firm, one wishes to count the number of workers a firm hires from different sectors. In this study, we define a sector as a group of industries that share a large number of skills and knowledge, and can be thought of as labour pools. However, as the industries in the South African industry classification are not equally skill-distant from one another, and industries within the same aggregation level are defined at different levels of detail, we cannot use a higher aggregate level in this classification to identify sectors. Therefore, we construct a new industry classification that groups industries according to their skill overlap. This consists of constructing and clustering the skill-relatedness network (Neffke and Henning 2013; O'Clery et al. 2019). The resulting partition is used as a new higher-level industry classification.

[1] 'Black people' includes all African, Coloured, and Indian people, as well as people of Chinese descent.

[2] A feminized labour force refers to the rapid and substantial increase in the proportion of women in paid employment.

To evaluate the impact of the Act on a firm's female labour flow diversity and female average wage, we adopt a regression discontinuity (RD) design. Our analysis exploits the clear cutoff created by the Act's adoption into the legislation that enforces all firms with 50 or more employees to comply with the Act. We therefore compare firms with slightly fewer than 50 employees, who are exempt from the Act, to those with slightly more than 50 employees, who must comply with the Act. The RD results reveal that the Act increases a firm's female labour flow diversity and a firm's female average wage. The corresponding impact for men was found to be negligible.

We further investigate whether the Act's impact differs between industries. We evaluate the relationship between the percentage of male employment within an industry and the industry's female labour inflow diversity and its gender wage gap. We find that male-dominant industries have a higher female labour inflow diversity and a smaller gender wage gap. This relationship is found to be significantly stronger among the group of firms who comply with the Act, compared to the group of firms exempt from the Act. Once again, no significant relationship is found for the male labour inflow diversity. These results suggest that male-dominant industries are more heavily impacted by the Act as they show greater adoption of the recruitment strategies used to restructure their workforce and increase female presence within their workforce.

In the next section we review the related literature on the EE Act, labour mobility, and labour networks. This is followed, in Section 3, with a discussion of the data used in this study. Next, the methodology adopted is presented in Section 4. This includes the construction of various networks, the quantification of inflow diversity, and the layout of our RD design and regression models. In Section 5 our results are presented, and finally, in Section 6, the conclusion of our study is discussed, along with potential avenues for future work.

2 Literature review

In this section, the related literature regarding the EE Act, labour mobility, and labour networks is reviewed.

2.1 The Employment Equity Act

The apartheid regime caused a high level of inequality within the South African labour market by systematically and purposefully restricting the majority of South Africans from economic and social opportunities. Access to skills, formal jobs, and self-employment was racially restricted. An inferior education system further divided the skills and positions obtained by various groups within the labour market (Burger and Jafta 2010). This has had a large effect on income distribution and the gender and racial representation within sectors, occupations, and workforce levels in the South African labour market (Chimhandamba 2010; Department of Public Service and Administration 1996).

Although much has been done to rectify these effects, the current, non-discriminatory South African labour market is still socially inequitable, as both black people and women are under-represented in the better-paying occupations and sectors, and over-represented in low-paid occupations and sectors. The 1996 Green Paper on Employment Equity (Department of Public Service and Administration 1996) showed high levels of labour market discrimination. It was found that race and gender (even after controlling for various factors such as education, age, occupation, and sector) were strong factors in determining an individual's probability of obtaining work, and predicting their corresponding remuneration. Whites earn 104 per cent more than blacks, and men receive 43 per cent higher wages than women who are similarly qualified and working in the same sectors and occupations (Department of Public Service and Administration 1996).

The EE Act, therefore, introduced firm-level affirmative action to enhance the re-entry of blacks and women into the mainstream of the economy, and accelerate their upward movement into higher-paying and higher-skilled occupations and sectors. The Act ensures that each firm constructs and abides by a comprehensive plan, focused on restructuring their workforce to allow for an appropriate representation of blacks and women within the labour force (conforming to the demographic representation of the country). Furthermore, a firm's plan should identify and remove any discrimination in employment policies and practices (Bowmaker-Falconer et al. 1997; Chimhandamba 2010).

The Act is compulsory for all *designated employers*. A designated employer is any South African firm with a workforce greater than 50 employees. However, if a firm has a workforce with fewer than 50 employees but generates an annual revenue above a certain threshold (dependent on the industry in which the firm operates), the firm is also classified as a designated employer (Department of Public Service and Administration 1996). The following industries are exempt from the Act: the South African Defence Force, the Secret Service, and the National Intelligence Service.

Advocates of the Act say that preferential policies break down negative views about previously disadvantaged individuals by allowing them to demonstrate their capabilities (Collins 1993). Many economists also argue that market forces alone are unable to solve the problem of discrimination and therefore an act changing structural labour market characteristics is vital (Visagie 1999). Critics, however, argue that the EE Act has led to brain drain (Horwitz 2013) as it incentivizes the emigration of the skilled minority population. The Act has also been blamed for reducing the productivity of firms by lowering the general standards of labour and thereby increasing the cost of doing business (Burger 2014; Kruger and Kleynhans 2014). It is also criticized for reducing foreign investment in South Africa (Dongwana 2016). Furthermore, it has led to many high-skilled vacant jobs or under-skilled employees, as there is limited labour supply meeting both the requirements of the Act and the requirements of the job (Horwitz 2013).

There is limited academic work analysing the impact of the EE Act on the labour market (Horwitz and Jain 2011). Most previous research is qualitative and case-study based (focusing on an industry, firm, or region (Public Service Commission 2006)). Existing studies that do have a quantitative component tend to focus on high-level aggregate statistics and do not consider lower-level structural properties of the labour market. For example, an increased representation of previously disadvantaged individuals has been observed within managerial positions and both higher-paid and higher-skilled occupations since the implementation of the Act (Public Service Commission 2006). To the authors' knowledge, there is no academic research that has investigated the impact of the Act on labour market flow patterns. Labour flows are a useful tool to evaluate the health of an economy, particularly regarding labour market participation and productivity. The EE Act is focused on increasing the participation of under-represented groups in the labour market, which implies specific changes in labour flow dynamics. Quantifying these changes is therefore a fundamental part of evaluating the Act's efficacy.

2.2 Labour mobility

In this section, we review the literature on labour mobility, focusing on the South African context and differences between genders.

Labour mobility in South Africa

Labour mobility has been an ongoing topic of interest among both social and economic researchers since the emergence of market societies (McNulty 1980). The study and regulation of labour markets is paradoxical as it involves the concern of both a well-functioning labour market and people's welfare.

Although labour mobility is a defining characteristic of an economy, it needs to be well understood within its context. High labour mobility is often attributed to a strong economy, because an immobile labour market leads to high rates of structural unemployment (Schioppa 1991). High mobility also enhances innovation and expansion by making it easier for firms to expand into new markets and attract qualified labour (Esping-Andersen and Regini 2000). It also creates resilience within the labour market by allowing workers to be more flexible and adaptable to economic shocks (Diodato and Weterings 2015). On the other hand, high labour mobility may be problematic for an economy. It may prevent the formation of specialized knowledge (Diodato and Weterings 2015), and lead to job insecurity and workers who struggle to cope with the impact of change (Pizzati and Funck 2002). Without policy intervention, high mobility is shown to enhance the downward vertical movement of the lower-educated workforce, which increases inequality within society.

Policy interventions are aimed at controlling the degree of labour mobility within an economy. These include the national level of education and skills, the national minimum wage, regulations around the hiring and firing of workers, bargaining powers and contracts negotiated by trade unions, the presence of zero-hour contracts, and unemployment protection grants, among many others (Esping-Andersen and Regini 2000; Pizzati and Funck 2002). The labour market is traditionally studied through a neoclassical framework. This consists of quantifying labour demand and supply in order to evaluate a labour market equilibrium. Labour demand is studied through worker flows and labour supply through job flows (gross creation and destruction of jobs). The impact of labour market regulations is then evaluated through determining its impact on the labour market equilibrium. The dominant view in the literature is that increased labour market regulations lowers worker flows (Bassanini et al. 2010; Pries and Rogerson 2005).

In South Africa, labour mobility has primarily been studied through survey data. Most of these studies have focused on changes in participation and employment rates (Casale et al. 2004), focusing on transitions in and out of formal employment (Banerjee et al. 2008; Cichello et al. 2005). The first study to quantify the labour demand and supply within the South African labour market was done by Kerr et al. (2014), who used the Quarterly Employment Survey (QES) firm data to quantify the level of job creation and destruction in firms in South Africa. It was found that firms typically create or destroy around 20 per cent of their total jobs annually. Kerr (2018) then continued his analysis using a new administrative tax dataset (the same dataset used within the current study) to quantify the flow of jobs, workers, and churning in South Africa. It was found that worker flows, between formal firms, were around 53 per cent for the 2011–14 period. This is substantially higher than what was previously thought due to the high levels of unemployment and the high labour regulations within the South African labour market (Banerjee et al. 2008; Go et al. 2009). Worker flows were also found to be highly heterogeneous across various factors. These include firm size, firm earning rates, worker earning rates, and firm industries.

Within our study, we are interested in how the EE Act has influenced inter-industry worker flows. Kerr investigated worker flows within 34 industry sectors (flows between firms in the same industry sector).[3] The largest worker flows were found within the following three industries: 93 per cent in 'manufacturing', 79 per cent in 'household services', and 72 per cent in 'hospitality'. On the other hand, the smallest worker flows were found within the following three industries: 20 per cent in 'public administration', 35 per cent in 'mining and quarrying', and 37 per cent in 'electricity, gas, and water' (Kerr 2018).

[3] The industry sectors were classified according to a high-level SARS (South African Revenue Service) industry classification measure.

Various socio-economic factors affect the mobility and participation rate of women within the labour market.[4] The main factors in the literature include the level of economic development, the level of female educational attainment, social dimensions (such as social norms influencing marriage, fertility, and the woman's role outside the household), access to credit, household and spouse characteristics, access to childcare and other supportive services, and institutional setting (e.g. laws, protections, and benefits) (Gaddis and Klasen 2014; Jaumotte 2004). The U-shaped relationship between economic development and women's labour force participation is one of the most well-studied relationships in the literature (Gaddis and Klasen 2014).[5]

Policy reforms focused on increasing the overall participation and mobility of female workers aim at influencing one of these above-mentioned factors. Long-term policy reforms often focus on increasing female education and improving female labour market conditions and norms. Some key short-term policy reforms include: allowing flexible working-time arrangements or part-time work, removing taxation policies that negatively influence work-sharing decisions (e.g. taxation where second earners in a household are taxed more heavily), enabling parental leave (up to a certain duration) and childcare subsidies, enhancing the growth of the service sector, and loosening immigration policies as they reduce the relative cost of childcare (Buchanan et al. 2011).

South Africa's female participation rate is 48.77 per cent for 2019, which is lower than the male participation rate of 62.59 per cent (Burger and Jafta 2010). However, Casale (2004) showed that the South African labour market has become feminized in the post-apartheid years. The female share of employment increased from about 38 per cent in 1994 to about 44 per cent in 2018 (Burger and Jafta 2010). The increase is believed to be particularly influenced by an increased level of education among women (Spaull and Broekhuizen 2017) and a reduction in gender biases within the labour market (Oosthuizen 2006). Various pieces of anti-discrimination legislation have been implemented by the South African government, including the Labour Relations Act No. 66 of 1995 (which sets guidelines for the interactions between employers and employees), the Basic Conditions of Employment Act No. 75 of 1997 (which regulates working conditions and sets a minimum wage for employees in different sectors), and the Employment Equity Act considered within this study (Burger and Jafta 2010; Leibbrandt et al. 2010; Ntuli 2007; Posel and Rogan 2014).

Despite an increase in female participation, differences in the quality of employment between genders persist. Women are over-represented in low-paying occupations and sectors and under-represented in high-paying occupations and sectors. A gender wage gap was found in the South African labour market (Burger and Jafta 2010; Casale and Posel 2011; Muller 2009; Ntuli 2007). The average wage gap has decreased from about 40 per cent in 1993 to about 16 per cent in 2014 (Mosomi 2019). However, Mosomi found that there was heterogeneity within the trend of the wage gap across the wage distribution. Most of the decline was found among the lowest-paid workers (below the 10th percentile), attributed to minimum wage policy implementation in low-paying industries. There has been no significant change in the gender wage gap at the median, which remains around 23–25 per cent. The 90th percentile showed a decline in the gender gap between 1993 and 2005; however, this trend has reversed in subsequent years (Mosomi 2019).

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[4] The labour force participation rate is the proportion of the country's working-age population that actively engages (either by working or seeking work) in the labour market (Jaumotte 2004).

[5] The U-shaped hypothesis between female participation rates and economic development states that female participation rates are highest in poor countries where women are engaged in subsistence activities; the participation rate then decreases among middle-income countries where most jobs are within industries that benefit men. However, as education levels improve and fertility rates fall, women re-enter the labour force in response to growing demand in the service sector (Gaddis and Klasen 2014).

2.3 Labour flow networks and network tools

The labour networks constructed and the network analysis tools used in this study are reviewed in this section.

Labour flow networks

Using networks as a modelling tool has become popular in biology, social sciences, and economics over the last decade. Network analysis has given us the tools to study complex systems.[6] It enables the understanding of the underlying interconnected structure of a system. The popularity of network analysis stems from the growing availability of micro-data and the increases in computational power that have made network construction and analysis possible in many cases (Guerrero and Axtell 2013). In this study, we construct three networks: the inter-firm labour flow network, the inter-industry labour flow network, and the skill-relatedness network (a normalized inter-industry labour flow network).

In labour economics, labour flow networks have primarily been used to understand the structure and topology of a labour market. Guerrero and Axtell (2013) were the first to construct an inter-firm labour network. This is a network in which the nodes represent firms and the edges represent the number of workers who transition between the corresponding firms.

Inter-industry labour networks, however, first emerged from the related diversification literature within evolutionary economic geography. The network has mainly been used for modelling and predicting regional diversification paths (Boschma 2017). Within this literature, Neffke and Henning (2013) were the first to construct an inter-industry labour network. This is a network in which each node represents an industry and each edge the number of workers who transition between the corresponding industries. A *skill-relatedness network* was then constructed in order to quantify the level of skill overlap between industries. This network is one in which each node also represents an industry; however, each edge now represents the skill-relatedness between the corresponding industries. The skill-relatedness is the skill and knowledge overlap between two industries quantified through normalized labour flows. Relying on labour flows as a measure of skill-relatedness is based on the assumption that workers have the incentive to move to industries where their skills are valued. Concurrently, firms are more willing to recruit workers from other industries who have relevant skills. Therefore, the greater the number of workers who move between two industries, the higher their skill similarity. However, the size of labour flows depends on the size of employment within the corresponding pair of industries. Therefore the worker flows are normalized to take employment size into account.

Skill-relatedness networks have been constructed for various countries, including Sweden (Neffke et al. 2011), Germany (Neffke et al. 2017), Ireland (O'Clery et al. 2019), the Netherlands (Diodato and Weterings 2015), and the United Kingdom (unpublished data). To the best knowledge of the authors, no skill-relatedness network has been constructed for an African economy.

Network analysis tools

This analysis uses three main network analysis tools: centrality measures, information entropy, and community detection. Each is briefly discussed.

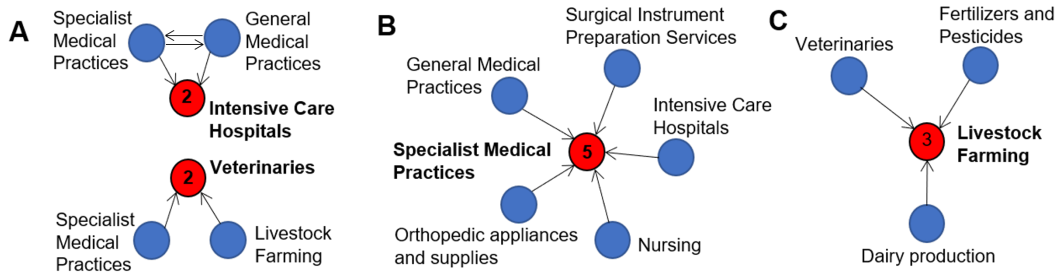
Centrality measures are often adopted in order to analyse the connectivity of a network. A centrality measure quantifies the relative importance of a node within a network by taking the structure of the network into account. There are many centrality measures within the literature that vary according to the amount of local or global information they include about the structure of the network. The *in-strength* is

[6] Here, a complex system is a system of many interrelated parts that influence its overall behaviour.

a local centrality measure. It sums the weight of all edges that point towards a node (only considering the direct neighbourhood of a node). It then ranks all nodes by their total strength. In our analysis we wish to measure the diversity of labour inflows. This measure cannot be used, as all industries within the inter-industry labour network are not skill-equidistant from each other. To elucidate this problem, consider the *intensive care hospitals* industry and the *veterinaries* industry shown in red in Figure 1(a). Both of these industries receive workers from two different industries and by using the in-strength centrality measure, should have similar labour inflow diversities. The *intensive care hospital* industry, however, receives workers from two industries that also share a high degree of labour flows. This industry is therefore less diverse. In-strength centrality is unable to consider a greater degree of network structure and can therefore not rank the two industries accordingly.

We could also consider a more global network centrality measure that takes a larger amount of the structure of the network into account (e.g.. the Katz centrality), but as all industries are not defined at equal levels of granularity this will not provide reliable results. To illustrate this problem, consider the *specialist medical practices* industry and the *livestock farming* industry shown in Figure 1(b) and (c). Note the difference in the level of detail with which each of these industries' neighbours are defined. Although the first industry receives workers from more industries, we cannot argue that it is more diverse as this could merely be a result of the heterogeneity of the level of detail with which the industries are defined.

Figure 1: Mock example of various industries and their direct neighbours within the inter-industry labour flow network illustrating that (a) all industries are not skill-equidistant and (b,c) are defined in varying levels of detail within the original industry classification



Notes: the industry considered is illustrated in red. The number inside the industry counts the number of different industries from which it receives workers.

Source: authors' illustration.

In order to measure the diversity of labour inflows reliably, we therefore adopt an entropy-based measure. Information entropy is traditionally found in statistical physics. It quantifies the amount of uncertainty about an event before it occurs, or the amount of information gained about the system once the event has occurred (Guevara et al. 2003). A low-probability event carries more information than a high-probability event when it occurs, and therefore a higher entropy. The amount of information that is obtained from each event becomes a random variable whose expected value is the information entropy. In the economic literature, Eagle et al. (2010) created an entropy-based measure to quantify the diversity of an individual's social interactions with different income levels in the population. The entropy measure was applied on a network consisting of cellular phone calls between individuals. Our study adopts a very similar entropy-based metric in which the diversity of labour inflows is measured according to the degree of different sectors from which an industry or firm hires workers.

In order to detect the various sectors, we construct a new industry classification by clustering the skill-relatedness network. Community detection techniques have been used extensively to study the structure and dynamics of biological, social, engineering, and economic networks (Girvan and Newman 2002). A community or cluster can be defined as a densely connected group of nodes with sparse connections to other clusters. The problem of community detection consists of partitioning the nodes within a network into several non-overlapping clusters. Most well-known community detection algorithms seek to find

a single partition under a particular optimization strategy (Newman 2003). Within this study, we use a dynamical community detection algorithm, the Markov stability algorithm (Delvenne et al. 2010). The algorithm is based on diffusion dynamics and uses its properties to unveil the modular structure of the network. It partitions the skill-relatedness network into 'dynamical skill-basins' (O'Clery et al. 2019), which are groups of industries in which workers freely transition but rarely leave.

3 Data

This study uses an administrative dataset constructed from anonymized tax records for the period 2011–14 (Ebrahim and Axelson 2019; Pieterse et al. 2018) to count inter-industry and inter-firm worker transitions. The dataset was recently made available to researchers by the National Treasury and SARS.

More specifically, the dataset was constructed from IRP5 tax certificates. These certificates are issued by an employer who is registered for pay as you earn (PAYE) tax, on behalf of an employee. Each certificate contains details of the employee, the employer, and the duration and terms of employment. Note that the dataset only contains workers and firms in the formal economy.[7] The reader is directed to Appendix A1 for a more detailed discussion on the data and the data-cleaning strategy used.

To construct an inter-industry labour flow network, we need to consider how firms are classified into industries. Within the dataset, a firm is assigned an industry classification code according to their primary economic activity. The industry codes are a four-digit code that abides by an internally set SARS classification system. Within the classification system there are 388 different four-digit industry codes. The codes closely correspond to the ISIC (International Standard Industrial Classification) Revision 4 classification.

In this study, we divide the firms into two groups according to whether a firm is exempt or complies with the EE Act. Recall that the Act only applies to firms with a workforce of more than 50 employees. However, firms with annual revenue above a certain threshold are still obliged to comply with the Act even if their workforce contains fewer than 50 employees. There are also various industries (the South African Defence Force, the Secret Service, and the National Intelligence Services) that are completely exempt from the Act. All firms and their workforces that are exempt from the Act are grouped as our control (exempt) group; all other firms and their workforces are grouped as our treatment (compliant) group. We have 95,156 firms within our control group and 17,138 firms within our treatment group. Although there are fewer firms within the treatment group, they contain many more employees.

4 Methodology

In this section we first discuss the construction of three different networks: the *inter-firm labour flow network*, the *inter-industry labour flow network*, and *the skill-relatedness network*. We then consider the construction of our *inflow entropy* measure. This also includes partitioning the skill-relatedness network and creating a new industry classification. Next, the variables used throughout the study are defined. Finally, details regarding the RD design and the multivariate regression analysis adopted are discussed.

[7] The exclusion of the informal economy does not significantly skew our results as only 30 per cent of all employment in South Africa is attributed to the informal economy. This is significantly less than in other developing countries (Magruder 2012).

4.1 Network construction

We use the administrative data, discussed in Section 3, to construct the inter-firm labour flow network, the inter-industry labour flow network, and the skill-relatedness network. The inter-firm labour flow network is a network in which each node represents a firm and each edge the number of workers who transition between the two corresponding firms. For the inter-industry labour flow network, each node represents an industry and each edge the number of worker transitions between the two corresponding industries. The skill-relatedness network is a network in which each node represents an industry and each edge is a measure of the skill similarity between the two corresponding industries. The skill similarity is calculated by comparing the number of worker transitions between two industries compared to what we expect at random. The differences among these three networks are summarized in Table 1.

Table 1: Properties of the three different networks constructed in this study

	Nodes represent	Edges represent	Size of network	Usage of network in this study
Inter-firm labour flow network	Firm	Average number of work transitions between firms per year	112,294	Quantify the diversity of labour flow for firms
Inter-industry labour flow network	Industry	Average number of work transitions between industries per year	388	Quantify the diversity of labour flow for industries
Skill-relatedness network	Industry	Skill overlap between industries	388	Construct a new industry classification

Source: authors' illustration.

Let $L_F(i,j,t)$ denote the observed labour flows from firm i to firm j between years t and $t+1$. We define the positive and non-symmetric adjacency matrix A_F for the inter-firm labour flow network as:

$$A_F(i,j) = \frac{1}{4} \sum_{t=2011:2014} L_F(i,j,t) \tag{1}$$

Furthermore, let $L_I(i,j,t)$ denote the observed labour flows from industry i to industry j between years t and $t+1$. We define the positive and non-symmetric adjacency matrix A_I for the inter-industry labour flow network as:

$$A_I(i,j) = \frac{1}{4} \sum_{t=2011:2014} L_I(i,j,t) \tag{2}$$

Both networks are weighted directed graphs.[8] In this study, these networks are used to calculate the labour flow diversity for a firm or industry.

We use various subgraphs of the inter-firm and inter-industry labour flow network within our analysis. We construct these subgraphs using the same method; however, we only use a subset of L_F or L_I, where we only include the transitions of workers who meet certain criteria. The criteria comprise whether the workers are male or female and whether they transition to firms who either comply or are exempt from the EE Act. The adjacency matrices of these subgraphs are denoted as $A_a^{b,c}$, where $a \in \{F,I\}$ indicates whether we are constructing an inter-firm or inter-industry labour flow network, $b \in \{M,F\}$ indicates whether we are considering male or female workers, and $c \in \{C,E\}$ indicates whether workers who transition to either compliant or exempt firms are used.

The third network constructed is the skill-relatedness network. The construction follows the method of Neffke and Henning (2013). First, the skill overlap between industries is quantified using the labour flows between them. Within this framework, the existence of large labour flows between a pair of industries shows that these industries are highly skill-related. However, the size of the two industries

[8] A weighted directed graph is a graph in which the edges have both direction and weight.

influences the size of the labour flows. Therefore, the labour flows are compared to a null model (the expected labour flows between two industries at random). The null model is a configuration model (Molloy and Reed 1995). Each edge is the number of labour flows you would expect at random when reconstructing the graph by shuffling its edges but keeping the total number of labour inflows and outflows of each industry constant.

The skill-relatedness between industry i and j between years t and $t+1$ is given by:

$$SR(i,j,t) = \frac{L_I(i,j,t)\sum_{ij} L_I(i,j,t)}{\sum_i L_I(i,j,t)\sum_j L_I(i,j,t)} \tag{3}$$

The value is effectively the level of labour flow that is observed between industry i and j beyond what is expected at random. Note that if the flow is larger than what is expected at random, then $SR(i,j,t) \in [1,\infty)$. However, if the flow is smaller than what is expected at random, then $SR(i,j,t) \in [0,1]$. This measure is highly skewed. Therefore, a transformation is applied to symmetrically map the values onto the interval $SR(i,j,t) \in [-1,1]$,

$$\tilde{SR}(i,j,t) = \frac{SR(i,j,t)-1}{SR(i,j,t)+1} \tag{4}$$

where the value of zero represents what is expected at random. Finally, the measure is averaged over the analysis period 2011–14,

$$M\tilde{SR}(i,j) = \frac{1}{4}\sum_{t=2011:2014} \tilde{SR}(i,j,t) \tag{5}$$

and made symmetric,

$$SSR(i,j) = \frac{M\tilde{SR}(i,j) + M\tilde{SR}(j,i)}{2} \tag{6}$$

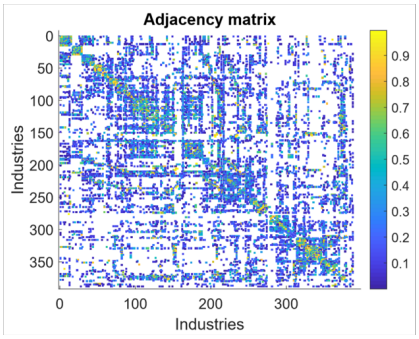
The skill-relatedness network is then constructed by only taking the positive part of $SSR(i,j)$. Therefore, the edges within the network only include labour flows that are greater than what is expected at random. We define the positive and symmetric skill-relatedness adjacency matrix A_{SR} as:

$$A_{SR}(i,j) = \begin{cases} SSR(i,j), & \text{if } SSR(i,j) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

A_{SR} is shown in Figure 2(a). We can see that the matrix is sparse, and there are clear clusters of values near the diagonals. This shows that there is high skill-relatedness between industries in the same sector (industries are ordered according to sector). The top 10 industry pairs with the highest skill-relatedness are shown in Figure 2(b). Note that these industries fall within a wide range of sectors. The skill-relatedness network is illustrated in Figure 2(c). In this figure, each node represents an industry, the size of a node represents the average size of the industry, and the weight of each edge represents its skill-relatedness. The node layout used is a spring algorithm called 'Force Atlas' (Jacomy et al. 2014) in Gephi, which positions industries that are more skill-related closer together. The algorithm simulates a physical system in which nodes, representing charged particles, repulse each other and edges, representing springs, attract their nodes. The various forces create a movement that converges to a balanced state that is then used as the final network configuration. Note that the network displays a high degree of clustering of related industries. We have added labels indicating the general position of sectors in the figure.

Figure 2: (a) The adjacency matrix of the skill-relatedness network. (b) The top 10 industry pairs of the skill-relatedness network by edge weight. (c) Visualization of the skill-relatedness network for South Africa

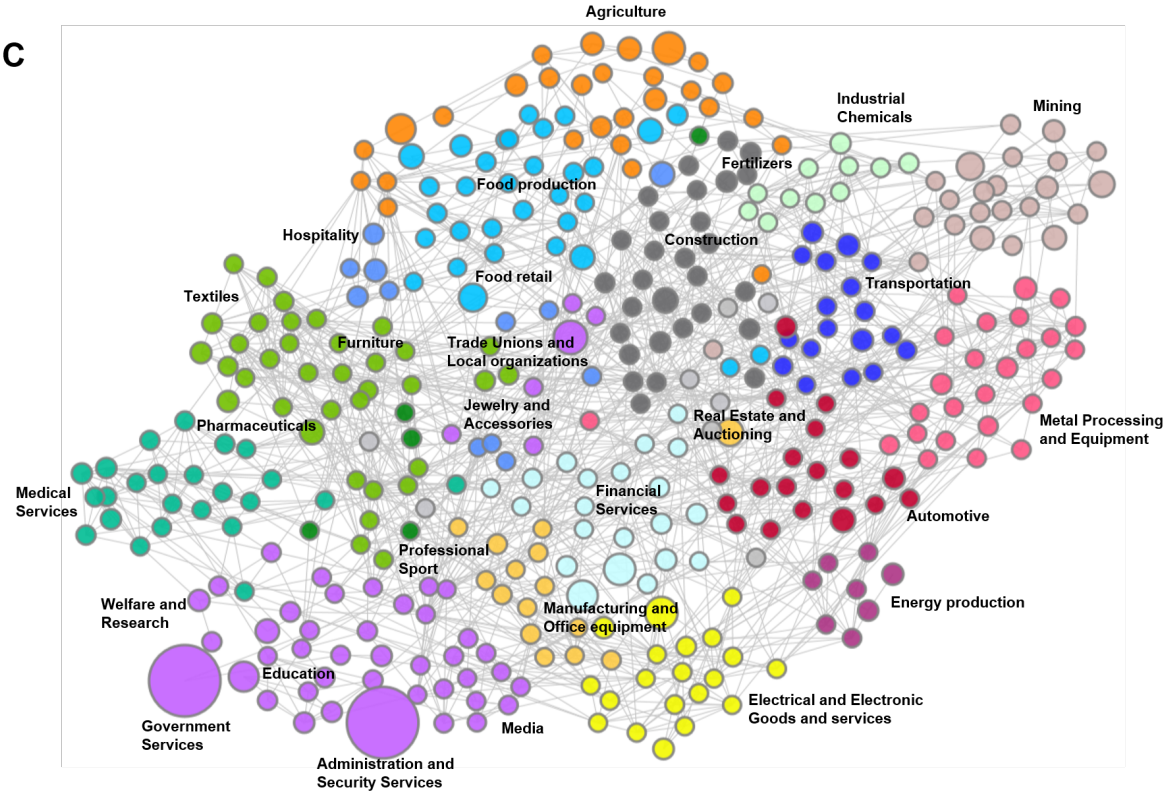
A



B

Industry i	Industry j
Veterinaries	Veterinary services (including kennels)
Motor cycles parts	Motor cycle dealers
Fishing	Commercial fishing
General and specialist dentists	Dental mechanics
Jewelry	Watch and clock repairers
Motor cycle parts	Bicycle dealers
Jewelry and related articles	Watch and clock retailers
Pet food shops	Veterinaries
Air conditioning and ventilation machinery	Heating and ventilation engineers
Jewelry and related articles	Jewelry

C



Notes: (c): each node represents an industry and each edge the skill-relatedness between the corresponding industry pair. Nodes are sized by the average employment over the 2011–14 period and coloured according to their industry cluster detected according to the Markov stability algorithm ($t = 0.5$). Only edges above a skill-related index of 0.6 are shown. The node layout is based on a spring algorithm called 'Force Atlas' in Gephi.

Source: authors' compilation based on data.

We use the skill-relatedness network in our study to identify the labour pools within the South African labour market and construct a new industry classification. We then use this for our inflow entropy measure. Recall from Section 2.3 that the original industry classification is not used because industries are not skill-equidistant and defined at different levels of granularity within this classification.

4.2 Constructing a new industry classification

We partition the skill-relatedness network into labour pools. We adopt the methodology of O'Clery et al. (2019), who use the Markov stability algorithm (Delvenne et al. 2010) as a dynamical clustering algorithm to unveil labour pools within the Irish labour market. The labour pools represent 'dynamical skill-basins in which workers can freely move but rarely leave' (O'Clery et al. 2019).

The Markov stability algorithm (Delvenne et al. 2010) is a dynamical community detection algorithm based on diffusion dynamics. It differs from a static community detection algorithm in that it produces a range of partitions at different scales (from a few large clusters to many well-defined clusters). This allows for a more natural understanding on how partitions are constructed. It also unveils the hierarchical structure of the labour market.[9] The range of partitions can also be considered as different aggregation levels within an industry classification.

The Markov stability algorithm is aimed at maximizing the probability that a random walker remains in the community in which it started during a time interval. It can be intuitively understood by considering a random walker wandering on a network (jumping from node to node). The propensity with which the walker jumps between two nodes is proportional to the corresponding edge's weight. To discover a community, the core idea is that if the walker gets trapped in a group of nodes for an extended period it indicates that there is high connectivity between the group of nodes and thereby a community exists. The longer the random walker is observed, the more it can wander and therefore the larger the node aggregations that result. Hence, changing the time results in different sizes of communities. For a detailed explanation of the workings of the Markov stability algorithm, the reader is referred to Appendix A2.

Within our study we choose a single partition as the new aggregation for our industry classification. To evaluate which of the partitions are most representative of the actual sectors within the economy, we calculate the robustness of each partition obtained and choose the one with the highest robustness. The robustness of a partition is a measure of the variation in the resulting partitions obtained when continually applying the community detection algorithm. We use the variation of information measure (Meila 2003) to quantify the variation. The partition with the lowest variation is chosen as our new industry classification. A partition containing 21 skill-related industry clusters is suggested by the Markov stability algorithm on the South African skill-relatedness network. The partition is illustrated in Figure 2(c) via the colour of the nodes. We also compare the modularity of our new industry classification and the original SARS industry classification. Modularity is a measure of how well a partition divides a network into communities by comparing the number of edges within a community compared to what is expected at random. Our new industry classification results in a higher modularity, and is therefore an improved partitioning of the network.

The resulting partition is used as the new industry classification that groups industries into sectors. Note that this classification considers both the level of skill-relatedness between industries (by quantifying the skill-relatedness between industries) and corrects for the heterogeneous SARS industry classification (by allowing industry clusters to be of varying sizes).

4.3 Constructing the inflow entropy measure

Finally, we can measure the diversity of labour flows for either a firm or an industry by measuring the array of different sectors (previously defined) from which a firm or industry hires workers. We therefore need to consider both how many different sectors a firm or industry's workforce flows from, and the number of workers that flow from each.

In order to quantify the diversity, we construct the *inflow entropy* measure. The entropy of a random variable, denoted by H, is a measure of uncertainty and quantifies how much we know about a variable before observing it. For a discrete random variable X, containing n possible states and a probability

[9] Note that the resulting partition from the Markov stability algorithm is only closely but not strictly hierarchical.

mass function $p(x)$, the entropy is defined as

$$H(X) = -\sum_{j=1}^{n} p(x_j)\log(p(x_j)) \tag{8}$$

The entropy value $H(X)$ lies within the interval $0 \leq H(X) \leq \log(n)$. $H(X)$ is minimal when X is deterministic (there is no uncertainty about the variable). The maximal value is obtained when the probability mass function is a uniform density, $p(x) = 1/n$.

We now apply entropy to our problem. We let $H(X_i^A)$ be the inflow entropy of node i in the network represented by adjacency matrix A. The discrete random variable X is the skill cluster or sectors from which node i receives workers. As there are 21 different skill clusters in our new classification, $n = 21$. Let $\text{sc}(i)$ denote the skill cluster to which node i belongs. We define

$$p(X_i^A = x_j) = \frac{e_{ji}}{\sum_{j=1}^{N} e_{ji}(1-\delta(\text{sc}(j),\text{sc}(i)))}$$

where δ is the Kronecker delta. This represents the fraction of node i's worker inflows that come from skill cluster j (excluding the flows in the skill cluster of node i). The inflow entropy is then given as

$$\text{H}(X_i^A) = -\sum_{j=1, j\neq \text{sc}(i)}^{21} \frac{e_{ji}}{\sum_{j=1}^{N} e_{ji}(1-\delta(\text{sc}(j),\text{sc}(i)))} \log\left(\frac{e_{ji}}{\sum_{j=1}^{N} e_{ji}(1-\delta(\text{sc}(j),\text{sc}(i)))}\right) \tag{9}$$

Therefore, the larger the array of different skill clusters a firm (or industry) workforce flows from, the higher their inflow entropy. We rewrite $\text{H}(X_i^A) = \text{H}_i(A)$, and use this notation in the rest of this paper. To obtain the male and female, as well as exempt and compliant, inflow entropies we apply this methodology to our various subgraphs.

4.4 Variable construction

Within this study, we investigate the impact of the EE Act on the diversity of labour flows (inflow entropy) and the gender wage gap. We investigate these factors on both a firm and industry level.

Recall from Section 4.3 that the inflow entropy is denoted as $H_i(A_a^{b,c})$, where i is the unit under investigation (either a firm or industry). Furthermore, $a \in \{F, I\}$ indicates whether a firm- or industry-level analysis is being adopted, respectively. Additionally, $b \in \{M, F\}$ indicates whether only male or female worker flows are considered, respectively. Finally, $c \in \{C, E\}$ indicates where flows into firms who comply with the Act or are exempt from the Act are being investigated.

Similarly, let the average wage be denoted as $W_a^{b,c}(i)$, where the subscripts and superscripts have the same meaning as in the inflow entropy measure. The gender wage gap, defined as the percentage difference between the male wage and the female wage, is then denoted as

$$WG_a^c(i) = (W_a^{M,c}(i) - W_a^{F,c}(i))/W_a^{M,c}(i)$$

Furthermore, the employment size within the time period t is denoted as $E_i(A_a^{b,c}, t)$ and the fraction of male employment $FME_i(A_a^c, t)$, where

$$FME_i(A_a^c, t) = E_i(A_a^{M,c}, t)/(E_i(A_a^{F,c}, t) + E_i(A_a^{M,c}, t))$$

When a superscript is omitted this is indicative that all states (the union of all states within the set) are considered. Next, we discuss the construction of our RD design to determine the impact of the EE Act in our study.

4.5 The RD design

An RD design is a well-known policy evaluation method that has emerged as one of the most credible non-experimental strategies for the analysis of causal effects (Cattaneo et al. 2019). An RD design exploits a discontinuity in the treatment assignment to identify causal effects (Cattaneo et al. 2019; Imbens and Lemieux 2008). We use an RD design within our study to determine the impact of the EE Act on a firm's male and female labour flow diversity and a firm's average gender wage gap.

Within our RD design, our independent variable is the size of employment within a firm. Following the legislation of the EE Act, firms with more than 50 employees comply with the Act, while those with fewer than 50 employees are exempt from the Act. Note that firms who have fewer than 50 employees but still abide by the Act as their annual revenue is above the threshold value are removed from our sample to prevent skewing of our results. The key feature of the design is that the probability of complying with the Act changes abruptly, from 0 to 1, at the 50-employees cutoff value. The discontinuous change in the probability is used to learn about the local effect of the Act on the dependent variable (which is the male and female inflow entropy ($H_i(A_F^{b,c})$ where $b \in \{M,F\}$ and $c \in \{C,E\}$) and the average male and female wage ($W_F^{M,c}(i)$ and $W_F^{F,c}(i)$ where $c \in \{C,E\}$)). Therefore, firms that are just below the cutoff (have a firm size of 49) are used as a comparison group for firms just above the cutoff (have a firm size of 51). As the sizes of these firms are very similar, it is expected that without the Act the outcome variable of these two groups should be very similar. A significant discontinuity found between these two groups is indicative of the impact of the Act.

To estimate the discontinuity, we fit a polynomial function to the data on each side of the cutoff. We adopt a local linear polynomial approach and choose a polynomial of order 1. We do not choose a higher-order polynomial, as these polynomials provide poor approximations at boundary points (this is also known as the Runge phenomenon in approximation theory (Trefethen 2013)). However, the problem with choosing a lower-order polynomial is that the size of the neighbourhood (the interval) that is considered within the analysis heavily influences the result. We therefore chose a bandwidth based on the algorithm of Imbens and Lemieux (2008), which chooses an optimal bandwidth that minimizes the asymptotic bias and variance of the local linear polynomial estimator. Therefore, in choosing a linear regression function with a suitable bandwidth, we allow for a fit that is less sensitive to over-fitting and boundary problems.

Furthermore, we adopt a triangular kernel function that assigns non-negative weights to each observation based on its distance to the cutoff. The largest weight is assigned to the values at the cutoff. The weight then symmetrically and linearly decreases as values are further from the cutoff (Cattaneo et al. 2019).

More formally, the independent variable x_i is equal to the firm's employment size. The dependent variable is denoted as y_i, which is equal to either the inflow entropy ($H_i(A_F^{b,c})$) or the average male and female wage ($W_F^{M,c}(i)$ and $W_F^{F,c}(i)$). The cutoff value is chosen at $c = 50$; the bandwidth $h = 10$. In our linear regression model, y_i is then also defined as a linear function of x_i as:

$$y_i = \begin{cases} \alpha_R + \beta_R x_i + \varepsilon_i, \text{ if } (c-h) \leq x_i < c \\ \alpha_L + \beta_L x_i + \varepsilon_i, \text{ if } c \leq x_i \leq (c+h) \end{cases} \tag{10}$$

The values of the coefficients are then determined by using the triangular kernel function for weighted least square regression by the following optimization function:

$$\min \varepsilon = (1 - |\frac{x_i - c}{h}|) \times (y_i - (\alpha + \beta x_i)^2) \tag{11}$$

Finally, the RD treatment effect is calculated by determining the vertical distance at the cutoff point:

$$\tau_{RD} = \lim_{x \xrightarrow{R} c} \mathbf{E}[y_i | x_i = c] - \lim_{x \xrightarrow{L} c} \mathbf{E}[y_i | x_i = c] \tag{12}$$

It is important to note that regardless of the ability of the RD design to show causal effects, the RD design is not a valid measure for representing treatment effects that occur for units much further away from the cutoff value. It is therefore unable to predict the overall relationship between the two variables (Cattaneo et al. 2019). Note that the RD design is done at the firm level. Next, we turn to the industry level.

4.6 Regression analysis

We are interested in investigating how the Act impacts different industries. We hypothesize that male-dominant industries will be more heavily impacted by the Act and result in a higher female inflow entropy and smaller gender wage gap.

We therefore investigate the relationship between the fraction of male employment (FME) and the male and female inflow entropy (H) using a multivariate linear regression model, given by

$$H(A_I^{b,c}) = \alpha + \beta_1 FME(A_I^c) + \beta_2 \log(E(A_I^{b,c})) + \beta_3 (E(A_I^{b,c}, 2014) - E(A_I^{b,c}, 2011)) + \beta_4 W_I^{b,c}) \quad (13)$$

where $b \in \{M, F\}$ and $c \in \{C, E\}$. We control for the impact of the average employment size, the employment growth across the time period (2011–14), and the average wage of each industry within the model. The logarithm of the change in employment is not used in order to allow for industries with negative growth to be included in the model.

Similarly, we investigate the relationship between the fraction of male employment (FME) and the gender wage gap (WG). The multi-variant linear regression model is given by

$$WG_I^c = \alpha + \beta_1 FME(A_I^c) + \beta_2 \log(E(A_I^c)) + \beta_3 (E(A_I^c, 2014) - E(A_I^c, 2011)) + \beta_3 \log(W_I^c) \quad (14)$$

where $b \in \{M, F\}$ and $c \in \{C, E\}$. We again control for the impact of the average employment size, the employment growth across the time period (2011–14), and the average wage of each industry.

The regression analysis is performed for both the treatment group (firms that comply with the Act) and the control group (firms that are exempt from the Act). We then compare the relationships between these groups.

5 Results

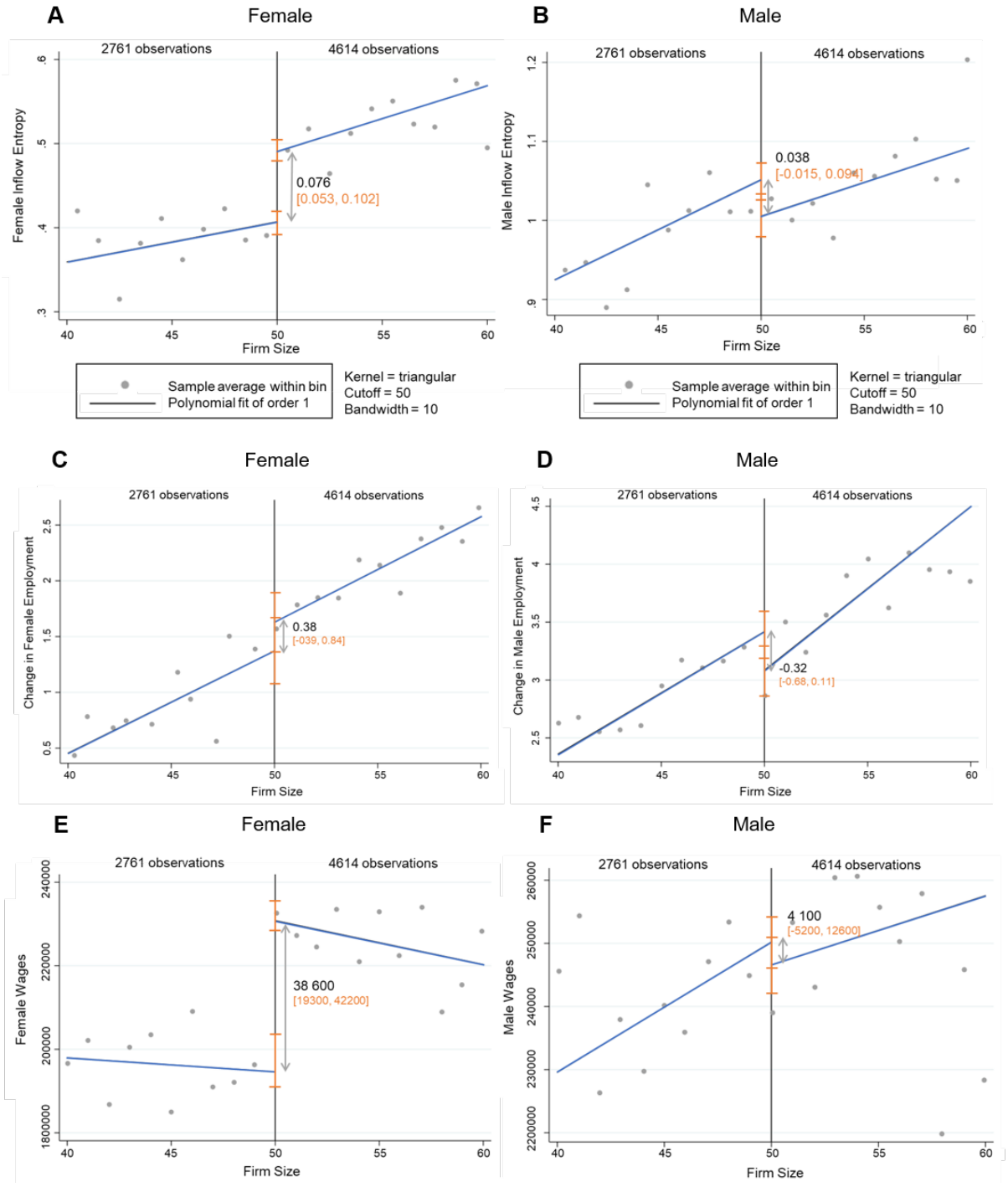
In this section, we investigate the impact of the EE Act on the diversity of male and female labour flows and the gender wage gap. We first investigate the impact at the firm level using an RD design. At the industry level, we then investigate whether male-dominant industries have been more heavily impacted by the Act and therefore display greater diversity of female labour flows and smaller gender wage gaps.

5.1 Evaluating the impact of the EE Act at the firm level

The resulting RD design, taking the male and female inflow entropy ($H_i(A_F^M)$ and $H_i(A_F^F)$) as our outcome variable (y_i) in Equation (10) is illustrated in Figure 3(a) and (b), respectively. Note that for both male and female workers, as the size of a firm increases, their inflow entropy also increases. This is because as a firm increases in size, it typically hires workers from more sectors in order to operate different divisions within the firm. However, as seen in the figure, a large, positive discontinuity is found for the female inflow entropy at the cutoff value of 50. Using Equation (12), we obtain a treatment effect of 0.076 with a 95 per cent confidence interval of $[0.053, 0.102]$ for the female inflow entropy. A much smaller, negative discontinuity is found for the male case. The treatment effect is 0.038 with a 95 per cent confidence interval of $[-0.015, 0.094]$. This shows that as a firm (with 50 employees) changes from

being exempt to complying with the EE Act, their average female inflow entropy increases by 0.076 and their male inflow entropy does not change.

Figure 3: Regression discontinuity plots showing the impact of the EE Act for firms with 50 employees on the following outcome variables: (a) female inflow entropy; (b) male inflow entropy; (c) change in female employment; (d) male employment; (e) female average wage; and (f) male average wage



Notes: the RD design uses a cutoff value of 50. It applies a local linear polynomial regression, with a triangular kernel function. A bandwidth of 10 is chosen according to the Imbens and Lemieux (2008) algorithm. Data points are binned for visualization purposes on the graphs, with the number of observations shown above the graph on each side of the cutoff line. The treatment effect, as well as its 95 per cent confidence interval, is also shown on each graph.

Source: authors' compilation based on data.

To ensure that the discontinuity observed previously is not caused by accelerated growth in larger firms, we apply the same RD design but use the growth of a firm as our outcome variable. We use the change in

employment size of each firm between 2014 and 2011 as a proxy for the growth of a firm: $E_i(A_F, 2014) - E_i(A_F, 2011)$. The resulting RD graphs for both male and female workers are shown in Figure 3(c) and (d). It can be seen that there is no discontinuity (a very small treatment effect with confidence interval containing 0) at the cutoff for both the change in male and female employment. Therefore, the Act appears to have no significant impact on the growth of employment within firms.

Next, we consider the impact of the EE Act on the male and female average wage. We again apply an RD design taking the male and female average wage (W_F^M and W_F^F) as the dependent variable in Equation (10). The results are illustrated in Figure 3(e) and (f), respectively. Note that observations are binned within the figure for visualization purposes. The number of observations considered in order to fit the polynomial on each side of the cutoff is shown above the graph. A large, positive discontinuity is observed for the female average wage at the cutoff, while a small, negative discontinuity is observed for the male average wage. The treatment effect of the male average wage has a 95 per cent confidence interval containing 0. Therefore, we find that as firms change from being exempt to complying with the EE Act, the average female wage increases by approximately ZAR38,600 (US$2,555), while the average male wage remains constant.

These results show that the female inflow entropy and the female average wage of a firm increases as it becomes compliant with the EE Act. To further investigate the impact of the Act, we turn to how it has influenced firms within different industries in the South African labour market.

5.2 Evaluating the impact of the EE Act on an industry level

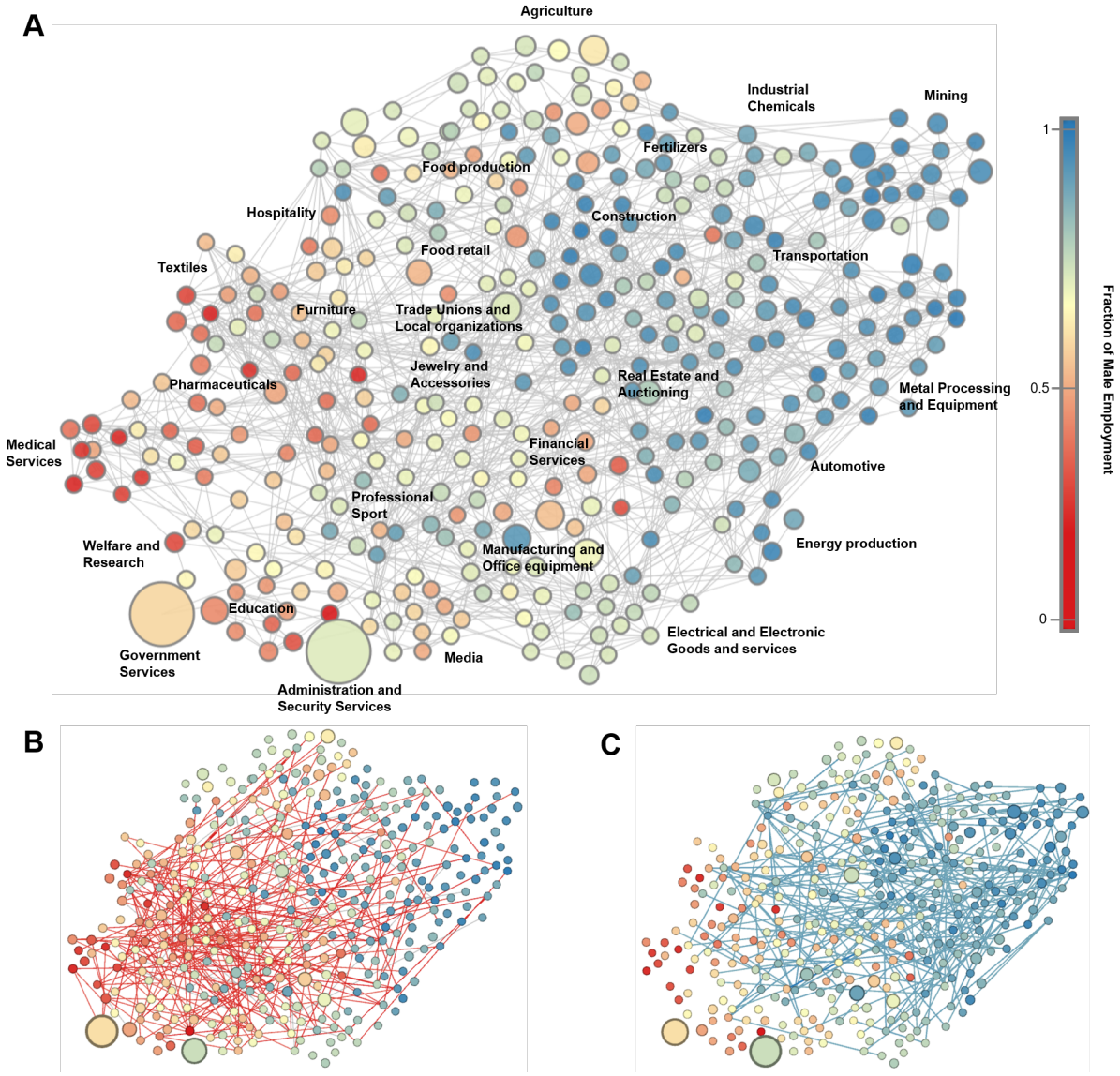
We now consider how the impact of the Act differs across industries. We hypothesize that the impact of the Act is greatest among male-dominant industries. Male-dominant industries will therefore have a larger female inflow entropy and a smaller gender wage gap. For better understanding, we first investigate the distribution of male and female workers within industries in the South African labour market.

A gender-polarized labour market

The South African labour market displays stark differences in terms of the skill and knowledge of the industries that employ male and female workers. This skill polarization can be seen in Figure 4(a), where the distribution of male and female employees within industries are shown using the layout of the skill-relatedness network constructed in Section 4.1. In the network, each node (industry) is coloured according to its percentage comprising male employment. Note that the network is split into female-dominant industries on the left and male-dominant industries on the right. Female workers are shown to be concentrated within services and low-skilled manufacturing industries, while male workers are shown to be concentrated in heavy and high-skilled manufacturing industries.

We can also show that the labour market is not only polarized from a node perspective, but also an edge perspective. Figure 4(b) and (c) shows the top 5 per cent of female and male inter-industry labour flows, respectively. Female workers are shown to mainly transition between industries on the left (female-dominant industries) of the network. Similarly, male workers mainly transition between industries on the right (male-dominant industries) of the network. There is limited movement of employees between male- and female-dominated industries. The observed labour flows further enhance the fragmentation and clear skill polarization of the labour market by gender.

Figure 4: The distribution of male and female workers on the South African skill-relatedness network



Notes: (a) each node represents an industry and each edge the skill overlap between the corresponding industries. The nodes are sized according to the industry's average employment size between 2011 and 2014, and coloured according to the percentage of male employment. (b, c) Each node represents an industry and its size for average female or male average employment between 2011 and 2014. The edges represent labour flows; only the top 5 per cent of female and male labour flows are shown.

Source: authors' compilation based on data.

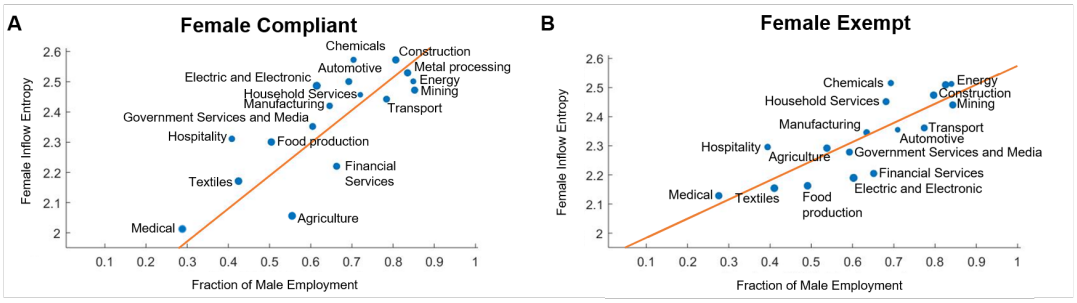
The impact of the EE Act on the diversity of labour flows

Next, we evaluate whether the male-dominant industries have larger female inflow entropy. The relationship between the percentage of male employment within an industry and the industry's male and female inflow entropy is evaluated using Equation (13), discussed in Section 4.6. We also compare this relationship between our treatment group (firms who comply with the Act) and our control group (firms who are exempt from the Act).

The resulting relationship between the percentage of male employment and the female inflow entropy of an industry, for our treatment and control groups, are shown graphically in Figure 5(a) and (b), respectively. Within the figure, industries are binned according to their sectors purely for visualization purposes. However, the trend line and the resulting regression tables quantifying the relationship and shown in Figure 5(c) and (d) is calculated using the 388 individual industries. It can be seen that there is

a positive and significant relationship between the percentage of male employment and the female inflow entropy. This is particularly pronounced in the case of the compliant group, with a regression coefficient of 0.6936, compared to the exempt group with a regression coefficient of 0.2467. The difference between these coefficients (0.4469) is larger than the standard error of both the coefficients (0.0941 and 0.0864, respectively). These relationships are therefore significantly different. These results show that male-dominant industries have a higher female inflow entropy, especially among the treatment group (firms that comply with the Act).

Figure 5: (a,b) Graph showing the relationship between the percentage of male employment and the female inflow entropy over the period 2011–14 for the group of firms who comply with the EE Act and those who are exempt. (c,d) Regression table quantifying the same relationship. (e–h) is the male case of (a–d)



C

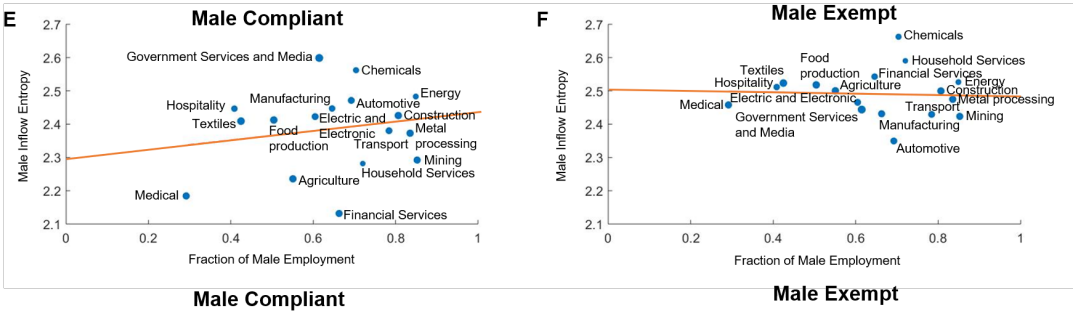
Female Compliant

	Female Inflow Entropy (Comply with EE Act)	
	(1)	(2)
Fraction of Male Employment	0.6156*** (5.4698)	0.6936*** (7.1579)
Log(Female Employment Size)		0.1001*** (8.8236)
Change in Female Employment Size		-8.5574e-06** (-3.0082)
Female Average Wage		8.5355e-07** (3.1579)
Constant	1.8017*** (27.361)	1,7913*** (22.458)
Observations	386	386
R-squared	0.2723	0.3672
Robust t-statistic in parentheses *** p<0.01, ** p<0.05, * p<0.1		

D

Female Exempt

	Female Inflow Entropy (Exempt from EE Act)	
	(1)	(2)
Percentage of Male Employment	0.1633 (1.3352)	0.2467*** (8.1886)
Log(Female Employment Size)		0.2116** (9.702)
Change in Female Employment Size		-1.93e-4** (-2.7145)
Female Average Wage		4.399e-07* (2.3471)
Constant	2.046*** (26.214)	0.1994* (1.7877)
Observations	386	386
R-squared	0.0046	0.136
Robust t-statistic in parentheses *** p<0.01, ** p<0.05, * p<0.1		



G

Male Compliant

	Male Inflow Entropy (Comply with EE Act)	
	(1)	(2)
Percentage of Male Employment	0.1332*** (3.965)	0.1276 (1.614)
Log(Male Employment Size)		0.0821*** (9.061)
Change in Male Employment Size		-3.444e-06** (-2.645)
Male Average Wage		-2.0671e-07 (-1.526)
Constant	1.9931*** (37.169)	1.4505*** (17.025)
Observations	386	386
R-squared	0.0393	0.225
Robust t-statistic in parentheses *** p<0.01, ** p<0.05, * p<0.1		

H

Male Exempt

	Male Inflow Entropy (Exempt from EE Act)	
	(1)	(2)
Percentage of Male Employment	0.0212 (1.920)	0.0668* (7.572)
Log(Male Employment Size)		0.1726*** (17.356)
Change in Male Employment Size		-5.938e-05 (-0.624)
Male Average Wage		-7.544e-07 (-2.916)
Constant	2.0392*** (28.942)	0.6950* (6.492)
Observations	385	386
R-squared	0.0095	0.26
Robust t-statistic in parentheses *** p<0.01, ** p<0.05, * p<0.1		

Notes: (a,b,e,f) industries are binned into skill clusters for visualization purposes. The line of best fit is calculated using the 388 industries and controlling for the variables shown in regression tables (e–h).

Source: authors' compilation based on data.

Similarly, the relationship between the percentage of male employment and the male inflow entropy of an industry, for our treatment and control group, is shown in Figure 5(e) and (f), respectively. The resulting regression table quantifying the relationship between these two variables is also shown in Figure 5(g) and (h). A weaker relationship between the percentage of male employment and the male inflow entropy is found in both the compliant and exempt groups, compared to the female case. The compliant group has a slightly larger relationship (with regression coefficient of 0.1276) than the exempt group (with regression coefficient of 0.0668). The difference between these coefficients (0.0608) is smaller than the standard error of these coefficients (0.0806 and 0.0749, respectively). Therefore, there is no evidence that these relationships are significantly different. The R-squared values for both of these fitted linear models are also very low.

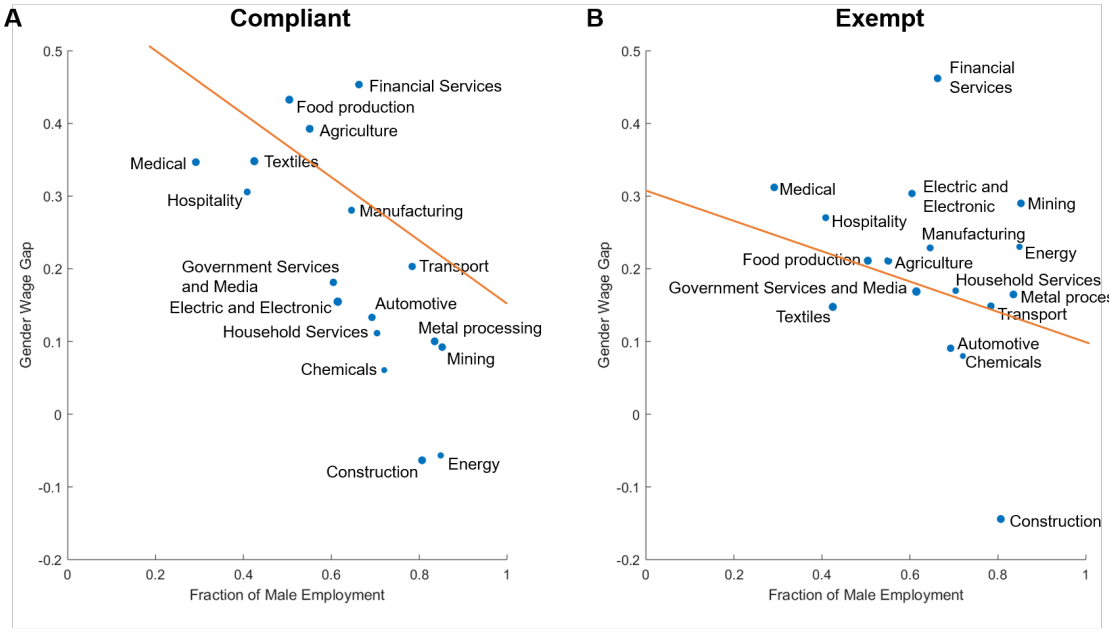
If we compare the relationship between the percentage of male employees and the male and female inflow entropy for both groups with each other, we can conclude that the strongest relationship is found between the percentage of male employment and the female inflow entropy for the group that complies with the EE Act. Male-dominant industries, therefore, have a higher diversity of female labour inflows. Although our results do not show causality, they suggest that male-dominant industries are most heavily impacted by the Act as they require the greatest restructuring of their workforce. They, therefore, recruit female workers from more diverse labour pools.

The impact of the EE Act on an industry's gender wage gap

Next, we evaluate whether male-dominant industries that comply with the EE Act have a smaller gender wage gap. Figure 6(a) and (b) shows the relationship between the percentage of male employment within an industry and the industry's gender wage gap, using Equation (14) as discussed in Section 4.6, for the treatment and control groups, respectively. The resulting regression tables quantifying the relationship and controlling for various factors are shown in Figure 6(c) and (d). We observe a general decrease in the gender wage gap as the percentage of male employment increases in an industry. This relationship is strongest within the group that complies with the Act, with a regression coefficient of -0.4886 compared to -0.2639. The difference between these coefficients (0.2247) is larger than the standard error of both regression coefficients 0.0715 and 0.0708, respectively. The difference between these relationships is significant.

Therefore, the gender wage gap is smaller among male-dominant industries. This relationship is also more pronounced among firms that comply with the EE Act. This adds evidence in support of our hypothesis that the EE Act has the largest impact on male-dominant industries as they require the greatest restructuring of their workforce. The firms within these industries are therefore increasing the female wage as a recruitment strategy to keep and attract female workers.

Figure 6: (a,b) Graph showing the relationship between the percentage of male employment and the gender wage gap over the period 2011–14 for both the group of firms who comply with the EE Act and those that are exempt. (c,d) The resulting regression table quantifying the relationship between the percentage of male employment and the gender wage gap, controlling for employment size, change in employment size, and average wage



C

Compliant

	Gender Wage Gap (Comply with EE Act)	
	(1)	(2)
Fraction of Male Employment	-0.4688*** (4.087)	-0.4886*** (-5.6306)
Log(Employment Size)		0.02923*** (3.3163)
Change in Employment Size		-7.824e-07 (-0.9546)
Average Wage		0.1052*** (3.9274)
Constant	0.5323* (2.6895)	0.6169* (1.901)
Observations	386	386
R-squared	0.1084	0.131
Robust t-statistic in parentheses *** p<0.01, ** p<0.05, * p<0.1		

D

Exempt

	Gender Wage Gap (Exempt of EE Act)	
	(1)	(2)
Fraction of Male Employment	-0.3088*** (12.372)	-0.26392*** (-5.1419)
Log(Employment Size)		-0.0068 (-0.6848)
Change in Employment Size		4.9943e-06 (0.69676)
Average Wage		0.0917*** (3.4745)
Constant	0.4323*** (4.6895)	0.3024*** (4.5874)
Observations	386	386
R-squared	0.0553	0.0768
Robust t-statistic in parentheses *** p<0.01, ** p<0.05, * p<0.1		

Notes: (a,b) industries are binned into sectors for visualization purposes. The line of best fit is calculated using the 388 industries and controlling for the variables shown in the regression tables (e,f).

Source: authors' compilation based on data.

6 Conclusion and future work

The goal of this paper was to determine the impact of the EE Act on labour mobility and the gender wage gap. We find that the EE Act caused an increase in both the diversity of female labour flows and the female average wage. No significant impact is found among the corresponding male factors. Next, we find that as the percentage of male employment within an industry increases, its female labour flow diversity also increases. This relationship is also significantly stronger among firms that comply with the EE Act compared to those that are exempt. We also find that as the percentage of male employment increases, the gender wage gap decreases. This relationship is significantly stronger among firms that comply with the EE Act. Although this does not show causality, it provides strong support for the fact that the EE Act has more heavily impacted male-dominant industries. Male-dominant industries therefore show the greatest implementation of recruitment strategies in order to feminize their work-

force: they have diversified recruitment to a larger number of sectors and increased the average female wage.

There are two main avenues for future work. The first includes investigating the impact of the 2004 EE Act amendment. The amendment included an increase in the annual revenue threshold that determines whether firms with fewer than 50 employees need to comply with the Act. Various firms that previously had to abide by the Act became exempt. The amendment therefore caused some firms with fewer than 50 employees to have less stringent labour regulations. Investigating the impact of the amendment on the structure of the labour market would therefore be an interesting avenue of further study. The second avenue includes investigating the impact of the increase in diversity of labour flows on growth and innovation. This study showed that the EE Act enhances the diversity of female labour flows for both firms and industries. Investigating the impact of this increase in skill diversity within a firm or industry on its growth and innovation could be very interesting. However, this analysis may require the availability of patent data in order to quantify innovation and would need to be matched to the data source used within this study.

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Appendix

A1 Data-cleaning strategy and validation

The employment panel within the individual-level panel, originally created by Ebrahim and Axelson (2019), was used for this study in order to count the labour flows between firms or industries. The CIT–IRP5 dataset, originally created by Pieterse et al. (2018), was also merged with this dataset in order to add firm-level characteristics to the labour flows. Only observations within the 2011–14 tax years were included within our study.

First, we apply a data-cleaning strategy to the employment panel dataset in order to ensure that all entries represent individuals within the South African labour force. The dataset is cleaned to ensure the following criteria hold for each observation:

1. be a natural person and thereby contain a personal identification code;
2. be part of the working-age population (individuals between aged 15–65);
3. be in full-time employment and therefore only having a single job at any given period;
4. receive remuneration greater than ZAR2,000 and less than ZAR10,000,000 per annum; and
5. work at a firm that has a valid industry classification code.

To determine how representative the cleaned dataset is of the South African labour force, it is compared to employment estimates in the South African Quarterly Labour Force Survey (QLFS). Table A1 shows the comparison of the number of formally employed individuals contained in both of these datasets for each year. Our dataset seems to cover a significantly smaller percentage of individuals within the first two years. However, thereafter the dataset contains approximately a 4.4 per cent larger employment size than the QLFS. This makes sense as the QLFS excludes all employment within the agriculture industry in its estimate.

Table A1: The size of formal employment in the cleaned employment panel dataset compared to the estimate of formal employment size in the Quarterly Labour Force Survey for each tax year

	Total formal employment (in cleaned dataset)	Total formal employment* (in QLFS)	% difference between cleaned dataset and QLFS
2011	6,299,958	9,182,600	–31.39
2012	7,960,501	9,395,400	–15.27
2013	9,791,577	9,534,700	2.69
2014	11,497,587	10,839,600	6.07

Notes: * this value excludes any employment from the agricultural industry.

Source: authors' compilation based on data and the QLFS data.

Next, we merge the CIT–IRP5 dataset to this cleaned dataset. We add the firm size, labour expenses, and total revenue variables to each entry in our cleaned dataset. Therefore, for each observation that consists of an employee working for a firm these additional characteristics are added about the firm in our dataset. Note that we define a firm on a company basis (using the firm's tax reference number) and not on a branch basis (using the PAYE number). We remove all firms that have either a firm size or a labour cost of ≤ 0, as these are not operating firms.

A2 Markov stability algorithm for community detection

Within this study, we adopt the Markov stability algorithm for community detection to detect the industry clusters within the skill-relatedness network. We formally define the workings of the algorithm as well as discuss its implementation in the South African skill-relatedness network.

The Markov stability algorithm is a dynamic community detection algorithm. It uses a dynamic random walk process on a network to study its structure. Intuitively, if we let a random walker wander on a network (jumping from node to node) and the walker remains trapped within a group of nodes over an extended period, this indicates that the group of nodes is tightly connected and a community exists. The algorithm also uses time as an intrinsic resolution parameter to obtain a range of partitions at different scales (many small clusters to a few large clusters). If the random walker is allowed to wander for a long period, it can move between more industries and thus the node aggregation is larger. Therefore, the Markov stability algorithm is aimed at maximizing the probability that a random walker remains in the community in which it started (the stability function) during a time interval that represents an intrinsic resolution parameter. Maximizing the stability function is the optimization strategy that this algorithm adopts.

In order to formally define the stability function, $r(t)$ at a particular resolution t, let $\mathbf{A}$ be the adjacency matrix and $\mathbf{D} = \text{diag}(\mathbf{A} \cdot \mathbf{1})$ the corresponding diagonal matrix where each (i,i) element in the matrix is the strength of node i. $\mathbf{D}^{-1}\mathbf{A}$ is then the transition matrix of the random walker. The updating rule that describes the random walkers diffusion process on a graph is then given by:

$$\mathbf{P}_{t+1} = \mathbf{P}_t \mathbf{D}^{-1}\mathbf{A} \tag{15}$$

where $\mathbf{P}_t$ is a vector showing the probability that a random walker is at each node at time t. If the graph is non-bipartite, non-directed and connected, the process converges to an equilibrium state irrespective of the starting position of the random walker. The stationary probability distribution is given by $\pi = \mathbf{d}^T/(2m)$.

Next, let $\mathbf{H}$ be an indicator matrix that encodes the resulting node partition:

$$\mathbf{H}(i,c) = \begin{cases} 1, & \text{if node } i \text{ is in community } c \text{ and} \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

Finally, the clustered auto co-variance matrix of the diffusion process can then be given by:

$$\mathbf{R}(t) = \mathbf{H}^T[\text{diag}(\pi)(\mathbf{D}^{-1}\mathbf{A})^t - \pi^T\pi]\mathbf{H} \tag{17}$$

Note that $R(t)_{ij}$ calculates the probability that a random walker who started in community i ends up in community j at time t discounted by the probability that two independent random walkers will be at community i and j in equilibrium. Therefore the diagonal entries of $R(t)$ represent the probability that a random walker remains in their initial community after t time steps. The stability function at time t is therefore given as:

$$r(t) = \text{trace}(\mathbf{R}(t)) \tag{18}$$

The Markov stability algorithm seeks to find an indicator matrix $\mathbf{H}^*$ that maximizes the stability function $r(t,\mathbf{H}^*)$. Note that t is used as an intrinsic resolution parameter. The longer the time interval (t), the more steps are considered by which the random walker wanders and thereby the larger the size of the detected communities. If a large enough time interval is used, then the system reaches equilibrium and the whole network is considered as a single community. Now, the problem of finding the optimal partition that maximizes the stability function is an NP-hard problem. Therefore, a heuristic is needed. Within this study, the Louvain algorithm (Blondel et al. 2008) is used, mainly for its computational speed. The Louvain algorithm (Blondel et al. 2008) works in two phases. In the first phase, each node is assigned to its own community. Then, for node i (chosen at random), the algorithm considers the stability gain that would occur if node i was removed from its community and placed in the community of its neighbour j. The algorithm considers the stability gain that would occur if node i were placed in each of its neighbours' communities. It then places node i into the community that allows for the maximum stability gain, but only if it is positive. If there is no positive gain, i remains in its community. This process is applied repeatedly and sequentially on all nodes until no change occurs. This completes

the first phase of the algorithm. The second phase consists of creating a new aggregated network in which each node is now defined as the communities of the previous network. The edge weights between nodes are given by the sum of the edges between communities. The first phase is then reapplied on the new network. The two phases are repeated, iteratively. Note that the algorithm is stochastic and therefore results in a different node partition each time the algorithm is applied. The algorithm is therefore often repeated multiple times and the partition allowing for the maximum stability used. We repeat the Louvain algorithm 10,000 times in our analysis.

The result of the Markov stability algorithm is a set of node partitions at different Markov times. However, in order to determine which of the partitions are most representative, we evaluate each partition's robustness. We calculate the robustness of the optimal partition at each Markov time by evaluating each pair of partitions at each Markov time using the *variation of information* measure (Meila 2003) and finding the average. The variance of information is given by:

$$\mathrm{VI}(c_1, c_2) = 2H(c_1, c_2) - H(c_1) - H(c_2) \tag{19}$$

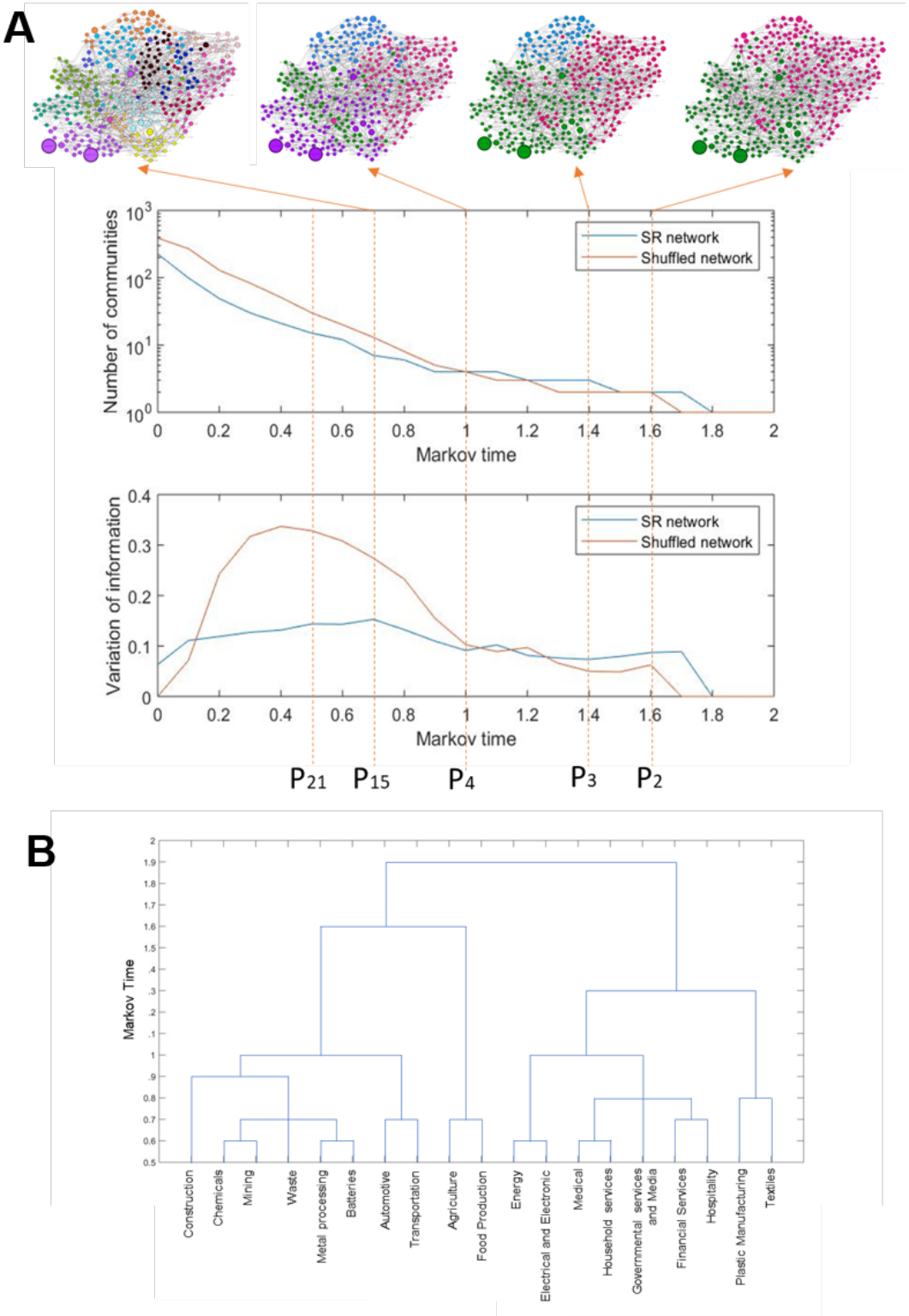
where c_i is a partition vector, $H(c_1, c_2)$ is the Shannon entropy of the joint partition, and $H(c_i)$ is the Shannon entropy of the partition vector c_i. Note that if two partitions are similar, they will have a low variation of information. Robust partitions are therefore found by identifying Markov times for which the variance of information value is low.

We apply the Markov stability algorithm to the South African skill-relatedness network. Figure 7(a) shows both the number of communities and the corresponding variance in information obtained. It can be seen that there are multiple Markov times for which the computed partition is robust (i.e. times for which the variation of information is at a local minimum). These partitions are denoted by P_n, where n is the number of communities and marked on the figure with orange dotted lines. The resulting community structures of the network from these partitions are also illustrated above the graph. The partition $P_2 1$ is used as the aggregated industry classification and as the industry clusters within this study.

We also compare the partitions with those detected on a random network to show the robustness of the results. The random network is constructed by randomly shuffling the edges in the network. The results from the random network are shown in orange in the figure. Note that for a large portion of the Markov time the variation of information of the skill-relatedness network is significantly less than that of the random network. Therefore, compared to a random network, the communities detected within the skill-relatedness network are robust. Note that for Markov times greater than 1, the partitions found are no longer stable as the results are roughly the same as for the shuffled network.

Using the method of O'Clery et al. (2019), we use the range of partitions to understand the nested structure of the South African labour market. We use the resulting range of partitions to indicate the different aggregation levels within our new industry classification. The various levels within the classification also show how different labour pools are aggregated. This is illustrated with a dendrogram in Figure 7(b). The dendrogram only shows classifications that contain fewer than 21 industries (thereby Markov times greater than $t = 0.5$). Furthermore, note that the partitions are not strictly hierarchical and a simple majority rule is used to assign clusters to parent clusters in coarser partitions. Two small (containing fewer than four industries) industry clusters containing non-related industries are considered as noise within the study and omitted from our analysis. Note that the persistence of a cluster within the dendrogram (a cluster that remains intact over an interval of time) shows that these communities are particularly strongly connected and well defined.

Figure 7: (a) A graph showing the number of communities and the variation of information of the node partition generated by the Markov stability algorithm at different Markov times for both the South African skill-relatedness network and a shuffled edge version of this network. (b) A dendrogram showing the merging process of the industry clusters



Notes: (a) the blue line represents the skill-relatedness network and the orange a version where the edges have been randomly shuffled. Robust partitions at various scales are shown with vertical dotted lines in orange and the corresponding partition of the network visualized above the figure.

Source: authors' compilation based on data.