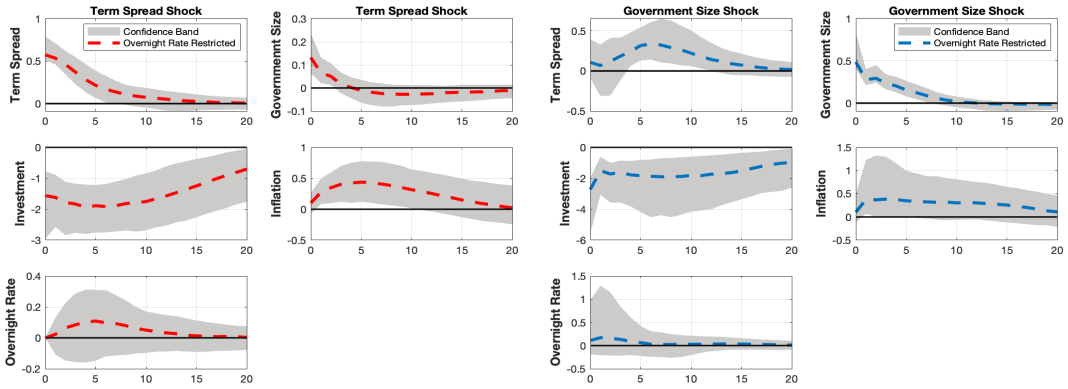
Online Appendix

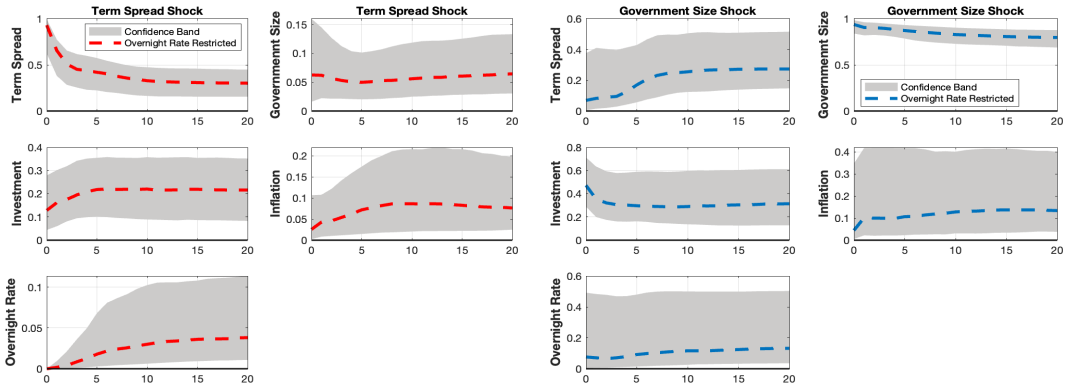
A Additional Results from Five Variable VAR Models

A.1 Model with Investment Instead of GDP



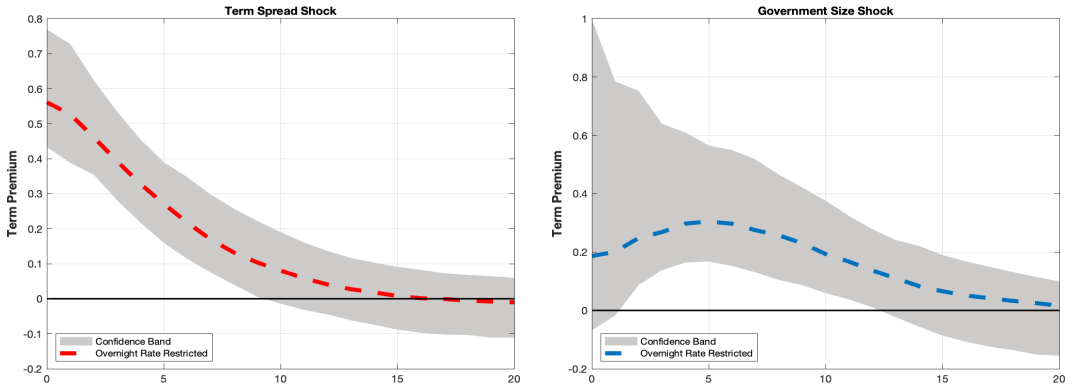
(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.1: Notes: We use five variables (term spread, government size, gross capital formation, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log gross fixed capital formation and log consumer prices due to the government size shock is -0.72 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

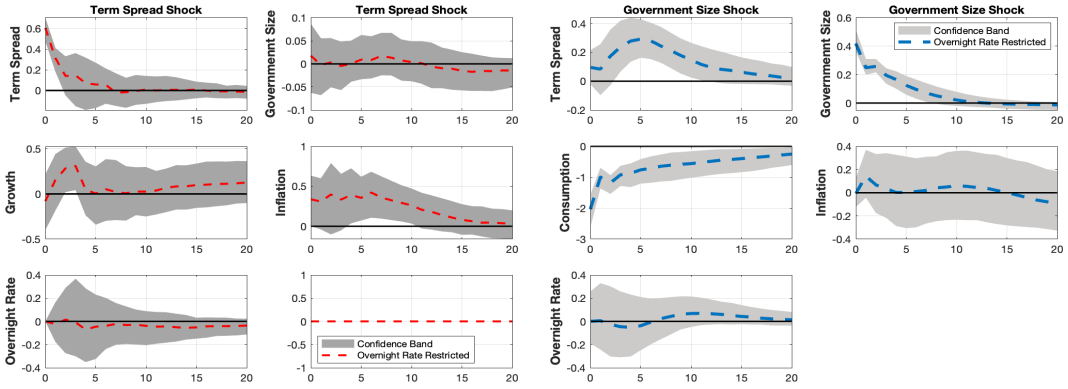
Figure A.2: Notes: We use five variables (term spread, government size, gross capital formation, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

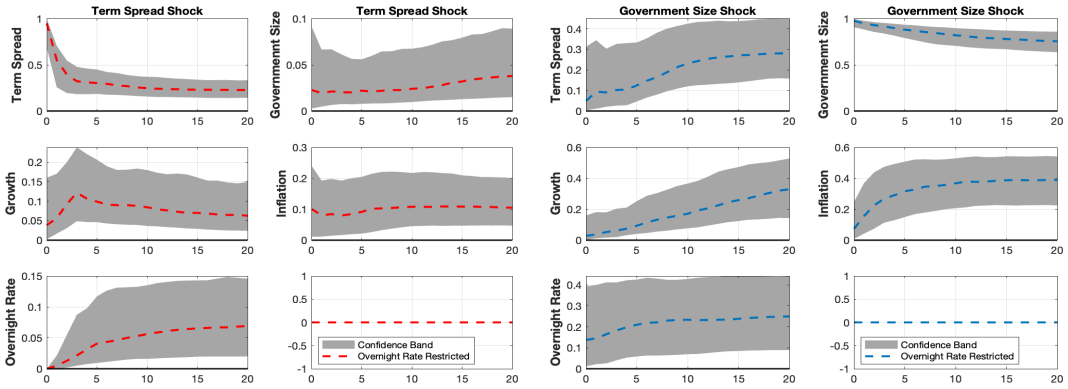
Figure A.3: We use five variables (term spread, size of the government, gross capital formation, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.2 Model with Private Consumption Instead of GDP



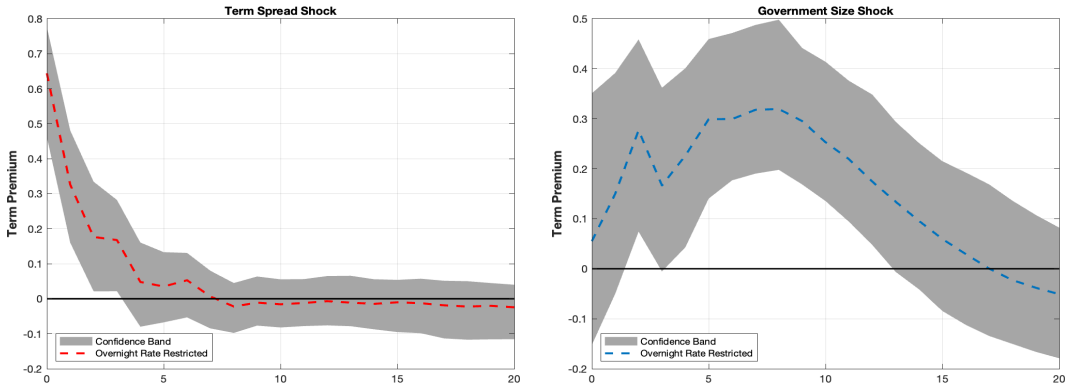
(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.4: Notes: We use five variables (term spread, government size, private consumption, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log private final consumption expenditure and log consumer prices due to the government size shock is -0.73 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

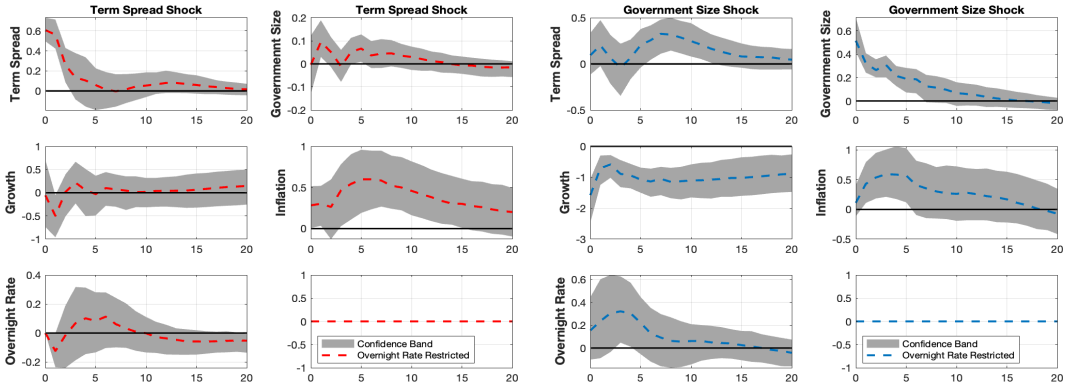
Figure A.5: Notes: We use five variables (term spread, government size, private consumption, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

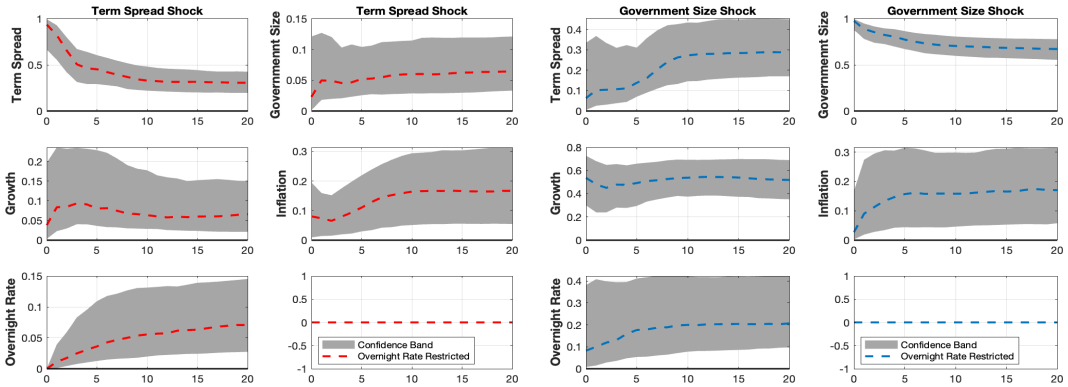
Figure A.6: We use five variables (term spread, size of the government, private consumption, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.3 Model with Alternative Term Spread



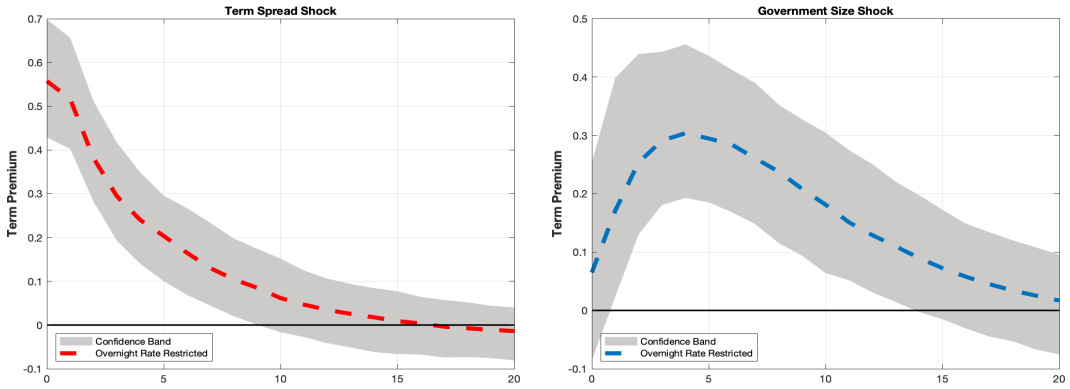
(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.7: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.72 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

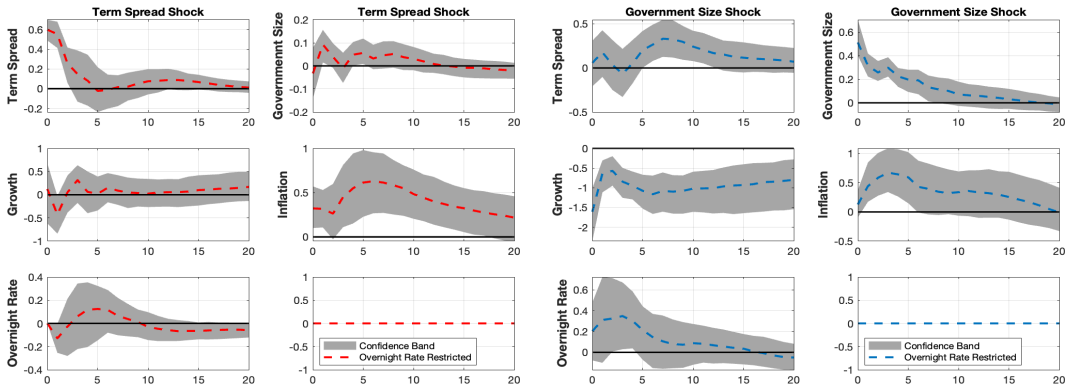
Figure A.8: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

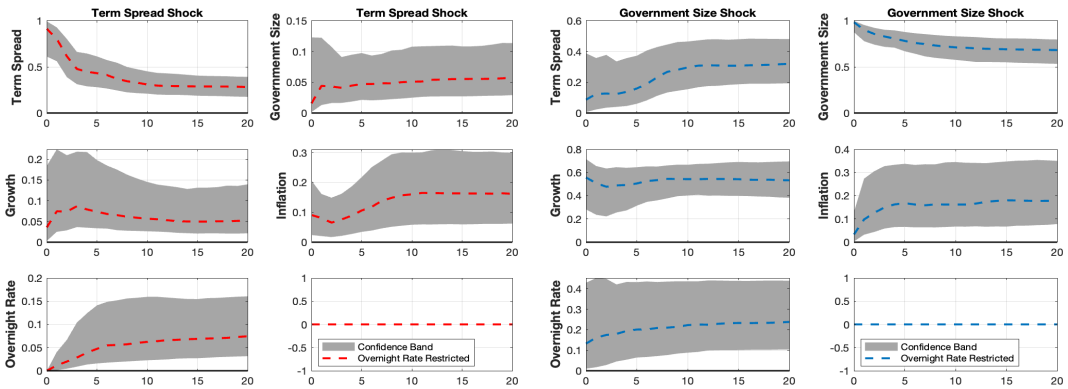
Figure A.9: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.4 Model with Alternative Term Spread and Size of the Government



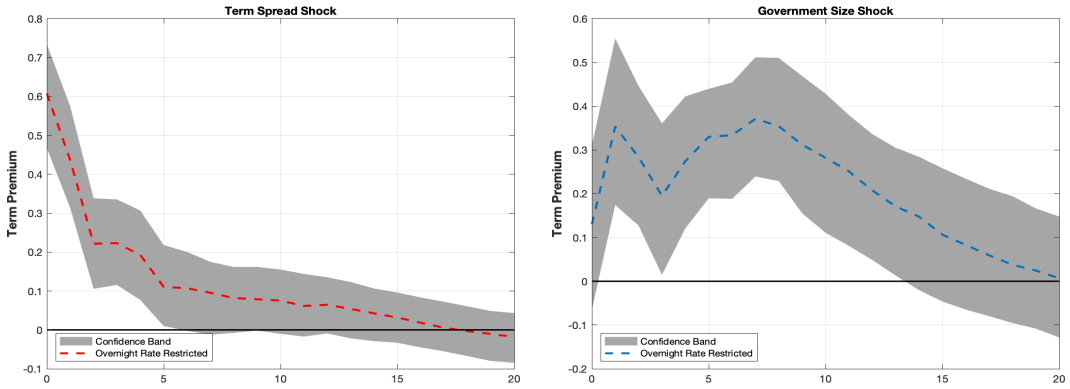
(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.10: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.69 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

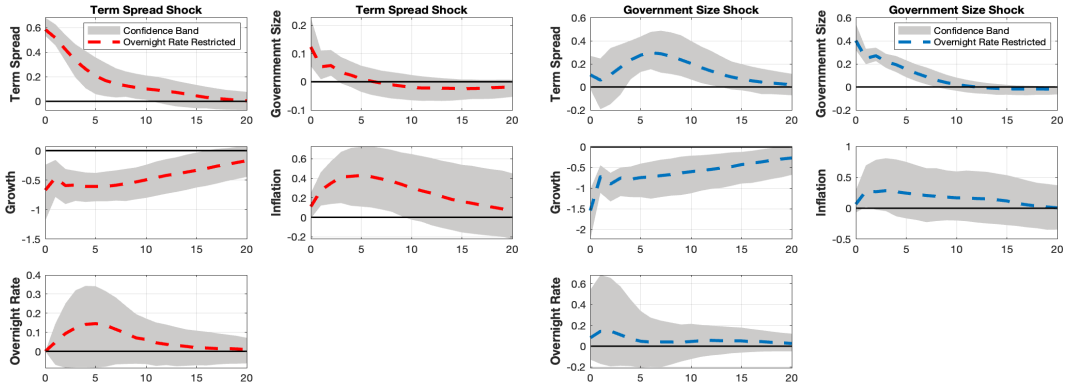
Figure A.11: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

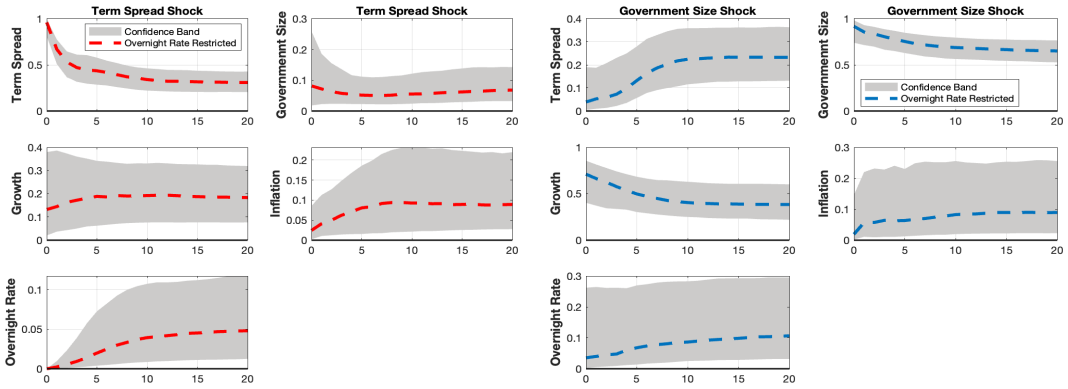
Figure A.12: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.5 Recessionary Government Size Shock



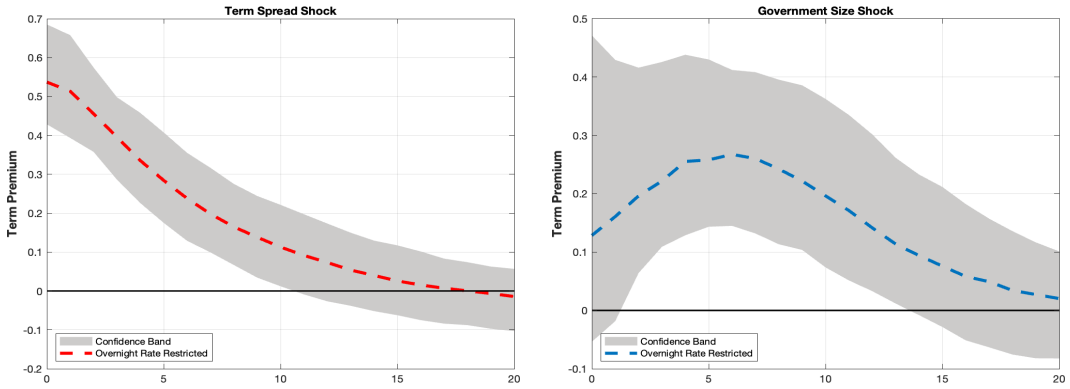
(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.13: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The recessionary government size shock is identified using additional restriction that government size shock decreases the output for two time periods $t = 0$ and $t = 1$. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.74 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

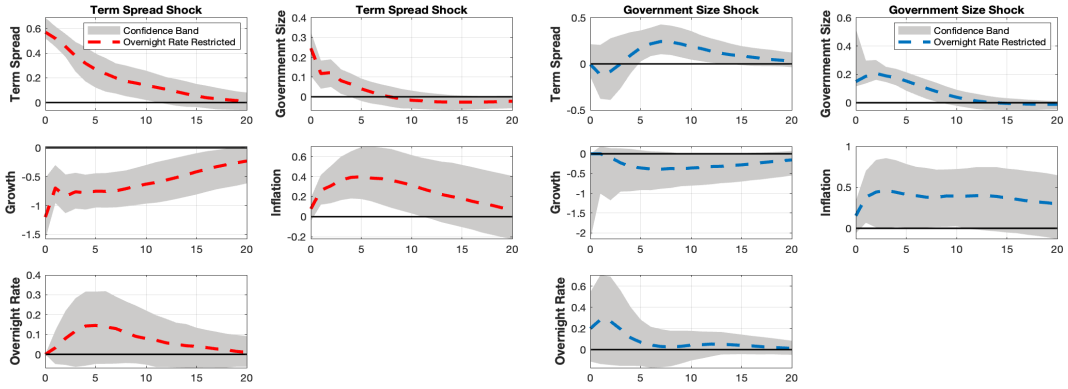
Figure A.14: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

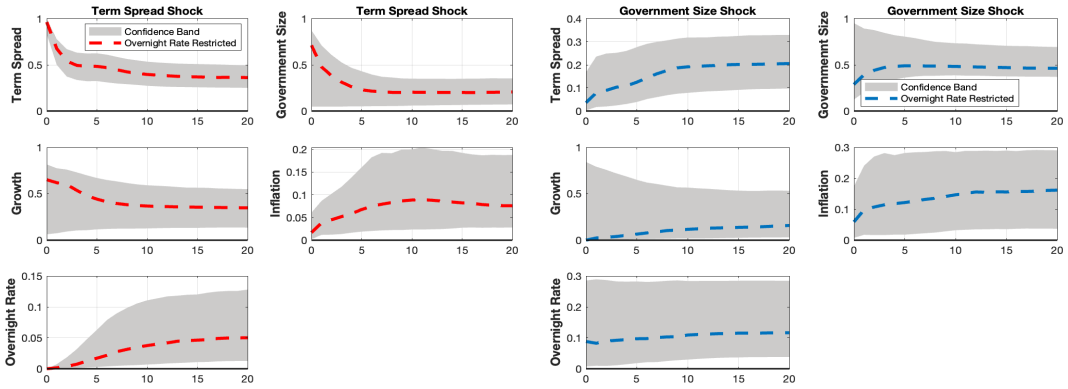
Figure A.15: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.6 Expansionary Government Size Shock



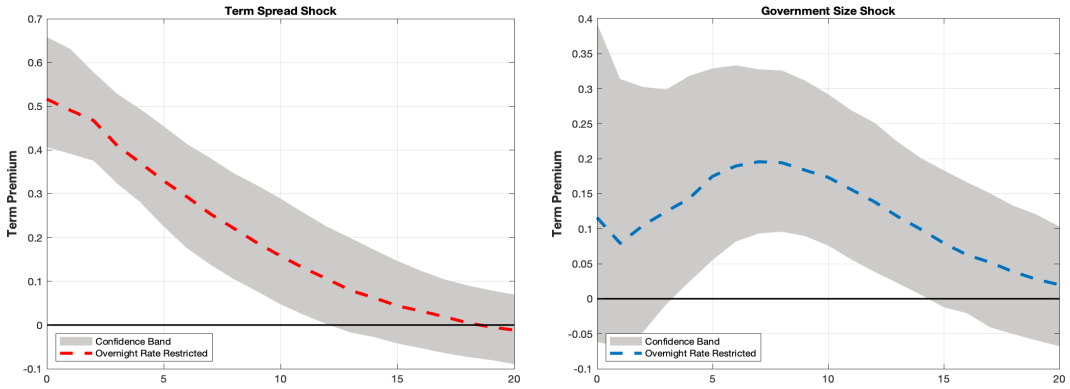
(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.16: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The expansionary government size shock is identified using additional restriction that government size shock increases the output for two time periods $t = 0$ and $t = 1$.The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.63 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

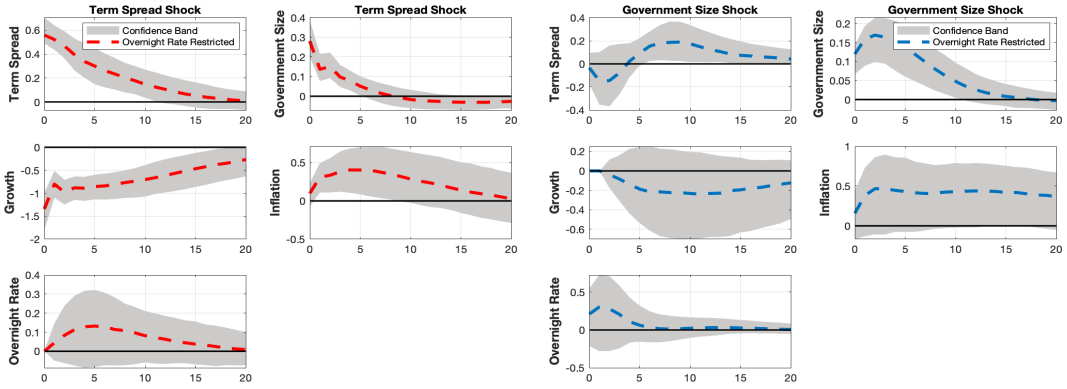
Figure A.17: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

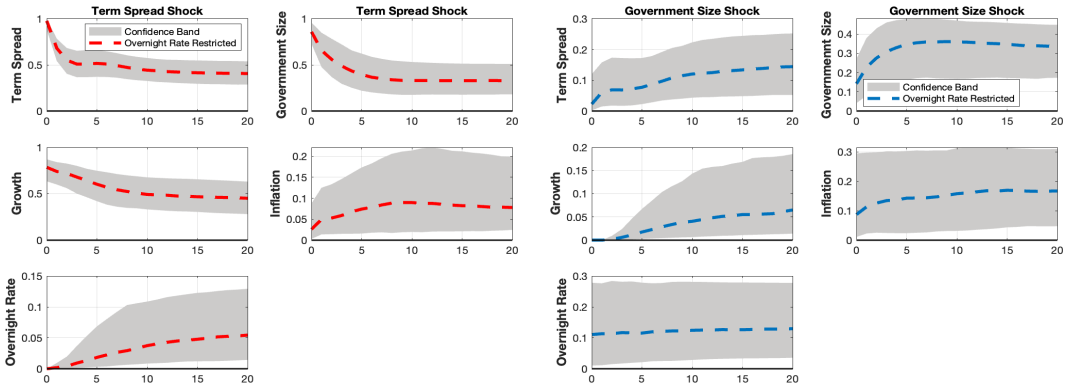
Figure A.18: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.7 Neutral Government Size or Government Expenditure Shock



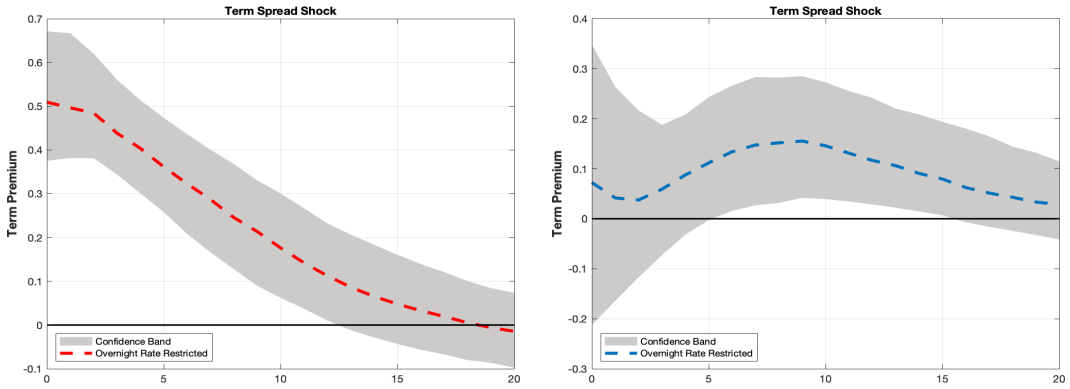
(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.19: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The neutral government size shock is identified using additional restriction that government size shock does not affect the output for two time periods $t = 0$ and $t = 1$. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.69 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End (b) Government Size Shock

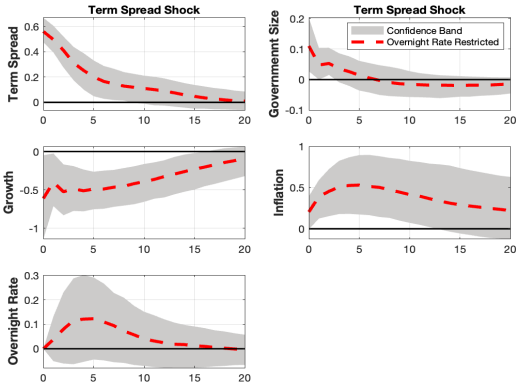
Figure A.20: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



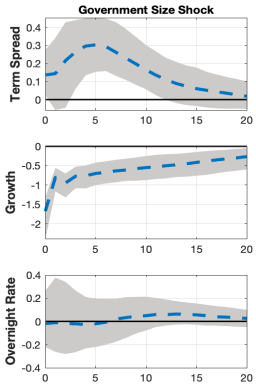
(a) Term Spread Shock at Long End (b) Government Size Shock

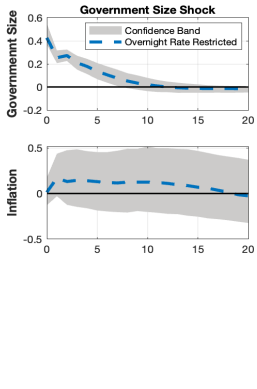
Figure A.21: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.8 Term Spread, Government Size and News Shocks

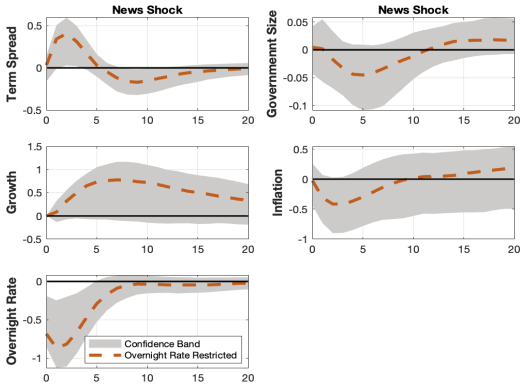


(a) Term Spread Shock at Long End



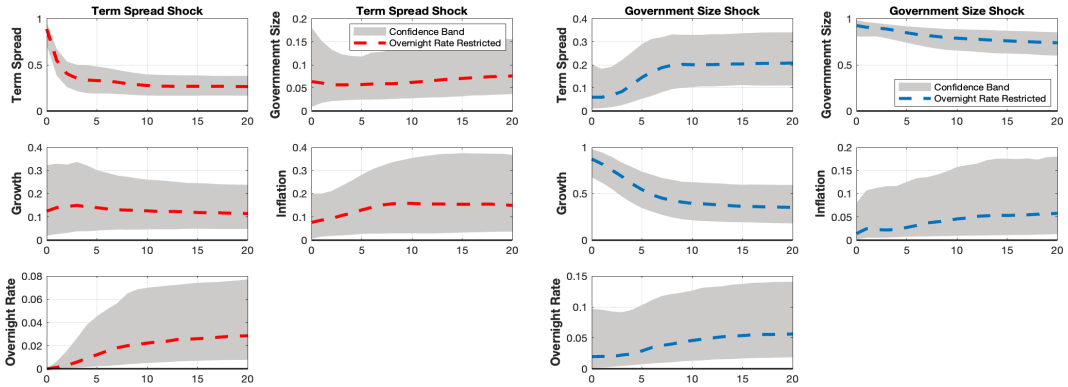


(b) Government Size Shock



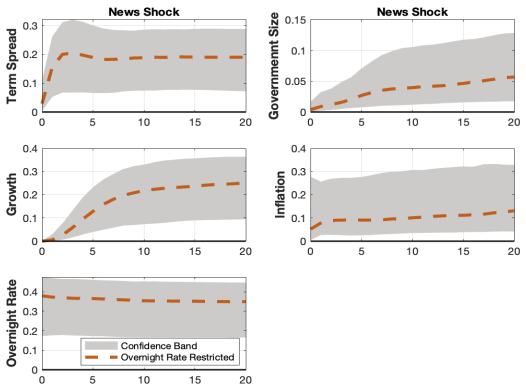
(c) News Shock

Figure A.22: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify three shocks, term spread shock originating at the long end, government size shock and new shock that maximize the sum of share of these three orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (c) gives the the responses of the variable due to news shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.69 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End

(b) Government Size Shock



(c) News Shock

Figure A.23: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify three shocks, term spread shock originating at the long end, government size shock and new shock that maximize the sum of share of these three orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. (c) the share of news shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

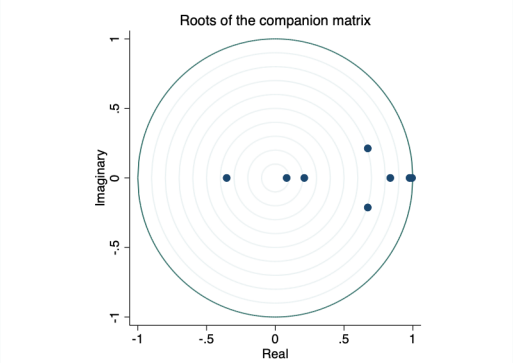
A.9 VAR: Diagnostics

Table A.1: Lag Length Selection: Four Variables VAR

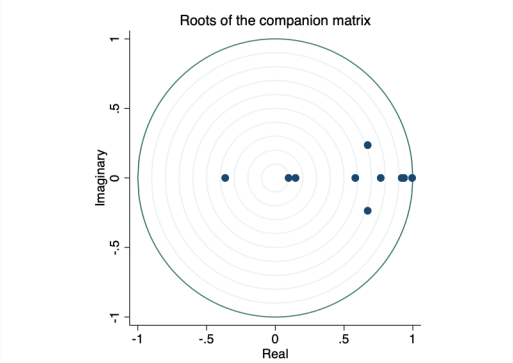
Lag	FPE	AIC	HQIC	SBIC
0	6.13573			
1	8.20E-09	-7.26575	-7.07508	-6.79615
2	4.4e-09*	-7.88946*	-7.54624*	-7.04416*
3	4.90E-09	-7.78783	-7.29208	-6.56685
4	5.60E-09	-7.66095	-7.01265	-6.06428

Table A.2: Lag Length Selection: Five Variables VAR

Lag	FPE	AIC	HQIC	SBIC
0	8.24713			
1	1.90E-10	-8.21213	-7.92612	-7.50772
2	8.8e-11*	-8.97042*	-8.44606*	-7.679*
3	9.60E-11	-8.88118	-8.11848	-7.00275
4	1.10E-10	-8.77899	-7.77794	-6.31355



(a) Four Variable VAR



(b) Five Variable VAR

Figure A.24: Notes: Stability test (a) We use four variables (term spread, GDP, consumer price and overnight rate) in the SVAR. (b) We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an three month rate. The size of the government is given by the ratio of the government final consumption expenditure to private final consumption expenditure. The sample period is 1993Q1-2023Q2.

B Coefficient of Risk Aversion in Generalized Recursive Preferences With Labor Income Tax

Proposition 12 in Swanson (2009) gives the absolute risk aversion for Epstein-Zin preferences given below:

$$R^a(a;\theta) = \frac{-V_{11}(a,\theta)}{V_1(a,\theta)} + \alpha \frac{V_1(a,\theta)}{V(a,\theta)}$$

The first derivative of the value function is given by:

$$V_1(a_t,\theta_t) = (1+r_t)u_1(c_t^*, l_t^*)$$

where we multiply by $(1+r)$ because assets in the beginning of time period t generate income in time period t. The second derivative of the value function is given by:

$$V_{11}(a_t, \theta_t) = (1 + r_t) \left[u_{11}(c_t^*, l_t^*) \frac{\partial c_t^*}{\partial a_t} + u_{12}(c_t^*, l_t^*) \frac{\partial l_t^*}{\partial a_t} \right]$$

Intra-temporal condition (consumption leisure decision) with labour income tax is given by:

$$-u_2(c_t^*, l_t^*) = w_t(1 - \tau) u_1(c_t^*, l_t^*)$$

Differentiating both sides, we obtain:

$$-u_{21}(c_t^*, l_t^*) \frac{\partial c_t^*}{\partial a_t} - u_{22}(c_t^*, l_t^*) \frac{\partial l_t^*}{\partial a_t} = w_t(1 - \tau) u_{11}(c_t^*, l_t^*) \frac{\partial c_t^*}{\partial a_t} + w_t(1 - \tau) u_{12}(c_t^*, l_t^*) \frac{\partial l_t^*}{\partial a_t}$$

which gives:

$$\frac{\partial l_t^*}{\partial a_t} = -\lambda_t \frac{\partial c_t^*}{\partial a_t}$$

where

$$\lambda_t = \frac{[u_{21}(c_t^*, l_t^*) + w_t(1 - \tau) u_{11}(c_t^*, l_t^*)]}{[u_{22}(c_t^*, l_t^*) + w_t(1 - \tau) u_{12}(c_t^*, l_t^*)]}$$

Swanson (2009) does not consider labour income tax. Budget constraint with labour income tax is given by

$$a_{t+1} = (1 + r) a_t + w_t(1 - \tau_t) l_t - c_t$$

Iterating it gives the life time budget constraint given by:

$$a_t = \sum_{s=0}^{\infty} \frac{1}{(1 + r)^{s+1}} \left[c_{t+s} - w_t(1 - \tau_t) l_{t+s} \right] + \lim_{s \to \infty} \left(\frac{1}{1 + r} \right)^{s+1} a_{t+s+1}$$

Using $\lim_{s \to \infty} \left(\frac{1}{1+r} \right)^{s+1} a_{t+s+1} \to 0$ gives:

$$\sum_{s=0}^{\infty} \frac{1}{(1 + r)^{s+1}} \left[c_{t+s} \right] = a_t + \sum_{s=0}^{\infty} \frac{1}{(1 + r)^{s+1}} \left[w_t(1 - \tau_t) l_{t+s} \right]$$

We multiply both sides by $(1 + r)$ and that gives:

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \left[c_{t+s}\right] = (1+r)a_t + \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \left[w_t(1-\tau_t)l_{t+s}\right]$$

Using $\frac{\partial l_t^*}{\partial a_t} = \frac{\partial l_{t+1}^*}{\partial a_t}$, see (Swanson (2009)) gives:

$$\frac{1+r}{r}\frac{\partial c_t^*}{\partial a_t} = (1+r) + w(1-\tau)\frac{1+r}{r}\frac{\partial l_t^*}{\partial a_t}$$

Using $\frac{\partial l_t^*}{\partial a_t} = -\lambda_t \frac{\partial c_t^*}{\partial a_t}$

$$\frac{1+r}{r}\frac{\partial c_t^*}{\partial a_t} = (1+r) + w(1-\tau)\frac{1+r}{r}\left(-\lambda_t\right)\frac{\partial c_t^*}{\partial a_t}$$

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1+w(1-\tau)\lambda}$$

Hence

$$R^a(a;\theta) = \frac{-\left(1+r_t\right)\left[u_{11} - u_{12}\lambda\right]\frac{\partial c_t^*}{\partial a_t}}{(1+r_t)u_1} + \alpha\frac{(1+r_t)u_1}{V(a,\theta)}$$

$V(a,\theta) = \frac{u}{1-\beta} = \frac{u}{1-\frac{1}{1+r}} = u(1+r)/r$

$$R^a(a;\theta) = \frac{-\left(1+r\right)\left[u_{11} - u_{12}\lambda\right]\frac{\partial c_t^*}{\partial a_t}}{(1+r)u_1} + \alpha\frac{(1+r)u_1}{u(1+r)/r}$$

$$R^a(a;\theta) = \frac{-u_{11} + u_{12}\lambda}{u_1}\frac{r}{1+w(1-\tau)\lambda} + \alpha\frac{u_1 r}{u}$$

Using equation 54 in Swanson (2012) and $A = c/r$ gives household's consumption-only coefficient of relative risk aversion given below:

$$R^c(a;\theta) = \frac{-u_{11} + u_{12}\lambda}{u_1}\frac{c}{1+w(1-\tau)\lambda} + \alpha\frac{u_1 c}{u}$$

use $u_{11} = -\phi c^{-\phi-1}$, $u_1 = c^{-\phi}$ and $u_{12} = 0$

$$R^c(a;\theta) = \frac{\phi c^{-\phi-1}}{c^{-\phi}}\frac{c}{1+w(1-\tau)\lambda} + \alpha\frac{c^{-\phi}c}{u}$$

$$R^c(a;\theta) = \frac{\phi}{1 + w(1-\tau)\lambda} + \alpha\frac{c^{-\phi}c}{u}$$

Use $-u_2 = w(1-\tau)u_1$, $\lambda = \frac{w(1-\tau)u_{11}}{u_{22}}$, $u_{22} = (-\eta)\,\chi_0\,(1-L_t)^{-\eta-1}$,$u_2 = -\chi_0\,(1-L_t)^{-\eta}$

$$R^c(a;\theta) = \frac{\phi}{1 - \frac{u_2}{(1-\tau)u_1}(1-\tau)\lambda} + \alpha\frac{c^{-\phi}c}{u}$$

Also $\lambda = \frac{w(1-\tau)u_{11}}{u_{22}}$

$$R^a(a;\theta) = \frac{\phi}{1 - \frac{u_2}{(1-\tau)u_1}(1-\tau)\frac{w(1-\tau)u_{11}}{u_{22}}} + \alpha\frac{c^{-\phi}c}{u}$$

$$R^a(a;\theta) = \frac{\phi}{1 - w(1-\tau)\frac{u_2}{u_1}\frac{u_{11}}{u_{22}}} + \alpha\frac{c^{-\phi}c}{u}$$

$$R^a(a;\theta) = \frac{\phi}{1 - w(1-\tau)\frac{u_{11}}{u_1}\frac{u_2}{u_{22}}} + \alpha\frac{c^{-\phi}c}{u}$$

We have $u_2 = -\chi_0\,(1-L_t)^{-\eta}$, $u_{22} = (-\eta)\,\chi_0\,(1-L_t)^{-\eta-1}$ and $u_{11} = -\phi c^{-\phi-1}$, $u_1 = c^{-\phi}$

$$\frac{u_2}{u_{22}} = \frac{-\chi_0\,(1-L_t)^{-\eta}}{(-\eta)\,\chi_0\,(1-L_t)^{-\eta-1}} = \frac{1-L}{\eta}$$

$$\frac{u_{11}}{u_1} = \frac{-\phi c^{-\phi-1}}{c^{-\phi}} = \frac{-\phi}{c}$$

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{\phi}{c}\frac{(1-L)}{\eta}w(1-\tau)} + \alpha\frac{c^{-\phi}c}{\frac{c^{1-\phi}}{1-\phi} + \chi_0\frac{(1-L)^{1-\eta}}{1-\eta}}$$

Since we have $-u_2 = w(1-\tau)u_1$

$$\chi_0\,(1-L_t)^{-\eta} = w(1-\tau)c^{-\phi}$$

Hence

$$R^c(a;\theta) = \frac{\phi}{1+\frac{(1-L)}{c}\frac{w(1-\tau)\phi}{\eta}} + \alpha\frac{c^{-\phi}c}{\frac{c^{1-\phi}}{1-\phi}+\frac{w(1-\tau)(1-L)c^{-\phi}}{(1-\eta)}}$$

$$R^c(a;\theta) = \frac{\phi}{1+\frac{(1-L)}{c}\frac{w(1-\tau)\phi}{\eta}} + \alpha\frac{c^{-\phi}c\,(1-\phi)}{c^{1-\phi}+\frac{w(1-\tau)(1-L)c^{1-\phi}(1-\phi)}{(1-\eta)c}}$$

$$R^c(a;\theta) = \frac{\phi}{1+\frac{(1-L)}{c}\frac{w(1-\tau)\phi}{\eta}} + \alpha\frac{(1-\phi)}{1+\frac{w(1-\tau)(1-L)(1-\phi)}{(1-\eta)c}}$$

$$R^c(a;\theta) = \frac{\phi}{1+\frac{(1-L)}{Lc}\frac{wL(1-\tau)\phi}{\eta}} + \alpha\frac{(1-\phi)}{1+\frac{wL(1-\tau)(1-L)(1-\phi)}{L(1-\eta)c}}$$

Using $wL(1-\tau) = c$

$$R^c(a;\theta) = \frac{\phi}{1+\frac{(1-L)}{L}\frac{\phi}{\eta}} + \alpha\frac{(1-\phi)}{1+\frac{(1-L)}{L}\frac{(1-\phi)}{(1-\eta)}}$$

Which is same as in footnote 23 of Rudebusch and Swanson (2012). ϕ gives intertemporal elasticity of substitution (IES where $\phi = \frac{1}{IES}$. The Frisch elasticity is given by:

$$\text{Frisch Elasticity} = \frac{U'_{L_t}}{L_t\left[U'_{L_t} - \frac{U'_{L_tC_t}}{U'_{L_tL_t}}\right]}$$

$$= \frac{-\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta}}{L_t\left(-\eta\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta-1} - \frac{0}{(-\phi)C_t^{-\phi-1}}\right)}$$

$$= \frac{-\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta-1}(1-L_t)}{L_t\left(-\eta\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta-1}\right)}$$

$$\text{Frisch Elasticity } = \frac{(1-L_t)}{\eta L_t}$$

$$R^c(a;\theta) = \frac{\phi}{1+\text{Frisch Elasticity }\times\phi} + \alpha\frac{(1-\phi)}{1+\text{Frisch Elasticity }\frac{\eta(1-\phi)}{(1-\eta)}}$$

C Model

C.1 Final Good Producer

The final good is produced by a firm in a perfectly competitive market, which combines the intermediate goods using the constant returns to scale technology given by:

$$y_t \leq \left[\int_0^1 y_t(i)^{(\phi_m-1)/\phi_m} di\right]^{\phi_m/(\phi_m-1)}$$

Where ϕ_m is the elasticity of substitution between intermediate goods $Y_t(i)$ with given price $P_t(i)$. Profit maximization leads to the following demand for intermediate goods:

$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\phi_m} y_t$$

Where

$$P_t = \left[\int_0^1 P_t(i)^{1-\phi_m} di\right]^{1/(1-\phi_m)}$$

$$P_t^{1-\phi_m} = \int_0^1 P_t(i)^{1-\phi_m} di$$

C.2 Intermediate goods producers

The production function of the intermediate goods producer is given by:

$$Y_t(i) = A_t \bar{K_t}(i)^{1-\theta} \left(Z_t L_t(i)\right)^{\theta}$$

where A_t is a stationary productivity shock given by:

In each period there is a probability (ξ) that the firm will not be able to change prices. So whenever the firm changes its price, it maximizes the expected sum of discounted

profits. The objective function is given by

$$\sum_{k=0}^{\infty} \xi^k \beta^k \lambda_{t,t+k} \left[P_t(i) - mc_{t+k} \right] Y_{t+k}(i)$$

The first order condition with respect to $P_t(i)$ is given by:

$$P_t(i) = \frac{\phi_m}{\phi_m - 1} \frac{\sum_{k=0}^{\infty} \xi^k \beta^k \Lambda_{t+k} mc_{t+k}(i) Y_{t+k}(i)}{\sum_{k=0}^{\infty} \xi^k \beta^k \Lambda_{t+k} Y_{t+k}}$$

where

$$mc_t(i) = \frac{W_t L_t(i)}{\theta Y_t}$$

We also introduce a markup shock in the model given by:

$$log(\phi_{mt}) = (1-\rho_\phi) log(\phi_m) + \rho_{\phi_m} log(\phi_{mt-1}) + \epsilon_{\phi_m,t} \quad 0 \leq \rho_{\phi_m} < 1 \qquad \epsilon_{\phi_m,t} \sim N(0, \sigma^2_{\phi_m})$$

C.3 Monetary policy

Central bank implements a monetary policy based on modified Taylor rule given by:

$$log\left(\frac{i_t}{i}\right) = \rho_i log\left(\frac{i_{t-1}}{i}\right) + (1-\rho_i)$$

$$\left[log\left(i\right) + log\left(\pi_t^4\right) + \rho_\pi log\left(\frac{\pi_t^4}{\pi}\right) + \rho_g log\left(\frac{y_t}{y}\right) \right] + \epsilon_{it} \quad \epsilon_{i,t} \sim N(0, \sigma_i^2)$$

where R and π are steady state value of R_t and π_t and y is the steady state value of stationary variable $y_t = \frac{Y_t}{Z_t}$. π_t^4 is the geometric moving average of inflation

$$log\left(\pi_t^4\right) = \theta_\pi log(\pi_{t-1}^4) + (1-\theta_\pi) log\left(\pi_t\right)$$

We estimate θ_π.

D Risk Aversion and Time Preferences

We estimate another set of models with a discount rate of $\beta = 0.98$. All the previous models were estimated with the discount rate $\beta = 0.99$. Given the structure of the model, lower β implies a higher steady-state interest rate. These results help us in understanding the relationship between discount rate and risk aversion. Booji and Parag (2009) & Ferecatu and Onculer (2022) argue a negative correlation between risk and time preference suggesting that risk averse agents are more patient and more willing to defer consumption. Results obtained in this paper reported in Table D.1 suggest that lowering β increases the coefficient of risk aversion both in recession and expansion compared to the estimates obtained using $\beta = 0.99$. These new estimates suggest risk aversion of coefficients of 11948 & 16105 in expansion and recession respectively. In other words, the time impatient households are also the more risk averse households and this is opposite of the relationship found in Booji and Parag (2009) & Ferecatu and Onculer (2022).

Table D.1: Estimated Parameters Using Impulse Response Matching

Coefficient	Expansionary	Recessionary
σ_k	0.0457	0.0627
ρ_k	0.8882	0.7952
ρ_τ	0.0726	0.0087
$\rho_{\tau b}$	-0.0176	-0.0457
$\rho_{\tau y}$	-0.0	-0.9369
θ_π	-0.0154	-0.9605
ϕ	12.4286	2.1724
η	4.5931	10.2267
α	-7694.9	-17226.0

Notes: We match the response of the term premium due to expansionary government size shock in the model with the response of the term premium due to government size shock in the structural vector auto-regression model. These estimates are obtained using $\beta = 0.98$. All other parameters are kept at calibrate values given in tables 1 and 2.

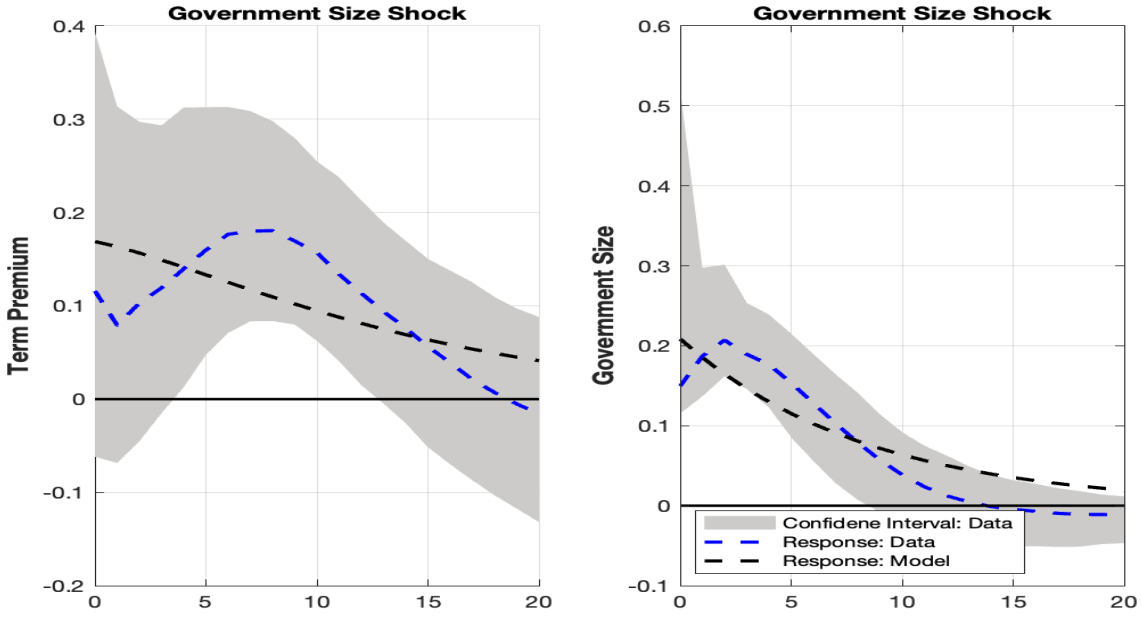


Figure D.1: Response of variables due to the expnasionary government size shock. The structural vector autoregression (data) is the five variable model with overnight rate. The model is the new Keynesian model with parameters given in tables 1, 2 and 4.

The reason for the difference could be the use of CRRA utility specification in these papers. Utility function in these papers do not have laour margin unlike the labour margin in this paper and that gives rise to differences in risk aversion. Also the traditional measure of risk aversion used in these paper is not the true measure of risk aversion as pointed by Swanson (2009). Also As mentioned before we use Epstein-Zin utility and this has distinct parameter to capture risk aversion apart from the parameter in CRRA utility function which is also inverse of elasticity of substitution. The estimates with lower β also changes the estimates of coefficient of Frisch elasticity of labour supply. With lower β, we find that the Frisch elasticity of labour supply is much higher in expansion than recession. Figures D.1 and D.2 give the response of term premium and government size shocks and corresponding responses from the model based on the estimated parameters.

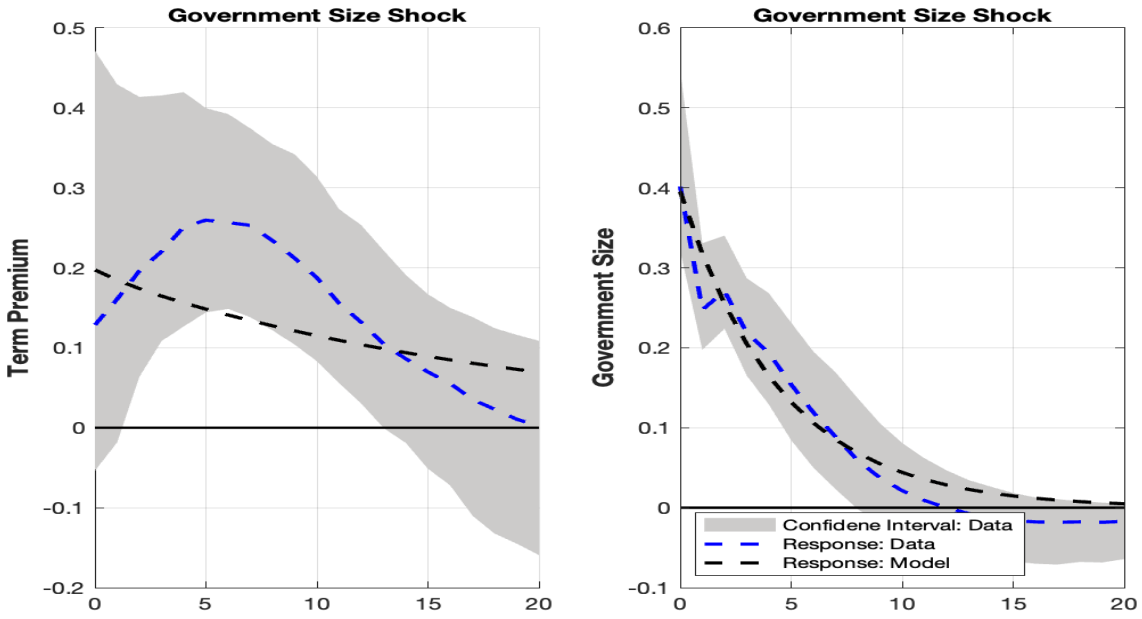


Figure D.2: Response of variables due to the recessionary government size shock. The structural vector autoregression (data) is the five variable model with overnight rate. The model is the new Keynesian model with parameters given in tables 1, 2 and 4.

E Data Sources

- 3-Month or 90-day Rates and Yields: Treasury Securities for South Africa
  https://fred.stlouisfed.org/series/IR3TTS01ZAM156N
- Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for South Africa
  https://fred.stlouisfed.org/series/IRLTLT01ZAM156N
- Immediate Rates: Less than 24 Hours: Central Bank Rates for South Africa
  https://fred.stlouisfed.org/series/IRSTCB01ZAM156N
- Private Final Consumption Expenditure in South Africa
  https://fred.stlouisfed.org/series/NAEXKP02ZAQ189S

- Gross Fixed Capital Formation in South Africa
  https://fred.stlouisfed.org/series/NFIRSAXDCZAQ
- Government Final Consumption Expenditure in South Africa
  https://fred.stlouisfed.org/series/NAEXKP03ZAQ652S
- Real Gross Domestic Product for South Africa
  https://fred.stlouisfed.org/series/NGDPRSAXDCZAQ
- Consumer Price Index: All Items for South Africa
  Growth rate previous period, Not Seasonally Adjusted https://fred.stlouisfed.org/series/CPALTT01ZA
- Consumer Price Index: All Items for South Africa
  Growth rate same period previous year, Not Seasonally Adjusted https://fred.stlouisfed.org/series/CPA
- General government gross debt for South Africa
  https://fred.stlouisfed.org/series/GGGDTAZAA188N
- Household Debt to GDP for South Africa
  https://fred.stlouisfed.org/series/HDTGPDZAQ163N