Dynastic measures of inter-generational mobility with empirical evidence from Indonesia

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Abstract: We suggest a simple and flexible criterion to assess inter-generational mobility. It accommodates different types of outcomes (continuous outcomes such as potential earnings, or discrete ones such as education groups) and captures dynastic improvements of such outcomes at different points of the initial distribution. We provide dominance characterizations—for instance, on the relative progress made by women vs men—that are consistent with social preferences upon desirable patterns of mobility. We suggest an application for Indonesia. Exploiting IFLS data to match parents observed in 1993 to their children in 2014, we provide one of the rare inter-generational mobility analyses based on a long panel in the context of a poor country. Results indicate that mobility in terms of education and potential earnings were markedly to the advantage of women. A large part of the population was lifted out of illiteracy, possibly due to large-scale education and school construction reforms. However, our mobility index also shows that educational mobility was regressive and has, in turn, reduced the progressivity of the mobility in terms of potential earnings.

Key words: inter-generational mobility, education, social welfare, Indonesia, gender

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1 Introduction

Inequality in a society is perhaps more bearable if it is accompanied by high economic mobility between generations. On this condition, parents can invest in their children’s education to ensure them a better future than their own. A growing academic literature aims to address this question, suggesting ways to measure and characterize patterns of inter-generational mobility (Jäntti and Jenkins 2015). Importantly, long panel datasets are becoming increasingly available, and provide researchers with the possibility to compare various sorts of achievements across generations. The literature on rich countries relies on this type of data, often from administrative sources, to produce increasingly refined analyses of inter-generational mobility (Black and Devereux 2011). The literature remains very scant for low- and middle-income countries precisely because of the limited availability of long and robust panels.

On the methodological side, inter-generational mobility studies often rely on a single-valued mobility elasticity stemming from the regression of children’s outcomes on parents’ outcomes. Jäntti and Jenkins (2015) emphasize the need for more general characterizations of inter-generational mobility. Recent studies go in this direction and report more comprehensive patterns based on mobility curves (e.g. Bratberg et al. 2017; Markussen and Røed 2019). However, these measures are specifically designed according to the variable chosen to represent individual economic status and on which mobility is calculated. There is no encompassing framework that allows measuring inter-generational mobility in a disaggregated way—that is, comparing mobility at different points of the initial distribution, while accommodating any type of achievement based on either continuous or discrete outcomes. Finally, there is a rich literature on dominance results for multidimensional inequality, but few operational methods to assess inter-generational mobility patterns within a normative framework.

This paper attempts to address both types of concern. On the methodological side, we suggest a simple criterion to evaluate and compare inter-generational mobility processes. It builds upon information on mobility experienced at the individual level—that is, for a parents–children dynasty defined according to the first-generation distribution—and relies on ‘dynastic curves’ to generate partial but robust rankings of mobility processes. The approach has several advantages. First, it enables us to assess if and how mobility differs at different points of the distribution, and can be implemented to compare the mobility of subgroups of the population (e.g. differential mobility patterns for women and men).

Second, the approach is consistent with a general definition of parents’ and children’s achievement. It can be used interchangeably to evaluate mobility when the achievement variable is either cardinal or only ordinal, making it general enough to assess the mobility experienced in different domains of well-being. Our application will focus on achievements based on cardinal outcomes (potential earnings) or ordinal outcomes (discrete education categories). The approach enables us to incorporate weighting mechanisms that account not only for the position of a given dynasty in each generation, but also for the distance that separates it from others. It allows us to go beyond re-ranking measures and to additionally account for structural changes in the distribution.

Third, our framework is closely linked to the inequality and social welfare literature, which helps to give a normative justification for its implementation. The normative support allows us to rationalize the use of mobility curves and to suggest and implement additional measurement tools (e.g. higher-order dominance results), which are particularly helpful when mobility curves alone do not allow us to rank mobility patterns. It leads to dominance characterizations—for instance, on the relative progress made by women versus men—that are consistent with social preferences upon desirable patterns of mobility.

On the empirical side, our application consists of one of the few inter-generational mobility analyses for low-/middle-income countries. We focus on education and earning mobility in Indonesia. To our knowledge, this is one of the rare developing countries for which a long representative panel data exists.
and can be used to measure inter-generational mobility. Thus, we exploit the features of the Indonesian Family Life Survey (IFLS) data, namely the long duration and the exceptionally low attrition of this household panel. It enables us to match parents in 1993 with their children in 2014 to extract and compare outcomes of both groups in each period. As argued above, this is relatively rare in the context of low- and middle-income countries, for which mobility studies are often based on specific households (e.g. cohabiting parents and children) or on recall questions about parents’ labour market outcomes and human capital.

The results point to broad improvement in education levels over a generation, lifting a large part of the population out of illiteracy. These changes are very likely due to the large-scale education policies implemented in Indonesia and extensively documented in the economic literature (see, *inter alia*, Akresh et al. 2018; Ashraf et al. 2020; Duflo 2001, 2004; Mazumder et al. 2019). Several reforms have indeed provided universal primary education and expanded access to secondary education in the second generation considered in our analysis.¹ Mobility patterns reveal that, independently of the outcome considered, women have progressed faster than men. Yet, a striking result is that educational mobility was regressive: progress in education was a lot more pronounced among economically advantaged dynasties. A decomposition analysis shows that these trends seriously limit the degree of progressivity observed in terms of potential earnings mobility. These results shed new light on the long-term implications of prominent education reforms and the way they reshape the distribution of human capital and that of potential labour market outcomes.

The structure of the paper is as follows. Section 2 suggests a brief survey of the related literature. Section 3 outlines the approach and suggests a normative characterization of patterns of mobility, including dominance results to make welfare-consistent comparisons across mobility processes. Section 4 presents the empirical application for Indonesia. Section 5 discusses the results of our inter-generational mobility analysis for education and earnings. Section 6 concludes.

## 2 A brief account of the literature

### 2.1 Methods to measure mobility

Most inter-generational mobility studies are based on a single linear parameter derived from regressions of children’s outcomes on parents’ outcomes and controls.² Such inter-generational elasticity coefficients are useful single-valued summary measures of mobility. They allow controlling for both life cycle effects and transitory shocks. They are widely used so that estimates can be compared across studies, across countries, and over time. However, these measures also show strong limitations. They are not informative on whether the society has faced upward or downward mobility. Thus, comparisons between different episodes of mobility performed using these indicators need to be interpreted with caution. Also, these measures are not sensitive to changes in the marginal distribution of income. Finally, they are not able to capture mobility differentials across different groups in the population or between different segments of the distribution.³

The recent literature adopts a more disaggregated approach that captures the mobility experienced by different parts of the distribution and, in doing so, provides a more detailed picture about the specific features of the mobility process under analysis (see, among others, Bratbert et al. 2017; Markussen and

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¹ This is the case of the INPRES reform, the One Roof School programme, and several scholarships programmes, as discussed in Section 5.


³ This last issue can in principle be overcome by the quantile regression technique, as used by Eide and Showalter (1999).
However, such measures are precisely designed according to the variable chosen to represent the achievement of an individual (and mobility thereof), while they could be more general and, for instance, distinguish between ordinal and cardinal outcomes (Klasen 2008; Klasen and Reimers 2017). We suggest a tool that can be used interchangeably to evaluate the mobility of cardinal and ordinal variables while providing normative support to our framework as well as dominance results.

2.2 Empirical results and challenges

Inter-generational mobility has been an active field of research (see the surveys by Björklund and Jäntti 2009; Black and Devereux 2011; Jäntti and Jenkins 2015; Solon 1999). In particular in the case of education mobility, early studies include Bowles (1972), Blake (1985), and Spady (1967). In more recent studies, Card et al. (2018) investigate education mobility in the USA, while Hertz et al. (2008) estimate country-level mobility coefficients across 42 countries. Because of the difficulty matching information on both parents and children, studies on low- and middle-income countries are limited. Alesina et al. (2020) use census data on cohabiting parents and children to explore education mobility across 26 African countries. Bossuroy and Cogneau (2013) study occupational mobility in five sub-Saharan African countries. Fontep and Sen (2020) estimate inter-generational persistence of occupation and education status in Cameroon, suggesting an interesting gender comparison.

Importantly, many of these studies do not rely on panel information and must find ways to retrieve information for both parents and children using specific selections (e.g. cohabiting families) or recall information.

2.3 Normative approaches to assess mobility

Our normative approach aims to provide dominance results and welfare foundations to characterize inter-generational mobility. Hence, it naturally relates to the tools adopted in the context of multidimensional inequality and intra-generational mobility (see the very detailed exposition from Jäntti and Jenkins 2015). Dominance checks extended to mobility patterns can find their origins in results for multivariate distributions of income (Atkinson and Bourguignon 1982; Gottschalk and Spolaore 2002; Markandya 1984). The framework suggested by these authors is commonly interpreted as an aggregation of intertemporal utilities defined over two periods. All relevant mobility is captured by the changes in individuals’ ranks, making social welfare results sensitive to mobility as reversals rather than mobility as origin dependence. In other words, the utilitarian social welfare framework does not incorporate evaluations of mobility in the form of individual income growth, which is the interesting dimension for inter-generational mobility. An exception is Bourguignon (2011), who shows that the Atkinson and Bourguignon (1982) results can be applied to comparisons of alternative ‘growth processes’ when the pair of marginal distributions relating to the first period are identical (this restriction unfortunately limits the applicability of the approach). Related to this, several contributions have defined social welfare

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4 Bratberg et al. (2017) specifically consider changes in income rank and income share between parents and children. Markussen and Røed (2019) look at rank and class correlation between parent and children for each vigintile of the parent distribution for different income and non-income variables.

5 Note also that from an empirical point of view, the aforementioned contributions focus on rich countries, benefiting from outstanding data to characterize mobility over several generations in a very representative way. Our work rather focuses on inter-generational mobility in a poor country for which only survey data are available.

6 Novel data on inter-generational mobility in 18 Latin American countries are provided by Neidhöfer et al. (2018). Several studies also focus on inter-generational mobility in India (Asher et al. 2018; Azam and Bhatt 2015; Emran and Shilpi 2015) and China (Emran and Sun 2015; Emran et al. 2020; Golley and Kong 2013).

7 Alesina et al. (2020) match individuals to their parents using data on cohabitants of different generations, which makes the final sample quite selective (less than 10 per cent of the initial data). Fontep and Sen (2020) retrieve parents’ information thanks to retrospective questions.

8 Aspects such as aversion to intertemporal fluctuations and to future income risk are specific to this interpretation and absent from the inter-generational perspective.
explicitly in terms of income mobility—that is, income changes rather than income levels. This literature, focusing again mainly on intra-generational mobility interpretations and applications, assumes that individual-level mobilities are represented by concepts of ‘distance’ between first- and second-period incomes (Fields et al. 2002; Jenkins and Van Kerm 2016; Van Kerm 2009). It has been extensively used to characterize where income growth has benefited specific segments of the population, notably the poor, using non-anonymous growth incidence curves (Berman and Bourguignon, 2021; Bourguignon 2011; Grimm 2007; Jenkins and Van Kerm 2016; Lo Bue and Palmisano 2020; Palmisano 2018; Van Kerm 2009). Our approach is closely related to this literature but adapted to the inter-generational perspective. We also go beyond the usual focus on income and suggest a flexible framework to accommodate any type of achievement (including non-cardinal measures such as discrete educational classes).  

3  Theoretical framework

We first describe the methodology used to measure inter-generational mobility, then discuss welfare-related characteristics of mobility patterns.

3.1 Measuring inter-generational mobility

Mobility means different things to different people (Fields 2008). We first present the definition of mobility that we adopt in this paper, then propose a set of tools to evaluate mobility in a way that is consistent with our definition. We interpret the degree of mobility as the extent to which individual achievements get better or worse across generations. Using the term ‘individual’ implies that we will compare the achievements of a person and her parent, i.e. of a specific dynasty, over the duration of a generation, retaining the principle of non-anonymity. To obtain an evaluation of such changes, we follow a three-step procedure that will define dynasties, achievements, and the mobility measure itself. Finally, we shall see that we endorse a relative approach to measuring mobility.

Dynasties

The first step corresponds to the definition of a rule that allows tracking individual achievements across generations, which we will refer to as a ‘dynasty’. We denote $t$ the generation of the parents and $t+1$ that of their children. Dynasties will be defined according to an outcome $z$ used to characterize the first-generation distribution. We will use per-capita expenditure in our application as it relates to notions of welfare (or living standards) and provides a very disaggregated picture of the population (at least compared to other variables such as income groups or education classes). Following the standard literature on mobility, the dynasty will correspond to the relative position of the parents in the initial distribution of $z$. Let the $z$-distribution observed for generation $t$ be represented by its cumulative distribution function (CDF) $F_z(z_t) = P(\tilde{z}_t \in \mathbb{R}_+: \tilde{z}_t \leq z_t)$. Each dynasty will simply be characterized by this probability, denoted $p_t \in [0,1]$, which returns the proportion of people observed below $z_t$ in generation $t$.

Achievements

The second step consists in the definition of an individual’s achievement. Note that in a general representation, the outcome used to measure achievements may differ from $z$. For instance, we may study the mobility in terms of educational achievements or earnings at different points of the distribution of liv-

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9 Note that a few contributions adopt transition matrices rather than rank comparisons when individual incomes are classified into discrete classes (see in particular Dardanoni (1993) for stochastic dominance results for rankings of mobility processes summarized by transition matrices).
ing standards. Different mobility studies adopt different definitions of achievement depending on the focus of the analysis (D’Agostino and Dardanoni 2009; Fields and Ok 1999). Two main approaches can be detected. The first interprets an individual’s achievement as something that is measurable in terms of level of any outcome variable. The second approach interprets an individual’s achievement as something that is only measurable in relation to the rank of an individual in the outcome distribution of interest. A measure of mobility based on an index of achievement constructed using the first approach would treat an ordinal variable (e.g. education categories) as a cardinal one. An important issue that characterizes ordinal data is that the mean is not order-preserving under scale changes (Allison and Foster 2004), whereas distributional orderings based on comparisons of CDFs are robust to changes in the scale. This mitigates for the use of CDF comparisons when dealing with ordinal data, as recently popularized by Cowell and Flachaire (2017, 2018) and Jenkins (2019, 2020). This is the line of reasoning that we adopt here and, hence, we shall follow the second approach.

We first consider a continuous outcome denoted by \( y \), which is typically the case of monetary variables such as individual earnings. We represent by \( y_t(p_t) : [0, 1] \rightarrow \mathbb{R}_+ \) the earnings of the parents (generation \( t \)) for a dynasty \( p_t \). In the same way, we denote by \( y_{t+1}(p_t) \) the earnings of the children (generation \( t+1 \)) of this dynasty \( p_t \). We denote \( f \) and \( F \) the probability density function and CDF of the outcome considered to characterize achievements. A measure of achievement for the parents and children of any dynasty \( p_t \) can respectively be written as follows:

\[
\forall p_t \in [0, 1], \quad \begin{align*}
    a_t^{(1)}(p_t) &= \int_0^{y_t(p_t)} f(y_t(s_t)) dy_t(s_t) \\
    a_{t+1}^{(1)}(p_t) &= \int_0^{y_{t+1}(p_t)} f(y_{t+1}(s_t)) dy_{t+1}(s_t)
\end{align*}
\]

In the space of a continuous measure such as earnings, for a dynasty \( p_t \), the parents’ achievement is represented by the fraction of individuals of generation \( t \) with earnings equal to or below the level \( y_t(p_t) \) achieved by the parents of dynasty \( p_t \). Their children’s achievement is measured by the fraction of individuals of generation \( t+1 \) with earnings equal to or below the level \( y_{t+1}(p_t) \) achieved by these children of dynasty \( p_t \).

Let us now consider a discrete outcome, which is typically the case of non-monetary variables that are ordinal but not cardinal, such as education classes (or health status, occupation types, etc.). Let there be an ordered set of \( K > 2 \) classes, each class \( k \) being associated with a latent outcome level—for instance, the educational attainment. Let \( k_t(p_t) \) (or \( k_{t+1}(p_t) \)) be the class occupied by parents (children) of dynasty \( p_t \), \( n_{t,k} \) (or \( n_{t+1,k} \)) the number of individuals in this class, and \( N_t \) (or \( N_{t+1} \)) the total number of individuals in the population of parents (children). Achievement for parents and children can be respectively expressed as follows:

\[
\forall p_t \in [0, 1], \quad \begin{align*}
    a_t^{(2)}(p_t) &= \frac{\sum_{i=1}^{k_t(p_t)} n_{i,t}}{N_t} \\
    a_{t+1}^{(2)}(p_t) &= \frac{\sum_{i=1}^{k_{t+1}(p_t)} n_{i,t+1}}{N_{t+1}}
\end{align*}
\]

In the space of discrete variables such as education classes, for a dynasty \( p_t \), the parents’ achievement is represented by the fraction of individuals of generation \( t \) who belong to the same or to a lower class than the class reached by these parents. Their children’s achievement is measured by the fraction of individuals of generation \( t+1 \) who belong to the same or to a lower class than the class reached by these parents.

\(^{10}\) Indeed, these measures more precisely point to the mobility of specific individuals (e.g. men versus women), which we can try to explain. In contrast, how the per-capita expenditure of their dynasty’s households evolve over time depends on many factors that would be extremely hard to disentangle (labour markets, marriage markets, the prevalence of customs such as matrilocality, our Indonesian context, etc.).
children of dynasty \( t \). Hence, our concept of achievement is independent of scale and allows us to treat cardinal and ordinal variables in a uniform way.

**Weighted achievements**

We also consider complementary definitions obtained by introducing weights in the achievement schemes previously defined. In the case of continuous outcomes, these are expressed as follows:

\[
\forall p_t \in [0, 1], \quad \begin{cases} 
  a_t^{(3)}(p_t) = \int_0^{y_t(p_t)} f(y_t(s_t))(F(y_t(p_t)) - F(y_t(s_t)))dy(s_t) \\
  a_{t+1}^{(3)}(p_t) = \int_0^{y_{t+1}(p_t)} f(y_{t+1}(s_t))(F(y_{t+1}(p_t)) - F(y_{t+1}(s_t)))dy(s_t)
\end{cases}
\]  

(3)

In the case of discrete outcomes, these are written as follows:

\[
\forall p_t \in [0, 1], \quad \begin{cases} 
  a_t^{(4)}(p_t) = \frac{\sum_{i=1}^{k_i(p_t)} n_{t,i} \sum_{s=i+1}^{k_i(p_t)} n_{s,j}}{N_t^2} \\
  a_{t+1}^{(4)}(p_t) = \frac{\sum_{i=1}^{k_{t+1}(p_t)} n_{t,i+1} \sum_{s=i+1}^{k_{t+1}(p_t)} n_{s,j}}{N_{t+1}^2}
\end{cases}
\]  

(4)

For a dynasty \( p_t \), individual achievement is represented by the fraction of individuals of generation \( t \) \((t + 1)\) attaining a level or class lower or equal to that of the parents (children) of this dynasty, now weighted by the distance that separate each of these individuals from the parents (children). The distance is simply the density of people between each of these individuals and the parents (children) of this dynasty, measured as a difference in ranks in the continuous approach of equation 3 or by summing intermediary classes in the discrete approach of equation 4.

**Mobility**

The last step consists in the construction of a measure of mobility, the Dynastic Curve (DynaC hereafter):

\[
d(a^{(s)}(p_t)) = a_{t+1}^{(s)}(p_t) - a_t^{(s)}(p_t), \forall p_t \in [0, 1] \quad \text{and} \quad s = 1, 2, 3, 4
\]  

(5)

DynaC allows measuring relative inter-generational mobility at a disaggregated level and in a consistent way for both cardinal and ordinal outcomes. DynaC associates to every dynasty \( p_t \) the difference between children’s achievement and their parents’ achievement. Since achievement is distribution-dependent, \( d(a) \) is a relative measure of mobility—that is, inter-generational mobility is evaluated according to how better or worse, with respect to their parents, children are positioned in the outcome distribution of their generation. For instance, if children of a dynasty \( p_t \) attain the same level of education as their parents (or progress only a little) while other dynasties progress a lot more, then children of dynasty \( p_t \) dominate fewer people than their parents used to and the mobility score of dynasty \( p_t \) is negative. Hence, we shall refer to ‘positive’ (‘negative’) mobility to convey relative upward (relative downward) mobility—that is, the fact that a dynasty attains a better (worse) rank in the second generation than in the first.

We can also suggest an interpretation of DynaC in the weighted case, i.e. for \( s = 3, 4 \). Assume the child in a given dynasty obtains the same unweighted achievement in two alternative distributions, i.e. he dominates the same number of people in both distributions. His weighted achievement needs not be the same. It will be higher in the distribution where the people the child dominates are further below. This is an interesting interpretation for mobility because it means that for a positive unweighted DynaC, which indicates that the dynasty improves its rank between a generation and the next, the progress made by this dynasty is all the faster as the weighted DynaC is large. Thus, while DynaC can be seen as a simple
measure of reranking in its unweighted formulation of the continuous case, the weighted approach adds more information related to the change in the shape of the distribution. In our example above, there is an increase in inequality below the dynasty in question. Take a somewhat opposite situation, for instance a dynasty with a zero unweighted DynaC and a negative weighted DynaC. It can be interpreted as a decline in inequality below this dynasty in the second generation, as the people dominated by this dynasty catch up and concentrate just below it. In other words, a dynasty can well preserve her rank (zero DynaC) but experience a negative weighted mobility through this reshaping of the distribution below it.\(^\text{11}\)

### 3.2 Normative justification

We now provide a normative support to the use of DynaC to evaluate and compare mobility episodes. Let \(D^{(t,t+1)}\) be the general process of mobility between two generations. We are interested in judging this process from a normative perspective. We assume that a social planner is endowed with cardinal preferences over mobility processes, denoted by \(W(D)\), and write as \(P\) the set of social preferences. A social planner with preferences \(W \in P\) may assess whether the mobility of a given process, say \(D^{(t,t+1)}\), is socially superior to immobility by evaluating whether \(W(D^{(t,t+1)})\) is larger than a benchmark process of immobility for all the dynasties. Considering two mobility processes \(D^{(t,t+1)}\) and \(D^{(t,t+1)}\), the planner may deem the first process socially preferred to the second if \(W(D^{(t,t+1)}) \geq W(D^{(t,t+1)})\). If the set of social preferences were known, we could directly conduct these types of assessments. We do not have this information and, hence, should in principle conduct a sensitivity analysis over a reasonable range of preferences.

In practice, we start by imposing some restrictions. Namely, we reformulate the dominance in terms of observables—that is, preferences over mobility processes are specified as functions of the observed distributions of achievements. To represent such preferences, we adapt the rank-dependent model proposed by Yaari (1987), which offers theoretical and empirical tractability. It assumes that social welfare can be written as a weighted average of all possible realizations, where the weights are a function of the rank of the realizations. Transposed to our mobility problem, it becomes the weighted average of mobility measures \(d(a^{(s)}(p_t))\) over all dynasties in the population, with a weight \(w(p_t) \geq 0\) assigned to the mobility of dynasty \(p_t\). Thus, the social evaluation of any inter-generational mobility process, indexed by \(\pi\), is written as:

\[
W(D^{(t,t+1)}) = \int_0^1 w(p_t) d_\pi(a^{(s)}(p_t)) dp_t \tag{6}
\]

The extent of mobility, as measured by the DynaC, is computed for one of the \(s = 1, 2, 3, 4\) definitions of achievement introduced in the previous section. Let us rewrite \(d_\pi(a^{(s)}(p_t))\) as \(d_\pi(p_t)\) hereafter to simplify notations. We will restrict to a set of social preferences

\[
P' = \{W : w(p_t) \geq 0 \forall p_t \in [0,1]\} \tag{7}
\]

such that the social marginal effect of each dynasty’s mobility is positive. We suggest four propositions, the proofs of which are reported in Appendix B.

\(^{11}\)The weights could alternatively be defined as a distance in levels of earnings or education. Our definition captures changes in the distribution while abstracting from any form of cardinalization, i.e. it uses density of population between discretized groups such as education classes. Related to this, further work could explore the link with recent studies on non-anonymous growth incidence curves (NAGIC), which are changes in levels (for instance income levels) of given dynasties while DynaC is a change in ranks. Berman and Bourguignon (2021) suggest a decomposition of NAGIC into a mobility effect (reranking) and a shape effect (i.e. a change in the whole cross-sectional distribution). Our weighted DynaC adds to the rank mobility a weight based on the change in shape (but focused on the part of the distribution below each dynasty).
Proposition 1: Given two mobility processes $D_\pi^{(t,t+1)}$ and $D_\omega^{(t,t+1)}$, $D_\pi^{(t,t+1)}$ is socially preferred to $D_\omega^{(t,t+1)}$, \( \forall W \in P^* \), if and only if
\[
d_\pi(p_t) \geq d_\omega(p_t), \forall p_t \in [0,1]
\]
(8)

Proposition 1 characterizes the first-order dominance criterion based on DynaC. It requires checking the DynaC dominance of one mobility process over the other for every dynasty. The intuition of this proposition is simple: if there is at least one dynasty experiencing a higher mobility in process $D_\pi$ than in process $D_\omega$, while there is no difference for the other dynasties, then the former process is socially preferred to the latter. When assessing whether a single process yields mobility, the proposition simply becomes $d_\pi(p_t) \geq 0, \forall p_t \in [0,1]$. With our relative mobility concept, it may seem difficult to obtain dominance (between two processes or between a single process and immobility). Indeed, assume a population composed only of fathers and sons: some sons will be better positioned in their distribution than their fathers in theirs—some dynasties will show positive (i.e. relative upward) mobility and others a negative one. Things are in fact more complicated in real-world data. Some children (e.g. daughters) may experience higher relative mobility compared to sons, so dominance may appear, at least over some portion of the dynastic distribution.

Nonetheless, if the DynaCs of two mobility processes are crossing, it is possible to follow the social choice tradition and suggest higher-order dominance results. These are the minimal refinements on the set of admissible preferences that may lead to an unambiguous assessment of the mobility processes. More than this, we use cumulative DynaCs to increasingly put more weight on the mobility of the poorest dynasties. We start with second-order dominance and general social weights that decline with the dynasty percentile (Proposition 2), then introduce convexity in the social weighting scheme to give an emphasis on the poorest of the poor (Proposition 3) and finally suggest a Rawlsian type of social valuation (Proposition 4).

Proposition 2: Given two mobility processes $D_\pi^{(t,t+1)}$ and $D_\omega^{(t,t+1)}$, $D_\pi^{(t,t+1)}$ is socially preferred to $D_\omega^{(t,t+1)}$, \( \forall W \in P^* \) for which $\frac{\delta w(p_t)}{\partial p_t} \leq 0$ for all $p_t \in [0,1]$, if and only if
\[
\int_0^{p_t} d_\pi(q_t) dq_t \geq \int_0^{p_t} d_\omega(q_t) dq_t, \forall p_t \in [0,1]
\]
(9)

If we allow social preferences to be more sensitive to the mobility experienced by the more disadvantaged dynasties in the initial period, a comparison between two alternative mobility processes can be carried out by testing for cumulative DynaC dominance, as suggested in this proposition. When assessing the mobility of a single process, we simply write $\int_0^{p_t} d(q_t) dq_t > 0 (\int_0^{p_t} d(q_t) dq_t < 0)$ for all $p_t$ as a situation of weak relative positive (negative) mobility according to the idea that there should be a priority for lifting up the poor.

Proposition 3: Given two mobility processes $D_\pi^{(t,t+1)}$ and $D_\omega^{(t,t+1)}$, $D_\pi^{(t,t+1)}$ is socially preferred to $D_\omega^{(t,t+1)}$ \( \forall W \in P^* \) for which $\frac{\delta w(p_t)}{\partial p_t} \leq 0$ and $\frac{\delta^2 w(p_t)}{\partial p_t^2} \geq 0$ for all $p_t \in [0,1]$ if and only if
\[
\int_0^{p_t} \int_0^{q_t} d_\pi(s_t) ds_t \geq \int_0^{p_t} \int_0^{q_t} d_\omega(s_t) ds_t, \forall p_t \in [0,1] \forall p_t \in [0,1]
\]
(10)

According to this proposition, we can perform a test based on a third-order upward DynaC dominance, which finds its justification in social preferences that give more weight to the mobility experienced by the poorest of the poor. Any progressive inheritance tax (or transfer) that improves the mobility of a poor individual by taking some mobility from a middle-class individual is preferred to a transfer of mobility.
from a rich to a middle-class individual.\textsuperscript{12} We conclude with a comparison criterion of a Rawlsian flavour and that might be useful in the presence of coarse data.

**Proposition 4:** Given two mobility processes $D_{\pi}^{(t,t+1)}$ and $D_{\omega}^{(t,t+1)}$, $D_{\pi}^{(t,t+1)}$ is socially preferred to $D_{\omega}^{(t,t+1)}$ for all $W \in P^*$ for which $w(p_t) = w(q_t) > 0 \forall p_t, q_t \in [0, \bar{p}]$ and $w(p_t) = 0 \forall p_t \in [\bar{p}, 1]$ if and only if

$$
\int_0^{\bar{p}} d_{\pi}(q_t) dq_t \geq \int_0^{\bar{p}} d_{\omega}(q_t) dq_t \quad (11)
$$

Selecting a threshold equal to $\bar{p}$ implies that we focus only on dynasties corresponding to households of the lower $\bar{p}$ percent of the first-generation distribution. Alternative thresholds can be selected to focus on different groups of dynasties.\textsuperscript{13} The test corresponding to this proposition can be interpreted as a ‘priority’ criterion—that is, it reflects the preferences of a social planner who endorses a ‘mobility priority for the worst off’. This echoes back to the maximin criterion à la Rawls: mobility is valuable if and only if poor dynasties experience an improvement, independently of how the rest of the population performs.

4 Empirical approach

4.1 Data

*Overview*

Our empirical analysis relies on the IFLS, which is an ongoing longitudinal survey of individuals, households, and communities. Based on an initial sample representing 83 per cent of the Indonesian population, the IFLS is conducted in 13 Indonesian provinces extending across the islands of Sumatra, Java, Kalimantan, Sulawesi, Bali, and West Nusa Tenggara. It has interesting features that make it particularly suited to our analysis. First, it covers a long time period, ranging from 1993 to 2014, which is appropriate for inter-generational mobility measurement. Second, it benefits from an exceptionally low attrition rate. Indeed, extensive efforts were provided to track respondents when collecting data in each of the five waves (1993, 1997, 2000, 2007, and 2014). It has allowed reaching a recontact rate of 92 per cent in the last wave (Strauss and Witoelar 2019; Strauss et al. 2016), which guarantees both cross-sectional and longitudinal representativeness of our sample for inter-generational mobility calculations. Third, it includes individual and household information on a large set of socioeconomic characteristics, such as education, income, and consumption expenditure, in addition to standard demographic variables.

For these reasons, the IFLS has been applied to many development studies, including those focusing on intra-generational mobility (see Grimm 2007; Lo Bue and Palmisano 2020). It has also been used to investigate the heterogeneous effect of the INPRES school construction programme on girls’ educational achievements between ethnic groups practising bride price and the others (Ashraf et al. 2018; Mazumder et al. 2019). We suggest here to exploit the length of the panel for inter-generational mobility measurement. As previously discussed, the ability to link information on parents and their children when both groups are about the same age is relatively rare in the context of low- and middle-income countries. Mobility across the two generations of interest in the IFLS will necessarily reflect the

\textsuperscript{12} Alternatively, it is possible to obtain a ranking criterion that is based on the idea of preserving the mobility of less-advantaged dynasties while, at the same time, focusing on the dynamics of the richest dynasties. This alternative proposition (3') is discussed in Appendix C.

\textsuperscript{13} If the threshold is equal to 1, the proposition reduces to the comparison of mean mobilities. When $\bar{p} = 0.5$, it is reminiscent of a recent approach introduced by Asher et al. (2018) to estimate inter-generational educational mobility with coarse data, which is typically the case of developing countries.
impact of the aforementioned reforms on the second generation’s potential earnings, but also the other changes affecting the earnings potential of men and women over time.

*Generation matching and selection*

We use the first and last waves of the IFLS. That is, we identify the generation of fathers and mothers in 1993 using IFLS 1 (cf. Frankenberg and Karoly 1995) and match them with their offspring observed in 2014 in IFLS 5 (cf. Strauss et al. 2016). We have two broad objectives. First, we aim at observing both generations at similar age ranges, or at least not too radically different. It is not an issue for education outcomes since parents and children are old enough for their education levels to be a fixed characteristic, but it is an issue for meaningful profiles of earnings mobility. We will also apply an age correction when predicting earnings hereafter. Second, to simplify interpretations, we prefer to define cohorts such that the first generation has left school after the implementation of INPRES and subsequent reforms, while the second generation is the one most affected by these large-scale policies.

As a result, we select dynasties where individuals are observed in an age range of 20–40 (born 1952–74 for parents and 1974–94 for children), with the exception of fathers, who are selected in a 20–50 age range. The asymmetry for fathers is justified as follows: males of the first generation tend to be older—and have children later—than their spouses, and restricting to 20–40 would reduce the sample size and the representativeness of the matched, dynastic sample. This leaves us with a sample of 2,164 daughters and 2,060 sons matched with 2,284 mothers and 2,284 fathers.

*4.2 Achievements and DynaC implementation*

The concept \( z \) used to define dynasties is the per-capita expenditure in households observed in 1993. It is convenient to use this welfare measure as a backdrop against which we can assess individual mobility in terms of earnings or education. Our analysis focuses on two types of achievements: one based on discrete outcomes \( k \) corresponding to education classes, the other based on continuous outcomes \( y \) corresponding to potential earnings.

Regarding the discrete outcome, based on the information reported in the IFLS data on the highest education level attended and on the grade completed for that level, we construct eight education classes, from ‘no education’ to ‘university degree’. These classes are used to implement baseline achievement measures of equation 2.

The continuous outcome is the potential earnings of each individual, used to implement achievements in equation 1. Given the number of adults without a paid job, potential earnings are more relevant than actual earnings, which would give a very truncated picture of inter-generational mobility. Moreover, predicted earnings help to assess how education mobility translates in monetary terms. Indeed, since education is one of the key determinants of earnings, predicting earnings would reflect a combination of relative education levels within each generation and different returns to education across generations.

To generate this outcome, we simply use observed earnings and run separate earnings estimations for

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\[\text{Further sensitivity checks could consist in applying the 20–40 age bracket to everyone or the 20–50 bracket to both mothers and fathers, but we do not expect radically different conclusions.}\]

\[\text{As justified before, we refrain from using it as an outcome. Additional reasons are empirical. First, there is a high potential for measurement error (see Grimm 2007). Second, for all children still living with their parents in 2014, there will be a strong persistence in this variable, which is not what we want to characterize. We rather focus on men’s and women’s individual mobility in terms of individual achievement (education, earnings potentials).}\]

\[\text{We will suggest counterfactual analyses along these lines to assess how educational mobility, compounded with changes in returns to education, may have contributed to earnings mobility at different points of the initial distribution.}\]
mothers, fathers, sons, and daughters.\textsuperscript{17} Using the four sets of estimations, we predict potential earnings for mothers, fathers, sons, and daughters, respectively, given their own characteristics at the time of observation, except age. Indeed, to compare the potential earnings of sons and fathers (daughters and mothers) at about the same age, we predict 2014 earnings for sons (daughters) using the age that their father (mother) had in 1993.\textsuperscript{18}

Descriptive statistics are reported in Table A2. Daughters and sons are observed aged 29 on average, which is lower but close to their mother’s age (32) and father’s age (37). Figure A1 shows a broad overlap between generations. However, there are mechanical differences imposed by the matching of generations at specific points in time (1993 and 2014) and demographic particularities (e.g. the fact that men are older than their spouses). This justifies the strategy described above to neutralize the role of age when predicting earnings. Further results in Table A2 focus on achievements. It turns out that, on average, our sample of individuals in the offspring generation seems to fare relatively better than their parents. Their economic conditions have improved, as the value of real per-capita expenditure has more than doubled. Second-generation average education level is about 5.9, corresponding to 10–11 years of schooling, which is almost twice as large as their parents’ educational achievement (i.e. 3–3.6 or roughly six years of schooling). Their potential earnings are also much improved: in real terms (i.e. when accounting for changes in living costs over 20 years) they are 5–6 times as high as their parents’ potential earnings.

4.3 Education reforms in Indonesia and absolute mobility

Education mobility in Indonesia is particularly interesting to examine, given the large-scale education reforms implemented by this country over the past decades. We start with a brief overview of the reforms potentially affecting the offspring in our analysis, and move to the description of mobility patterns.

\textit{Education reforms}

A series of programmes have been implemented since the 1970s and have certainly contributed to boost enrolment rates and increase access to education. In 1973, the Indonesian government launched the ‘Sekolah Dasar INPRES’ programme, a large-scale school construction programme whose effectiveness has been well documented in many studies (see, \textit{inter alia}, Akresh et al. 2018; Ashraf et al. 2020; Duflo 2001, 2004; Mazumder et al. 2019). Between 1973–74 and 1978–79, the number of primary schools in the country more than doubled, leading to a remarkable increase in enrolment rates among children aged 7–12, from 69 per cent in 1973 to 83 per cent by 1978. In our selection, the parents will have been hardly affected by this reform when they were themselves children, while all their children would be in the right age group to fully benefit from the reforms.\textsuperscript{19}

Education enrolment expansion continued through the 1990s and the early 2000s, the years of the Asian economic ‘miracle’ marked by remarkable progress in poverty reduction and economic growth (Bolt et al. 2018; Timmer 2018). By the early 2000s, Indonesia achieved a nearly universal net primary

\textsuperscript{17}These regressions account for a first-step selection equation using a relatively standard instrument, namely the total resource of other family members. Estimation results are reported in Appendix Table A1: they essentially show the (monotonic) effect of education classes in the main equation (earning potentials), as well as the significant role of the instrument (and its expected negative sign) in the participation equation.

\textsuperscript{18}Without this adjustment, earnings mobility would partly reflect the fact that parents are observed at older ages and hence at higher earnings levels (a differential that might vary along the cross-sectional distribution).

\textsuperscript{19}Indeed, those who could benefit from the early phase of INPRES were those aged five or younger in 1974, which represents less than 3 per cent (12 per cent) of the fathers (mothers) in our sample. In contrast, all the children in our selection where born after 1974 and hence were primary-school-aged at the time of the second phase of INPRES school construction.
enrolment rate, while junior high school enrolment rates had reached about 60 per cent, compared to 17 per cent in 1975 (Granado et al. 2007).\(^{20}\) Efforts to increase access to education, especially among the most disadvantaged children, have also been made since the late 1990s through the launch of a series of scholarship programmes.\(^{21}\)

*Changes in education levels and absolute mobility*

Figure 1 shows the distribution of education levels for both generations in our selected data. The first observation is that the new generation has basically escaped from illiteracy, possibly thanks to the previously discussed reforms. While 40 per cent of the parents in our dynastic sample had no education or only incomplete primary schooling, their offspring are almost all in higher education classes. Almost none of them have no education, and less than 5 per cent have uncompleted primary schooling. The completion of junior high school has almost doubled, while a majority of children now have a (senior) high school degree or go to university.

We check who has benefited from these tremendous improvements in education. Using the matched generations, we represent the share of (absolute) education mobility in Figure 2. Ranking dynasties according to parental education levels, we report the proportion of upward mobility, downward mobility, or immobility (left axis). The graph essentially points to the upward mobility for all children of parents who had less than a high school degree, and a relative immobility for the top dynasties (with 80 per cent of the university-graduated parents sending their children to university as well). We also depict the average difference in education levels between the two generations (right axis). As expected, it shows larger improvement among dynasties of low-educated parents and a declining profile.

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\(^{20}\) In 2005, the government launched the ‘One Roof School Program’ aimed to facilitate transitions from primary to secondary school by building junior high schools on the same sites as primary schools, especially in remote areas (ILO 2011).

\(^{21}\) This includes the 1998 ‘School Grants Program’, which was effective in preventing schools drop-outs especially among primary-school-aged children from poor rural families (Sparrow 2004). Another programme was implemented in 2001–005, the ‘Compensation for Fuel Subsidy Decreased Program’, to help children from poor families through scholarships (Bantuan Khusus Murid, BKM), which covered both primary and secondary education.
5 Empirical results

We now turn to our main results based on DynaC measures. As per equation 5, the DynaC curves simply show for all dynasties the difference between children’s and parents’ relative achievements, their achievement being the proportion of people they dominate in their own distribution. We present results in two steps. First, we describe DynaC and cumulated DynaC curves across dynasties to assess whether the overall mobility process over the 1993–2014 period has been progressive or not, with potential implications for the role played by education reforms. We do so for each type of outcome, namely education classes (discrete) and potential earnings (continuous). We disentangle the contribution of education mobility on potential earnings mobility. Second, we present DynaC dominance results when comparing the relative mobility of men and women. Our baseline relies on like-for-like comparisons (daughters to mothers and fathers to sons), but we also check the sensitivity of dominance results to alternative definitions, including a reference point that is common to both men and women.

5.1 Relative mobility across dynasties

Relative education mobility by initial education class

We first represent education DynaCs across parents’ education classes. Figure 3 shows a declining pattern with ‘positive’ mobility only for dynasties of parents in the lowest education categories (classes 1 and 2 for no education and incomplete primary school). This seems consistent with previous results indicating how education reforms have greatly reduced illiteracy in Indonesia. More than this, it tells us that dynasties where parents were at the lowest levels have progressed more than others on the education ladder. Mechanically, dynasties with parents at higher level have smaller margins of progress (recall that
parents with junior high school or above were dominating more than 70 per cent of their distribution in 1993).

Figure 3: Relative education mobility (DynaC by parent education levels)

Note: educational level correspondence: (1) no education, (2) incomplete primary school, (3) completed primary school, (4) incomplete junior high school, (5) completed junior high school, (6) incomplete senior high school, (7) completed senior high school, (8) university.
Source: authors’ elaboration based on IFLS 1 and IFLS 5.

Relative education mobility

While the DynaC mobility by parental education class is interesting, most of the non-anonymity is lost with such a large grouping. Moreover, faster education mobility among the lowest levels is expected, especially given the fact that a larger majority of first-generations were in this situation (67 per cent had, at best, completed primary schooling in 1993). Hence, we turn to our main results, depicting DynaC curves over the range of dynasties \( p_t \) based on per-capita expenditure in first-generation households. For the sake of exposition, we shows vigintile averages of dynastic mobility measures. We are interested to know where mobility was the highest and, in particular, whether large-scale educational policies have targeted the poorest. Figure 4(a) shows a very different pattern compared to Figure 3. When considering mobility across the living standard distribution, we observe a regressive pattern. It is first due to the fact that, as recalled above, most of the first generation was uneducated: the low-educated parents were to be found in all quantiles of per-capita expenditure and not just among the poor. Then, dynasties progressing the most were those with low-education parents but also higher living standards. This is illustrated in Figure A4(a) in Appendix A, which depicts DynaC curves for dynasties of low-educated parents (classes 1 and 2) versus others (parents with education classes 3–8). The former show positive mobility scores while the latter group does not progress as much (hence negative mobility measures). These results tend to indicate that INPRES and subsequent reforms have improved education levels more substantially among the wealthier.

In the next section we address dominance between two groups (men and women) using Propositions 1–4. At this stage, we can examine dominance with respect to the immobility benchmark and establish

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22 Note that the distance travelled could be evaluated differently when the starting point is ‘no education’ compared to when it is at a higher level. We keep this type of normative refinement for further research.

23 They combine possibly very different situations across the different persons composing each vigintile, in particular quite different patterns between men and women, as we shall see.
whether the overall mobility profile for education was socially superior. In Figure 4(a) there is no dominance of mobility over immobility since we observe a crossing of the zero line around the third vigintile. Among the poorest dynasties, the mobility measure is negative, reflecting a slower pace of educational improvement compared to other dynasties. We will see that these negative values are due to men, whose educational position tends to decline relative to that of their fathers. Moving to second-order dominance, we see in Figure 4(b) that immobility prevails over a larger segment (i.e. up to the fifth vigintile) when priority is put on the poorest and further still in Figure 4(c) (up to the seventh vigintile) where priority goes to the poorest of the poor.

Relative mobility of potential earnings: the role of education

In Figure 5 we report DynaC results for the continuous outcome (i.e. potential earnings). Figure 5(a) depicts the DynaC curve (solid line) and a counterfactual (dashed line) obtained when setting children’s education level to their parents’ level. The main DynaC curve shows a regressive pattern (up to the 12th vigintile) that turns progressive thereafter. Negative values at the top indicate that children from wealthier backgrounds progressed relatively less quickly in terms of earnings-generating capacity than the rest of the population. Importantly, while the overall pattern is fairly flat, the dashed curve shows that progressivity would be more pronounced if education levels had not changed between generations. In other words, the regressive nature of education reforms translates here into a reduced extent of progressivity in terms of potential earnings mobility.

Relative mobility of potential earnings: dominance and weighted DynaC

DynaCs based on continuous variables are equivalent to a change in rank between generations, as used in the recent literature focusing on re-ranking measures (cf. Section 2). Our framework goes a few steps further by adding dominance results and the possibility to weight achievement measures. First, we question the dominance of the mobility process over immobility. Figure 5(a) is inconclusive because the DynaC curve crosses the zero line (around the 17th vigintile). Yet, if we put the emphasis on the poor, positive mobility scores may be what prevails. This is indeed what we see in Figure 5(b) with second-order dominance over immobility. There is a fortiori dominance of the third order in Figure 5(c). Then, weighted DynaC curves based on the achievement formulas of equation 3 allow us to account for structural changes in the distribution between parents and their children. Dynastic mobility is all the larger as the rank improvement across generation is larger. Weighted and unweighted DynaCs are presented together in appendix Figure A5, but only to inspect patterns (mobility levels are not comparable). Adding weights leads in this case to flatter trends, with in particular a lower degree of regressivity in the first half of the living standard distribution. As discussed in the theory section, if a dynasty is characterized by a zero weighted DynaC but a negative unweighted Dynac, there is a reduction in inequality below this dynasty, i.e. the distance between this dynasty and its followers shrinks as the latter are catching up. This is the situation we observe for instance at the 17th vigintile, i.e. when the DynaC curve crosses the zero line.
Figure 4: Relative education mobility (DynaCs, baseline)

Note: DynaCs are represented by vigintiles of per-capita expenditure. Dynasties are constructed by comparing daughters (sons) with mothers (fathers).

Source: authors’ elaboration based on IFLS 1 and IFLS 5.
5.2 Gender heterogeneity and dominance in relative mobility

Relative education mobility

We now move to the decomposition of the DynaC results by gender. Results are reported in Figure 6. In Figure 6(a), DynaC mobility is only slightly progressive for men up to the 12th vigintile and steadily regressive for women. These joint trends are consistent with the overall regressivity in Figure 4. As before, we can also inspect results by parental education level: Figure A4(b) and (c) confirms that for both men and women, those from richer families progress faster in the bulk of low-educated families.

Figure A2 shows that, overall, boys and girls now attain very similar education levels while mothers were less educated than fathers. Hence, when daughters are compared to mothers and sons to fathers in our baseline (like-for-like matching), it is expected that the relative position of women increases more than that of men. This is indeed what we observe in Figure 6(a). From Proposition 1, we obtain a clear dominance of women’s mobility over men’s. Daughters have systematically improved their relative position compared to their mothers, while only the sons of the top five vigintiles are better ranked than their fathers. Because Proposition 2 (resp. 3) is based on a subset of social preferences compared to Proposition 1 (resp. 2), dominance of the first order implies dominance of second and third orders, as can be checked in Figure 6(b) and (c).
Relative education mobility with a common reference

A normative dimension—not discussed in the theory section because it is very much related to the empirical implementation—p pertains to whom sons and daughters are compared to. With the like-for-like matching, we exacerbate gender differences in mobility. As discussed, girls’ ranks are compared to those of their mothers, who had the lowest education achievement in the previous generation. Thereby, daughters are deemed very mobile in this configuration. Alternatively, we can use a first-generation achievement that is common to both daughters and sons, for instance the mean education of both parents. We expect that girls’ mobility scores mechanically decrease and those of boys increase compared to the baseline. This is indeed the case in Figure 7(a). Importantly, what changes is the levels, while DynaC patterns are similar to what we saw in the baseline, confirming the overall regressivity of educational mobility. DynaCs of men and women are crossing so there is no longer an unambiguous ranking of
their mobility processes. With men now achieving more mobility than women in the lowest vigintiles, dominance of male mobility goes further in Figure 7(b) and even further in Figure 7(c) so that there is almost full dominance of the third order.

Figure 7: Relative education mobility (DynaC by dynasty of per-capita expenditure, same reference point for sons and daughters)

![Graphs showing relative education mobility](image)

Note: parental achievement refers here to the rank of the best education level between the father and the mother.
Source: authors' elaboration based on IFLS 1 and IFLS 5.

This reversal in our conclusions about mobility dominance between men and women is not a concern: as indicated, it is a normative characterization that is not absolute but rather dependent to a large extent on the first-generation achievement of reference that is used in the analysis. Other results are stable, in particular the conclusion about the overall regressivity of educational mobility. It holds when using many alternative reference points, such as the father’s achievement or the highest achievement among the parents.24

24 The educational advantages provided by parents are often defined by the highest level of human capital within the family (Erikson 1984). It is also possible to argue that the parent with less education is more relevant for children’s attainment, under
We end our analysis of educational mobility with the dominance conditions defined in Proposition 4, for which results are reported in Table A3. This proposition reflects strong preferences for mobility among the ‘worst-off’ in an absolute sense. We suggest four different thresholds corresponding to vigintiles 1, 3, 5, and 10 of the per-capita expenditure dynasties. In line with graphical results, we observe that the mobility profiles of daughters are significantly better than those of sons at all these cutoffs, and especially at the higher ones, in the like-for-like matching (panel A). Using the mean education achievement of the parents as a common reference point, we observe the same reversal as on the graphs when all the weight is put on lower segments of the dynastic distribution. In the presence of social preferences over mobility profiles à la Rawls, one can judge the mobility of men to be superior to that of women (Panel B) in this case.  

Relative mobility of potential earnings

We report DynaC curves for potential earnings in Figure 8(a). Patterns are broadly regressive for women (except at the top) and mildly progressive for men. We disentangle again the role of education by comparing the DynaC curves to the counterfactuals where children’s education is set to their parents’ level. We obtain the same result as in Figure 5 for men and women: regressivity in educational mobility tends to limit the extent of progressivity in potential earnings mobility. For women, mobility would be relatively neutral (and still progressive at the top) if girls had the same education levels as their mothers. For men, progressivity would be accentuated if they had the education levels of their fathers. These results translate in monetary terms our findings regarding educational mobility and the underlying role of education reforms. Figure 8 also points to an unequivocal dominance of women’s mobility patterns over men’s. As for education, this conclusion depends on the reference point used in the first generation.

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25 In the presence of utilitarian social preferences, one would probably judge the mobility of women to be superior to that of men.

26 Additional (unreported) results show a similar reversal when the father’s or the highest earnings achievement is used as a common reference for both men and women. This reversal is even more radical than in the case of education due to the fact that distributions of potential earnings are even more contrasted across groups than those of education. As can be seen in Figure A3, potential earnings of fathers are markedly larger than those of mothers while those of sons and daughters broadly overlap.
6 Conclusion

We provide a new tool for the comparison and evaluation of inter-generational mobility processes. Our theoretical framework is specifically tailored to provide a general characterization of inter-generational mobility. It can be implemented to generate partial but robust rankings of mobility processes for different subgroups of the population (e.g. gender, ethnic group) and using all types of individual achievements (independently of the cardinal or ordinal scale of those outcomes). Our approach can incorporate a weighting scheme which—differently from classic measures of rank changes—can account for structural changes in the parents’ and offspring’s distributions.

We apply our theoretical framework to the IFLS data to empirically analyse the inter-generational patterns of mobility in Indonesia, both in terms of educational attainment and potential earnings, between 1993 and 2014. For both outcomes, relative mobility patterns were markedly to the advantage of women. A large part of the population was lifted out of illiteracy, possibly due to the large-scale education re-
forms implemented in Indonesia. However, our DynaC approach also shows that educational mobility was regressive: dynasties that have progressed the most were those of the low-educated parents at the top of the living standard distribution. These patterns also tend to seriously limit the degree of progressivity of the mobility in terms of potential earnings.

The theoretical and empirical results proposed in this paper are encouraging and open up new lines of research. From a theoretical perspective, two extensions seem particularly promising. One concerns the assessment of inter-generational mobility across more than two generations. The other extension would consist in adapting the model to allow for a multidimensional evaluation of mobility. From an empirical perspective, the mobility scheme proposed in this paper could be applied to other countries for which long panels exist (mainly rich countries). Cross-country comparisons would be a particularly interesting aspect, for instance to characterize how the relative mobility of poor dynasties (in terms of education, earnings, or health) varies across countries and to identify the policies that may have contributed to strong relative mobility in some countries.

References


Appendix A: Additional results

Figure A1: Age distribution by cohort and gender

Note: sample size: 4,955 for offspring; 2,558 for fathers; and 2,492 for mothers.
Source: authors’ elaboration based on IFLS 1 and IFLS 5 data.

Figure A2: Education distribution by cohort and gender

Note: educational level correspondence: (1) no education, (2) incomplete primary school, (3) completed primary school, (4) incomplete junior high school, (5) completed junior high school, (6) incomplete senior high school, (7) completed senior high school, (8) university. Sample size: 4,955 for offspring; 2,558 for fathers; and 2,492 for mothers.
Source: authors’ elaboration based on IFLS 1 and IFLS 5 data.
Figure A3: Potential earnings distribution by cohort and gender

Note: sample size: 4,955 for offspring, 2,558 for fathers; and 2,492 for mothers.
Source: authors’ elaboration based on IFLS 1 and IFLS 5 data.
Figure A4: Relative education mobility, DynaCs by first-generation education level

Source: authors’ elaboration based on IFLS 1 and IFLS 5.
Figure A5: Relative potential earnings mobility, weighted DynaC

Source: authors’ elaboration based on IFLS 1 and IFLS 5.
Table A1: Heckman regressions

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<th>Mother</th>
<th>Father</th>
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<td>(0.0870)</td>
<td>(0.105)</td>
<td>(0.229)</td>
</tr>
<tr>
<td>Education: university</td>
<td>0.756***</td>
<td>0.993***</td>
<td>0.712***</td>
<td>0.861**</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.0989)</td>
<td>(0.194)</td>
<td>(0.388)</td>
</tr>
<tr>
<td>Rural</td>
<td>−0.266***</td>
<td>−0.0237</td>
<td>−0.219***</td>
<td>−0.301*</td>
</tr>
<tr>
<td></td>
<td>(0.0727)</td>
<td>(0.0658)</td>
<td>(0.0643)</td>
<td>(0.164)</td>
</tr>
<tr>
<td>Married</td>
<td>0.0111</td>
<td>0.257***</td>
<td>0.369***</td>
<td>0.209</td>
</tr>
<tr>
<td></td>
<td>(0.0793)</td>
<td>(0.0670)</td>
<td>(0.133)</td>
<td>(0.820)</td>
</tr>
<tr>
<td><strong>Selection</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real PCE</td>
<td>−1.58e–06***</td>
<td>−1.37e–06***</td>
<td>−8.29e–07***</td>
<td>−4.19e–07***</td>
</tr>
<tr>
<td></td>
<td>(5.31e–08)</td>
<td>(4.88e–08)</td>
<td>(8.73e–08)</td>
<td>(1.35e–07)</td>
</tr>
<tr>
<td>Mills ratio</td>
<td>−0.930</td>
<td>−1.401</td>
<td>−0.839</td>
<td>−3.602</td>
</tr>
<tr>
<td>t-stat</td>
<td>−24.13</td>
<td>−32.08</td>
<td>−5.56</td>
<td>−1.42</td>
</tr>
<tr>
<td>Observations</td>
<td>3,485</td>
<td>3,339</td>
<td>2,451</td>
<td>2,558</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.212</td>
<td>0.181</td>
<td>0.177</td>
<td>0.188</td>
</tr>
<tr>
<td>Ethnicity fixed effects</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Province fixed effects</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

Note: PCE, per capita expenditure. Standard errors in parentheses. * p < 0.1, ** p < 0.05, *** p < 0.01.

Source: authors’ elaboration based on IFLS 1 and IFLS 5 data.

Table A2: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Children</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
<td>4,224</td>
</tr>
<tr>
<td>Rural</td>
<td>0.4</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
<td>4,224</td>
</tr>
<tr>
<td>Daughter’s age</td>
<td>28.4</td>
<td>4.9</td>
<td>20</td>
<td>40</td>
<td>2,164</td>
</tr>
<tr>
<td>Son’s age</td>
<td>28.4</td>
<td>5.0</td>
<td>20</td>
<td>40</td>
<td>2,060</td>
</tr>
<tr>
<td>Daughter’s level of education</td>
<td>5.9</td>
<td>1.9</td>
<td>1</td>
<td>8</td>
<td>2,164</td>
</tr>
<tr>
<td>Son’s level of education</td>
<td>5.9</td>
<td>1.9</td>
<td>1</td>
<td>8</td>
<td>2,060</td>
</tr>
<tr>
<td>Daughter’s log predicted annual earnings</td>
<td>15.3</td>
<td>0.7</td>
<td>12.6</td>
<td>17.4</td>
<td>2,164</td>
</tr>
<tr>
<td>Son’s log predicted annual earnings</td>
<td>15.8</td>
<td>0.7</td>
<td>5.2</td>
<td>18.1</td>
<td>2,060</td>
</tr>
<tr>
<td>Log annual PCE</td>
<td>15.8</td>
<td>0.6</td>
<td>14.0</td>
<td>18.9</td>
<td>4,224</td>
</tr>
<tr>
<td><strong>Parents</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mother’s age</td>
<td>31.5</td>
<td>5.1</td>
<td>20</td>
<td>40</td>
<td>4,220</td>
</tr>
<tr>
<td>Father’s age</td>
<td>36.5</td>
<td>6.0</td>
<td>20</td>
<td>50</td>
<td>4,223</td>
</tr>
<tr>
<td>Mother’s level of education</td>
<td>3.0</td>
<td>1.8</td>
<td>1</td>
<td>8</td>
<td>4,224</td>
</tr>
<tr>
<td>Father’s level of education</td>
<td>3.6</td>
<td>2.1</td>
<td>1</td>
<td>8</td>
<td>4,224</td>
</tr>
<tr>
<td>Mother’s log predicted annual earnings</td>
<td>12.9</td>
<td>0.4</td>
<td>12.1</td>
<td>14.7</td>
<td>3,836</td>
</tr>
<tr>
<td>Father’s log predicted annual earnings</td>
<td>13.6</td>
<td>0.4</td>
<td>12.5</td>
<td>15.2</td>
<td>3,885</td>
</tr>
<tr>
<td>Log annual PCE</td>
<td>14.8</td>
<td>0.6</td>
<td>12.1</td>
<td>18.3</td>
<td>4,224</td>
</tr>
</tbody>
</table>

Note: PCE, per capita expenditure (IDR, real, base: 2002).

Source: authors’ elaboration based on IFLS 1 and IFLS 5 data.
Table A3: Education mobility: dominance results of Proposition 4

<table>
<thead>
<tr>
<th></th>
<th>Sons</th>
<th>Daughters</th>
<th>t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: baseline (like-for-like)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 1$</td>
<td>-0.009</td>
<td>-0.001</td>
<td>-28.315</td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 3$</td>
<td>-0.010</td>
<td>0.009</td>
<td>-33.950</td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 5$</td>
<td>-0.015</td>
<td>0.009</td>
<td>-37.968</td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 10$</td>
<td>-0.034</td>
<td>0.047</td>
<td>-59.000</td>
</tr>
<tr>
<td><strong>Panel B: same reference point (mean educ.)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 1$</td>
<td>-0.001</td>
<td>-0.006</td>
<td>28.742</td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 3$</td>
<td>0.005</td>
<td>-0.004</td>
<td>27.653</td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 5$</td>
<td>0.005</td>
<td>-0.012</td>
<td>36.820</td>
</tr>
<tr>
<td>Threshold: $\bar{\rho} = 10$</td>
<td>0.013</td>
<td>0.006</td>
<td>11.462</td>
</tr>
</tbody>
</table>

Note: $t$-test of statistical significance obtained through 300 bootstrap replications. Same reference point refers to father’s and mother’s average education level.

Source: authors’ elaboration based on IFLS 1 and IFLS 5 data.
Appendix B: Proofs of Propositions 1–4

B1 Proof of Proposition 1

We seek sufficient and necessary conditions such that:

\[ \Delta W = \int_0^1 w(p_t)(d_\omega(p_t)dp_t - d_\pi(p_t)dp_t) \geq 0, \text{ for all } W \in P^x \] (12)

Let \( \delta(p_t) = d_\omega(p_t) - d_\pi(p_t) \) so equation 12 is rewritten as:

\[ \Delta W = \int_0^1 w(p_t)\delta(p_t)dp_t \geq 0 \] (13)

For the sufficiency condition, note that \( w(p_t) \geq 0 \) for all \( p_t \in [0,1] \), so that \( \delta(p_t) \geq 0 \) for all \( p_t \in [0,1] \) implies that \( \int_0^1 w(p_t)\delta(p_t)dp_t \geq 0 \). For the necessary condition, let \( \Delta W \geq 0 \), but assume that \( \delta(p_t) < 0 \) for some \( p_t \in [0,1] \). Following lemma 1 in Chambaz and Maurin (1998), there exists a set of values \( z(p) \in V^+ \) and \( \rho(p) \in V^+ \) such that: 

\[ \int_0^1 z(p)\delta(p_t)dp_t \leq 0. \]

Define \( z(p) = w(p_t) \), since \( z(p) \in V^+ \) (hence \( z(p) > 0 \) for all \( p \)), substituting in equation 13 gives \( \Delta W \leq 0 \), which is a contradiction.

B2 Proof of Proposition 2

We look for sufficient and necessary conditions such that:

\[ \Delta W = \int_0^1 w(p_t)d(p_t)dp_t \geq 0, \text{ for all } W \in P^x \] (14)

for which \( \frac{\partial w(p_t)}{\partial p_t} \leq 0 \) for all \( p_t \in [0,1] \). For the sufficiency part, we integrate equation 14 by parts:

\[ w(p_t) = 1 \int_0^1 \delta(p_t)dp_t - \int_0^1 w'(p_t) \int_0^{p_t} \delta(q_t)dq_t dp_t \] (15)

Since \( w(p_t) = 1 \) for all \( p_t \in [0,1] \), \( \int_0^{p_t} \delta(q_t)dq_t \geq 0 \) for all \( p_t \in [0,1] \) implies \( w(p_t) = 1 \int_0^1 \delta(p_t)dp_t \geq 0 \). Furthermore, since \( w'(p_t) \leq 0 \) for all \( p_t \in [0,1] \), we have \( \int_0^1 w'(p_t) \int_0^{p_t} \delta(q_t)dq_t dp_t \leq 0 \). Thus, \( \Delta W \geq 0 \).

For the necessity part let \( \Delta W \geq 0 \), but assume that \( \int_0^{p_t} \delta(q_t)dq_t < 0 \) for some \( p_t \in [a,b] \subset [0,1] \). Rewrite equation 15 as follows:

\[ w(p_t) = 1 \int_0^1 \delta(p_t)dp_t + \int_0^1 -w'(p_t) \int_0^{p_t} \delta(q_t)dq_t dp_t \] (16)

Denote \( -w'(p_t) = \alpha(p) \). By lemma 2 in Chambaz and Maurin (1998), \( \int_0^1 \alpha(p) \int_0^{p_t} \delta(q_t)dq_t \leq 0 \) for all \( \alpha(p) \in V^+ \) and \( p_t \in [a,b] \subset [0,1] \). Suppose \( \int_0^{p_t} \delta(q_t)dq_t \geq 0 \) for all \( p_t \in [0,1] \setminus [a,b] \), the second term of equation 16 becomes negative. Then it is always possible to find combinations of \( w(p_t) \) and \( \delta(p_t) \) such that:

\[ \left| w(p_t) = 1 \int_0^1 \delta(p_t)dp_t \right| < \left| \int_0^1 -w'(p_t) \int_0^{p_t} \delta(q_t)dq_t dp_t \right| \] (17)

which results in \( \Delta W < 0 \), a contradiction.
For a given

\[ \text{equation 20 as follows:} \]

\[ \Delta W = \int_0^1 w(p_t) \delta(p_t) dp_t \geq 0 \] (18)

\[ \forall W \in P^* \text{ for which } \frac{\delta w(p_t)}{\delta p_t} \leq 0 \text{ and } \frac{\delta^2 w(p_t)}{\delta p_t^2} \geq 0 \text{ for all } p_t \in [0, 1]. \]

For sufficiency, consider equation 15 and use the following notation \( \Psi(p_t) = \int_0^{p_t} \delta(q_t) dq_t. \) Integrating by parts the second component:

\[ w(1) \Psi(1) - w'(1) \int_0^1 \Psi(p_t) dp_t + \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \] (19)

Since \( w''(p_t) \geq 0 \forall p_t \in [0, 1], \) \( \int_0^{p_t} \Psi(q_t) dq_t \geq 0 \) for all \( p_t \in [0, 1] \) implies \( \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \geq 0; \) since \( w(1)' \leq 0, \) it also implies that \(-w(1) \int_0^1 \Psi(p_t) dp_t \geq 0; \) last, given that \( w(1) \geq 0, \) \( w(1) \Psi(1) \geq 0. \) Thus, \( \int_0^{p_t} \Psi(q_t) dq_t \geq 0 \) for all \( p_t \in [0, 1] \) is sufficient for \( \Delta W \geq 0. \)

For the necessity part, let \( \Delta W \geq 0, \) but assume that \( \int_0^{p_t} \Psi(q_t) dq_t < 0 \) for some \( p_t \in [\alpha, \beta] \subset [0, 1]. \)

\( \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \leq 0 \) for all \( w(p_t)'' \in V^+ \) and \( p_t \in [\alpha, \beta]. \) Assuming that \( \int_0^{p_t} \Psi(p_t) \leq 0 \) for all \( p_t \in [0, 1] \setminus [\alpha, \beta], \) then \(-w'(1) \Psi(1) + \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \leq 0. \)

Now, it is always possible to find a combination of \( w(1) \) and \( \Psi(1) \) such that \( |w(1) \Psi(1)| < \left| -w'(1) \int_0^{p_t} \Psi(p_t) dp_t + \int_0^1 w''(p_t) \int_0^{p_t} \Psi(q_t) dq_t dp_t \right|, \)

which would result in \( \Delta W < 0, \) a contradiction.

**B4 Proof of Proposition 4**

We want to find sufficient and necessary conditions for

\[ \Delta W = \int_0^1 w(p_t) \delta(p_t) dp_t \geq 0 \] (20)

\[ \forall W \in P^* \text{ for which } w(p_t) = w(q_t) > 0 \forall p_t, q_t \in [0, \bar{p}] \text{ and } w(p_t) = 0 \forall p_t \in [\bar{p}, 1]. \]

For a given \( \bar{p}, \) since \( w(p_t) = w(q_t) > 0 \) for all \( p_t, q_t \in [0, \bar{p}] \) and \( w(p_t) = 0 \) for all \( p_t \in [\bar{p}, 1], \) rewrite equation 20 as follows:

\[ \Delta W = w \int_0^{\bar{p}} \delta(p_t) dp_t \geq 0 \] (21)

Given \( w > 0, \) \( \int_0^{\bar{p}} \delta(p_t) dp_t \geq 0 \) is necessary and sufficient for \( \Delta W \geq 0. \)
Appendix C: An additional proposition

**Proposition 3':** Given two mobility processes \( D_{w}^{(i,j+1)} \) and \( D_{a}^{(i,j+1)} \), \( D_{w}^{(i,j+1)} \) is preferred to \( D_{a}^{(i,j+1)} \) \( \forall W \in P^* \) for which \( \frac{\delta w(p_i)}{\delta p_i} \leq 0 \) for all \( p_i \in [0, 1] \), \( \frac{\delta^2 w(p_i)}{\delta p_i^2} \geq 0 \) for all \( p_i \in [0, \bar{p}] \), \( \frac{\delta^2 w(p_i)}{\delta p_i^2} \leq 0 \) for all \( p_i \in [\bar{p}, 1] \) if and only if

\[
(i) \quad \int_{0}^{\bar{p}} \int_{0}^{q_0} d_{\pi}(s_i)ds_i \geq \int_{0}^{p_i} \int_{0}^{q_0} d_{\omega}(s_i)ds_i \forall p_i \in [0, 1] \forall p_i \in [0, \bar{p}] \tag{22}
\]

\[
(ii) \quad \int_{p_i}^{1} \int_{0}^{q_0} d_{\pi}(s_i)ds_i \geq \int_{p_i}^{1} \int_{0}^{q_0} d_{\omega}(s_i)ds_i \forall p_i \in [0, 1] \forall p_i \in [\bar{p}, 1] \tag{23}
\]

This is an alternative to Proposition 3. It suggests a test based on third-order upward (downward) DynaC dominance for all dynasties ranked lower (higher) or equal to \( \bar{p} \). It finds its justification in the presence of a social planner that wants to preserve the mobility of the poorest among the poor while, at the same time, avoiding that the distances among the richest dynasties grow further apart (see Aaberge (2009) for a discussion on the application of this principle in standard inequality measurement).

C1 Proof of Proposition 3'

We seek sufficient and necessary conditions for:

\[
\Delta W = \int_{0}^{1} w(p_i)\delta(p_i)dp_i \geq 0 \tag{24}
\]

\( \forall W \in P^* \) for which \( \frac{\delta w(p_i)}{\delta p_i} \leq 0 \) for all \( p_i \in [0, 1] \), \( \frac{\delta^2 w(p_i)}{\delta p_i^2} \geq 0 \) for all \( p_i \in [0, \bar{p}] \), \( \frac{\delta^2 w(p_i)}{\delta p_i^2} \leq 0 \) for all \( p_i \in [\bar{p}, 1] \).

For a given \( \bar{p} \), rewrite equation 24 as follows:

\[
\Delta W = \int_{0}^{\bar{p}} w(p_i)\delta(p_i)dp_i + \int_{\bar{p}}^{1} w(s_i)\delta(s_i)ds_i \geq 0 \tag{25}
\]

For sufficiency, use the notations \( \Psi(p_i) = \int_{0}^{p_i} \delta(q_i)dq_i \), \( \Psi(\bar{p}) = \int_{0}^{p_i} \delta(p_i)dp_i \) and \( \Theta(s) = \int_{\bar{p}}^{s} \delta(r_i)dr_i \) and \( \Theta(1) = \int_{\bar{p}}^{1} \delta(r_i)dr_i \). We integrate equation 25 by parts twice to have:

\[
w(\bar{p})\Psi(\bar{p}) - w'(\bar{p}) \int_{0}^{\bar{p}} \Psi(p_i)dp_i + \int_{0}^{\bar{p}} \Psi(q_i)dp_i \geq 0 \tag{26}
\]

\[
w(1)\Theta(1) - w'(1) \int_{\bar{p}}^{1} \Theta(s_i)ds_i + \int_{\bar{p}}^{1} \Psi''(s_i) \int_{\bar{p}}^{s_i} \Theta(r_i)dr_i ds_i \geq 0
\]

The last component of the above equation can be rewritten as follows:

\[
\int_{\bar{p}}^{1} w''(s_i) \left[ \int_{\bar{p}}^{1} \Theta(r_i) - \int_{s_i}^{1} \Theta(r_i) \right] dr_i ds_i = \int_{\bar{p}}^{1} w''(s_i) \int_{\bar{p}}^{1} \Theta(r_i)dr_i ds_i - \int_{\bar{p}}^{1} w''(s_i) \int_{s_i}^{1} \Theta(r_i)dr_i ds_i.
\]

Noting that \( \int_{\bar{p}}^{1} w''(p_i)dp_i = w'(1) - w'(\bar{p}) \), for \( w'(1) = 0 \) we have:

\[
-w'(\bar{p}) \int_{\bar{p}}^{1} \Theta(r_i)dr_i - \int_{\bar{p}}^{1} w''(s_i) \int_{s_i}^{1} \Theta(r_i)dr_i ds_i
\]

\( \Delta W \) can now be rewritten as follows:

\[
w(\bar{p})\Psi(\bar{p}) - w'(\bar{p}) \int_{0}^{\bar{p}} \Psi(p_i)dp_i + \int_{0}^{\bar{p}} w''(p_i) \int_{0}^{p_i} \Psi(q_i)dp_i \tag{27}
\]
w(1)\Theta(1) - \int_{\bar{p}_t}^{1} w''(s_t) \int_{st}^{1} \Theta(r_t) dr_t ds_t

Since \( w''(s_t) \leq 0 \) for all \( s_t \in [\bar{p}_t, 1] \), \( \int_{st}^{1} \Theta(r_t) dr_t \geq 0 \) for all \( s_t \in [\bar{p}_t, 1] \) implies \(-\int_{\bar{p}_t}^{1} w''(s_t) \int_{st}^{1} \Theta(r_t) dr_t \geq 0 \). Hence, \( \int_{st}^{1} \Theta(r_t) dr_t ds_t \geq 0 \) for all \( s \in [\bar{p}_t, 1] \) is sufficient for the sum of the last two components of equation 26 to be positive. The sufficiency for the positivity of the first three terms has been proved in Proposition 3; for this proposition just assume that \( p_t \in [0, \bar{p}_t] \). Putting together the arguments: \( \int_{0}^{p_t} \int_{0}^{q_t} \delta(x_t) dx_t dq_t \geq 0 \) for all \( p_t \in [0, \bar{p}_t] \) and \( \int_{s_t}^{1} \int_{st}^{1} \delta(r_t) dr_t ds_t \geq 0 \) for all \( s \in [\bar{p}_t, 1] \) imply \( \Delta W \geq 0 \). For the necessity part, let \( \Delta W \geq 0 \), but assume that \( \int_{s_t}^{1} \Theta(r_t) dr_t < 0 \) for some \( s_t \in [\alpha, \beta] \subset [\bar{p}_t, 1] \). \( \int_{p_t}^{1} w''(p_t) \int_{q_t}^{1} \Theta(q_t) dq_t dp_t \leq 0 \). Now, it is always possible to find a combination of \( w(1) \) and \( \Theta(1) \) such that \( |w(1)\Theta(1)| < \left| \int_{0}^{1} w''(p_t) \int_{0}^{p_t} \Psi(q_t) dq_t dp_t \right| \). Putting together these results with those obtained for Proposition 3 (letting them hold for all \( p \in [0, \bar{p}_t] \)) would result into \( \Delta W < 0 \), a contradiction.