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**WALRAS' LAW**

**Don Patinkin**

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*The Hebrew University of Jerusalem  
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WIDER  
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Finland

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## WALRAS' LAW

Walras' Law (so named by Lange, 1942) is an expression of the interdependence among the excess-demand equations of a general-equilibrium system that stems from the budget constraint. Its name reflects the fact that Walras, the father of general-equilibrium economics, himself made use of this interdependence from the first edition of his Eléments d' économie politique pure (1874, §122) through the fourth (1900, §116), which edition is for all practical purposes identical with the definitive one (1926). I have cited §116 of this edition because it is the one cited by Lange (1942, p.51, fn.2), though in a broader context than Walras' own discussion there (see below). In this section, Walras presents the argument for an exchange economy. In accordance with his usual expository technique (cf. his treatment of the tâtonnement), he repeats the argument as he successively extends his analysis to deal first with a simple production economy and then with one in which capital formation also takes place (ibid, §§206 and 250, respectively).

For reasons that will become clear later, I shall derive Walras' Law in a more general - and more cumbersome - way than it usually has been.

Basically, however, the derivation follows that of Arrow and Hahn (1971, pp.17-21), with an admixture of Lange (1942) and Patinkin (1956, chapters I-III and Mathematical Appendix 3:a).

Let  $x_i^h$  be the decision of household  $h$  with respect to good  $i$  ( $i = 1, \dots, n$ ), where "goods" also include services and financial assets (securities and money). If  $x_i^h \geq 0$ , it is a good purchased by the household; if  $x_i^h < 0$ , it is a good (mainly, labour, or some other factor-service) sold. Similarly, let  $y_i^f$  be the decision of firm  $f$  with respect to good  $i$  : if  $y_i^f \geq 0$ , it is a good produced and sold by the firm (i.e., a product-output); if  $y_i^f < 0$ , it is a factor-input.

Assume that firm  $f$  has certain initial conditions (say, quantities of fixed factors of production) represented by the vector  $k^f$  and operates in accordance with a certain production function. Following Patinkin (1956), let us conduct the conceptual individual-experiment of confronting the firm with the vector of variables  $v$  (the nature of which will be discussed below) while keeping  $k^f$  constant and asking it to designate (subject to its production function) the amounts that it will sell or buy of the various goods and services. By repeating this conceptual experiment with different values of the respective elements of  $v$ , we obtain the behaviour functions of firm  $f$ ,

$$(1) \quad y_i^f = y_i^f(v; k^f) \quad (i = 1, \dots, n).$$

For  $y_i \geq 0$ , this is a supply function; for  $y_i < 0$ , it is a demand function for the services of factors of production. Profits (positive or negative) of firm  $f$  are then

$$(2) \quad R^f = \sum_i p_i y_i^f(\mathbf{v}; \mathbf{k}^f),$$

Let  $d^{hf}$  represent the proportion of the profits of firm  $f$  received by the household  $h$ . Its total profits received are then  $\sum_f d^{hf} R^f$  and its budget restraint is accordingly

$$(3) \quad \sum_i p_i x_i^h = \sum_f d^{hf} R^f,$$

which assumes that households correctly estimate the profits of firms (cf. Buiter, 1980, p.7; I shall return to this point below). As with the firm, let us, mutatis mutandis, conduct individual-experiments with household  $h$  (with its given tastes), subject to its budget restraint (3) by varying the elements of  $\mathbf{v}$ , while keeping its initial endowment (represented by the vector  $\mathbf{e}^h$ ) constant. This yields the behaviour functions

$$(4) \quad x_i^h = x_i^h(\mathbf{v}; \mathbf{e}^h) \quad (i = 1, \dots, n).$$

For  $x_i \geq 0$ , this is a demand function for goods; for  $x_i < 0$ , it is a supply function (e.g., of factor-services).

Substituting from (2) and (4) into (3) then yields

$$(5) \quad \sum_i p_i x_i^h(\mathbf{v}; \mathbf{e}^h) = \sum_f d^{hf} \sum_i p_i y_i^f(\mathbf{v}; \mathbf{k}^f),$$

which holds identically for all  $\mathbf{v}$ ,  $\mathbf{e}^h$ , and  $\mathbf{k}^f$ , and  $p_i$ . Summing up over all households then yields

$$(6) \quad \sum_h \sum_i p_i x_i^h(\mathbf{v}; \mathbf{e}^h) = \sum_h \sum_f d^{hf} \sum_i p_i y_i^f(\mathbf{v}; \mathbf{k}^f),$$

which we rewrite as

$$(7) \quad \sum_i p_i \sum_h x_i^h(\mathbf{v}; \mathbf{e}^h) = \sum_i p_i \sum_f \left( \sum_h d^{hf} \right) y_i^f(\mathbf{v}; \mathbf{k}^f).$$

On the assumption that firm  $f$  distributes all its profits,

$$(8) \quad \sum_h d^{hf} = 1 \quad \text{for all } f,$$

so that (7) reduces to

$$(9) \quad \sum_i p_i [X_i(\mathbf{v}; \mathbf{E}) - Y_i(\mathbf{v}; \mathbf{K})] = 0 \quad \text{identically in all } \mathbf{v}, \mathbf{E}, \mathbf{K}, \text{ and } p_i,$$

where

$$(10) \quad X_i(\mathbf{v}; \mathbf{E}) = \sum_h x_i^h(\mathbf{v}; \mathbf{e}^h) \text{ and } Y_i(\mathbf{v}; \mathbf{K}) = \sum_f y_i^f(\mathbf{v}; \mathbf{k}^f)$$

represent the aggregate demand and supply functions, respectively, for good  $i$ ;  $\mathbf{E}$  is a vector containing all the  $\mathbf{e}^h$ ; and  $\mathbf{K}$  a vector containing all the  $\mathbf{k}^f$ . If  $X_i(\mathbf{v}; \mathbf{E}) - Y_i(\mathbf{v}; \mathbf{K}) > 0$ , an excess demand is said to exist in the market; if  $X_i(\mathbf{v}; \mathbf{E}) - Y_i(\mathbf{v}; \mathbf{K}) < 0$ , an excess supply; and if  $X_i(\mathbf{v}; \mathbf{E}) = Y_i(\mathbf{v}; \mathbf{K})$ , equilibrium.

Equation (9) is a general statement of Walras' Law. Its most frequent application in the literature has been (as in Walras' Elements itself) to the general-equilibrium analysis of a system of perfect competition, in which the behaviour functions of firms are derived from the assumption that

they maximize profits subject to their production function, and those of households are derived from the assumption that they maximize utility subject to their budget restraint. In this context, the vector  $\mathbf{v}$  is the price vector  $(p_1, \dots, p_{n-1})$ , with the  $n$ -th good being money and serving as numéraire (i.e.,  $p_n = 1$ ), so that there are only  $n-1$  prices to be determined. Ignoring for simplicity vectors  $\mathbf{E}$  and  $\mathbf{K}$ , which remain constant in the conceptual market-experiment, equation (9) then becomes

$$(11) \quad \sum_{i=1}^n p_i [X_i(p_1, \dots, p_{n-1}) - Y_i(p_1, \dots, p_{n-1})] = 0$$

identically in the  $p_i$ .

(Though it does not bear on the present subject, I should note that under the foregoing assumptions, and in the absence of money illusion, each of the demand and supply functions is homogeneous of degree zero in  $p_1, \dots, p_{n-1}$  and in whatever nominal financial assets are included in  $\mathbf{E}$  and  $\mathbf{K}$  (e.g., initial money holdings): see Patinkin, 1956, chap. II:4 and Mathematical Appendix 2:a). Thus Walras' Law states that no matter what the  $p_i$ , the aggregate value of excess demands in the system equals the aggregate value of excess supplies. This is the statement implicit in Lange's presentation (1942, p.50).

Walras himself, however, sufficed with a particular and narrower application of this statement, and was followed in this by, inter alia, Hicks (1939, chaps. IV:3 and XII:4-5), Modigliani (1944, pp.215-16) and Patinkin (1956, chap. III:1-3). Assume that it has been shown that a certain price vector  $(p_1^{\circ}, \dots, p_{n-1}^{\circ})$  equilibrates all markets but the  $r$ -th. Since (11) holds identically in the  $p_i$ , it must hold for this price vector too. Hence substituting the  $n-1$  equilibrium conditions into (11) reduces it to

$$(12) \quad p_j^{\circ} [X_j(p_1^{\circ}, \dots, p_{n-1}^{\circ}) - Y_j(p_1^{\circ}, \dots, p_{n-1}^{\circ})] = 0.$$

Thus if  $p_j^{\circ} > 0$ , the price vector  $(p_1^{\circ}, \dots, p_{n-1}^{\circ})$  must also equilibrate the  $j$ -th market, which means that only  $n-1$  of the equilibrium equations are independent. In this way Walras (and those who followed him) established the equality between the number of independent equations and the number of price-variables to be determined. (Though such an equality is not a sufficient condition for the existence of a unique solution with positive prices, it is a necessary - though not sufficient - condition for the peace-of-mind of those of us who do not aspire to the rigour of mathematical economists.)

It should however be noted that at the end of §126 of Walras' Elements (1926), there is a hint of Lange's broader statement of the Law: for there Walras states that if at a certain set of prices "the total demand for some commodities is greater (or smaller) than their offer, then the offer of some of the other commodities must be greater (or smaller) than the demand for them"; what is missing here is the quantitative statement that the respective aggregate values of these excesses must be equal.

Since the contrary impression might be gained from some of the earlier literature (cf., e.g., Modigliani, 1944, pp.215-16), it should be emphasized that no substantive difference can arise from the choice of the equation to be "dropped" or "eliminated" from a general-equilibrium system by virtue of Walras' Law. For identity (11) can be rewritten as

$$\begin{aligned}
 (13) \quad & X_j(p_1, \dots, p_{n-1}) - Y_j(p_1, \dots, p_{n-1}) \\
 = & -(1/p_j) \sum_{\substack{i=1 \\ i \neq j}}^n p_i [X_i(p_1, \dots, p_{n-1}) - Y_i(p_1, \dots, p_{n-1})]
 \end{aligned}$$

identically in the  $p_i$ .

Thus the properties of the "eliminated" equation are completely reflected in the remaining ones. Correspondingly, no matter what equation is "eliminated", the solution for the equilibrium set of prices obtained from the remaining equations must be the same. (From this it is also clear that the heated "loanable-funds versus liquidity-preference" debate that occupied the profession for many years after the appearance of the General Theory, was largely misguided; see Hicks, 1939, pp.157-62, and Patinkin, 1956, chap.XV:3, and 1958, pp.300-302, 316-17.)

In his influential article, Lange (1942, pp.52-53) also distinguished between Walras' Law and what he called Say's Law, and I digress briefly to discuss this. Let the first  $n-1$  goods represent commodities and the  $n$ -th good money. Then Say's Law according to Lange is

$$(14) \quad \sum_{i=1}^{n-1} p_i X_i(p_1, \dots, p_{n-1}) = \sum_{i=1}^{n-1} p_i Y_i(p_1, \dots, p_{n-1})$$

identically in the  $p_i$ .

That is, the aggregate value of commodities supplied at any price vector  $(p_1, \dots, p_{n-1})$  must equal the aggregate value demanded: supply always creates its own demand.

On both theoretical and doctrinal grounds, however, I must reject Lange's treatment of Say's Law. First of all, Lange himself demonstrates

(ibid.,p.62) that identity (14) implies that money prices are indeterminate. In particular, subtracting (14) from (11) yields

$$(15) \quad X_n(p_1, \dots, p_{n-1}) = Y_n(p_1, \dots, p_{n-1}) \quad \text{for all } p_i.$$

That is, no matter what the price vector, the excess-demand equation for money must be satisfied, which in turn implies that money prices cannot be determined by market forces. But it is not very meaningful to speak of a money economy whose money prices are indeterminate even for fixed initial conditions as represented by the vectors **E** and **K**. So if we rule out this possibility, we can say that Say's Law in Lange's sense implies the existence of a barter economy. Conversely, in a barter economy (i.e., one in which there exist only the  $n-1$  commodities) Say's Law is simply a statement of Walras' Law. Thus from the above viewpoint, a necessary and sufficient condition for the existence of Say's Law in Lange's sense is that the economy in question be a barter economy: it has no place in a money economy (Patinkin, 1956, chap.VIII:7).

Insofar as the doctrinal aspect is concerned, identity (14) cannot be accepted as a representation of Say's actual contention. For Say's concern was (in today's terminology) not the short-run viewpoint implicit in this identity, but the viewpoint which denied that in the long run inadequacy of demand would set a limit to the expansion of output. In brief, and again in today's terminology, Say's concern was to deny the possibility of secular stagnation, not that of cyclical depression and unemployment. Thus, writing in the first quarter of the nineteenth century, Say (1821, p.137) adduces evidence in support of his thesis from the fact "that there should now be bought and sold in France five or six times as many commodities, as in the miserable reign of Charles VI" - four centuries earlier. Again, in his

Letters to Malthus (1821, pp.4-5) Say argues that the enactment of the Elizabethan Poor Laws (codified at the end of the sixteenth century) proves that "there was no employ in a country which since then has been able to furnish enough for a double and triple number of labourers" (italics in original). Similarly, Ricardo, the leading contemporary advocate of Say's loi des débouchés, discusses this law in chapter 21 of his Principles (1821), entitled "Effects of Accumulation on Profits and Interest"; on the other hand, he clearly recognizes the short-run "distress" that can be generated by "Sudden Changes in the Channels of Trade" (title of chap. 19 of his Principles. For further discussion see Patinkin, 1956, Supplementary Note L.)

Let me return now to the general statement of Walras' Law presented in equation (9) above. This statement holds for any vector  $v$  and not only for that appropriate to perfect competition. In particular, Walras' Law holds also for the case in which households and/or firms are subject to quantity constraints. In order to bring this out, consider the macroeconomic model of a disequilibrium economy presented in chapter XIII:2 of Patinkin (1956) and illustrated by Figure 1. In this figure,  $w$  is the money wage-rate,  $p$  the price level,  $N$  the quantity of labour,  $N^d = Q(w/p, K_0)$  the firms' demand curve for labour as derived from profit-maximization as of a given stock of physical capital  $K_0$ ; and  $N^s = R(w/p)$  is the supply curve of labour as derived from utility maximization subject to the budget constraint (these perfect-competition curves are what Clower (1965, p.119) subsequently denoted as "notional curves"). Assume that because of the firms' awareness that at the real wage rate  $(w/p)_1$  they face a quantity constraint and will not be able to sell all of the output corresponding to their profit-maximizing input of labour  $N_1$ , they demand only  $N_2$  units of labour, represented by point P in Figure 1. This

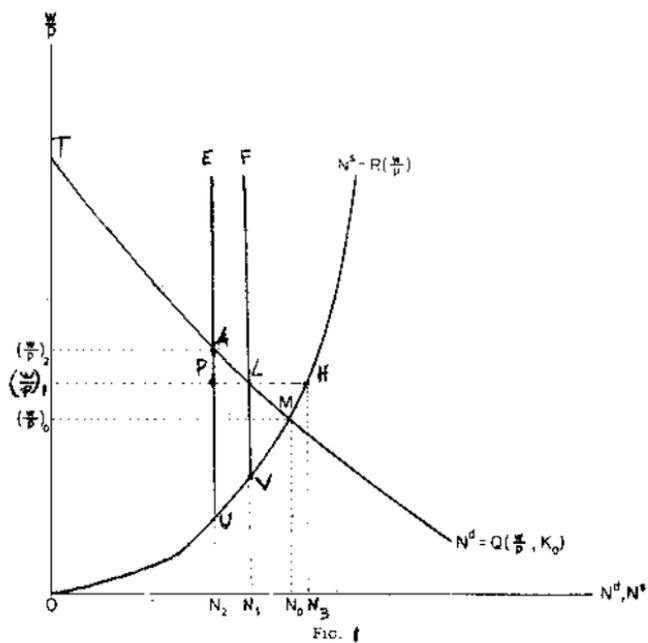


FIG. 1

constraint also operates on workers, who can sell only the foregoing quantity of labour instead of their optimal one  $N_3$ , represented by point H. In brief, at point P, both firms and workers are off their notional curves. In order to depict this situation, the notional curves must accordingly be replaced by quantity-constrained ones: namely, the kinked demand curve  $TAN_2$  and kinked supply curve OUE. Note that for levels of employment before they become kinked, the curves coincide respectively with the notional ones (but see Patinkin, chap.XIII:2, fn.9, for a basic analytical problem that arises with respect to the kinked demand curve  $TAN_2$ ).

The obverse side of these constraints in the labour market are corresponding constraints in the commodity market. In particular, as Clower (1965, pp. 118-21) has emphasized, the demands of workers in this market are determined by their constrained incomes. Clower also emphasizes that it is this quantity constraint which rationalizes the consumption function of Keynes' General Theory, in which income appears as an independent variable. For in the absence of such a constraint, the individual's income is also a dependent variable, determined by the optimum quantity of labour he decides to sell at the given real wage rate in accordance with the labour-supply function  $N^S = R(w/p)$  in Figure 1; and he makes this decision simultaneously with that with respect to the optimum quantities of commodities to buy. If, however, his income is determined by a quantity constraint which prevents him from selling his optimum quantity of labour, the individual can decide on his demands for commodities only after his income is first determined. This is the so-called "dual decision hypothesis" (Clower, *ibid.*). To this I would add (and its significance will become clear below) that the quantity restraint also rationalizes the form of Keynes' liquidity preference function, for this too depends on income (General Theory, p.199). Furthermore, if the behaviour functions in the markets for labour,

commodities, and money balances are thus quantity-restrained, so too (by the budget restraint) will be that for bonds - the fourth market implicitly (and frequently explicitly) present in the Keynesian system. (The theory of the determination of equilibrium under quantity restraints - in brief, disequilibrium theory - has been the subject of a growing literature, most of it highly technical; for critical surveys of this literature, see Grandmont (1977), Drazen (1980), Fitoussi (1983), and Gale (1983, chap.1).)

In the General Theory (chap.2), Keynes accepted the "first classical postulate" that the real wage is equal to the marginal product of labour, but rejected the second one, that it always also measures the marginal disutility of labour. In terms of Figure 1 this means that while firms are always on their demand curve  $N^d = Q(w/p, K_0)$ , workers are not always on their supply curve  $N^s = R(w/p)$ . Thus, for example, at the level of employment  $N_2$ , the labour market will be at point A on the labour-demand curve, corresponding to the real wage rate  $(w/p)_2$ ; but the marginal utility of the quantity of commodities that workers then buy with their real-income  $(w/p)_2 N_2$  is greater than the marginal disutility of that level of employment. And Keynes emphasizes that only in a situation of full-employment equilibrium - represented by intersection point M in Figure 1 - will both classical postulates be satisfied.

Consider now the commodity market as depicted in the usual Keynesian-cross diagram (Figure 2). The 45° line represents the amounts of commodities which firms produce and supply as they move along their labour-demand curve from point T to M. Thus  $Y_0$  represents the output (in real terms) of  $N_0$  units of labour. Note too that the negative slope of the labour-demand curve implies that the real wage declines as we move rightwards along the 45° line.

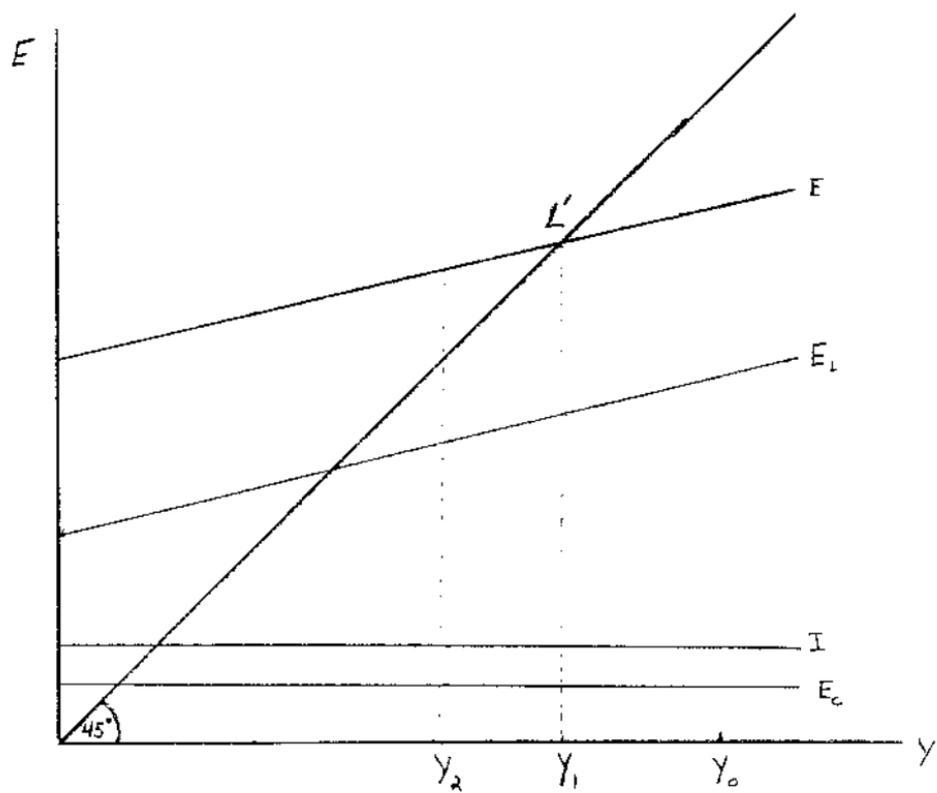


Fig. 2

Curve E represents the aggregate demand curve, which is the vertical sum of the consumption function of workers ( $E_L$ ) and capitalists ( $E_C$ ), respectively, and of the investment function (I). For simplicity, these last two are assumed to be constant. The fact that curve E does not coincide with the 45° line reflects Keynes' assumption that in a monetary economy, Say's Law (in his sense, which is the macroeconomic counterpart of Lange's subsequent formulation) does not hold (ibid., pp.25-26).

Consider now the consumption function of workers. The income which they have at their disposal is their constrained income as determined by the labour-demand curve in Figure 1. Thus assume that  $Y_1$  and  $Y_2$  in Figure 2 are the outputs corresponding to the levels of employment  $N_1$  and  $N_2$ , respectively. The corresponding incomes of workers at these levels are  $(w/p)_1 \cdot N_1$  and  $(w/p)_2 \cdot N_2$ . On the assumption that the elasticity of demand for labour is greater than unity, the higher the level of employment the greater the income of workers and hence their consumption expenditures. From Figure 2 we see that at income  $Y_2$  there is an excess demand for commodities. This causes firms to expand their output to, say,  $Y_1$ , and hence their labour-input to, say,  $N_1$ , thus causing the constrained labour-supply curve to shift to the right to the kinked curve OUVLF. By construction,  $Y_1$  is the equilibrium level of output.

What must now be emphasized is that Walras' Law holds in this situation too - provided we relate this Law to excess-demand functions of the same type. Thus if within our four-good Keynesian model we consistently consider notional behaviour functions, the excess supply of labour LH in Figure 1 corresponds to an excess demand for commodities which is generated by workers' planned consumption expenditures at the real wage rate  $(w/p)_1$  and level of employment  $N_3$  as compared with firms' planned output at that

wage rate and lower level of employment  $N_2$ ; and there will generally also exist a net excess planned demand for bonds and money. Alternatively, if we consistently consider constrained functions, then constrained equilibria exist in both the labour market (point L), the commodity market (point L'), the bond market, and the money market. Similarly, the broader form of Walras' Law states that a constrained (say) excess supply in the commodity market corresponds to a constrained net excess demand in the bond and money markets, while the labour market is in constrained equilibrium. In brief, a sufficient condition for the validity of Walras' Law is that the individual's demand and supply functions on which it is ultimately based are all derived from the same budget constraint, whether quantity-constrained or not. (This is the implicit assumption of Patinkin's (1956, p.229; 1958, pp.314-16) application of Walras' Law to a disequilibrium economy with unemployment, and the same is true for Grossman (1971, pp. 956-60) and Barro and Grossman (1971 and 1976, p.58).)

I must admit that the validity of Walras' Law in this Keynesian model depends on our regarding the kinked curve OUVLF as a labour-supply curve, and that this is not completely consistent with the usual meaning of a supply curve or function. For such a function usually describes the behaviour of an agent under constraints which leave him some degree of freedom to choose an optimum, whereas no such freedom exists in the vertical part of OUVLF. However, I prefer this inconsistency to what I would consider to be the logical - and hence more serious - inconsistency that lies at the base of the rejection of Walras' Law, and which consists of lumping together behaviour functions derived from different budget constraints.

It is thus clear that the foregoing constrained equilibrium in the labour market is not an equilibrium in the literal sense of representing a balance of opposing market forces, but simply the reflection of the passive adjustment by workers of the amount of labour they supply to the amount demanded by firms (cf. Patinkin, 1958, pp.314-15). From this viewpoint, the constrained equilibrium in the labour market always exists and simply expresses the fact that, by definition, every ex post purchase is also an ex post sale. In contrast, as we have seen in the discussion of Figure 2 above, the corresponding constrained equilibrium in the commodity market is a true one: for, in accordance with the usual Keynesian analysis, were the level of  $Y$  to deviate from  $Y_1$ , automatic market forces of excess demand or supply would be generated that would return it to  $Y_1$ . And a similar statement holds, mutatis mutandis, for the constrained equilibria in the bond and money markets.

Note, however, that in the commodity market too there is an ex post element. This element is a basic, if inadequately recognised, aspect of the household behaviour implied by Clower's "dual decision hypothesis": namely, that households' constrained decisions on the amount of money to spend on commodities is based on their ex post knowledge of the amount of money received from the constrained sale of their factor services. And to this I again add that a similar statement holds for their constrained decisions with reference to the amounts of bonds and money balances, respectively, that they will want to hold. (Note that an analysis in terms of constrained decisions can also be applied to the case in which households do not correctly estimate firms' profits in equation (3) above, and are consequently forced to base their effective (say) consumption decisions on the ex post knowledge of these profits.)

In his treatment of an economy with constrained functions, Clower (1965, pp.122-23) has claimed that under these conditions Walras' Law does not hold. This is not true for the Law as hitherto discussed. What Clower seems to have in mind, however, is that though the excess supply of labour LH in Figure 1 is notional, it nevertheless exerts pressure on workers to reduce their money wages; in contrast, the notional excess demand for commodities corresponding to LH (see above) cannot - because of their constrained incomes - lead households to exert expansionary pressures on the commodity market. Thus there exists no effective excess demand for commodities to match the effective excess supply of labour. Accordingly, no "signal" to the market is generated that will lead to the expansion of output and consequent reduction of unemployment (cf. also Leijonhufvud, 1968, pp.81-91). And it is the absence of such a "signal" that Leijonhufvud (1981, chap.6) subsequently denoted as "effective demand failure."

This "failure", however - and correspondingly the failure of Walras' Law to hold in Clower's sense - is not an absolute one: for though there is no direct signal to the commodity market, an indirect one may well be generated. In particular, the very fact that the constrained equilibrium in the labour market does not represent a balancing of market forces means that the unemployed workers in this market are a potential source of a downward pressure on the money wage rate. And if this pressure is to some extent effective, the resulting decline in money wages will generate an increase in the real quantity of money, hence a decrease in the rate of interest, hence an increase in investment and consequently in aggregate demand - and this process may be reinforced by a positive real-balance effect (see chapter 19 of the General Theory, which, however, also emphasizes how many weak - and possibly perverse - links there are in this causal chain). Thus a sufficient condition for absolute "effective demand

failure" is the traditional classical one of absolute rigidity of money wages and prices.

An analogy (though from a completely different field) may be of help in clarifying the nature of the foregoing equilibrium in the labour market. Consider a cartel of (say) oil-producing firms, operating by means of a Central Executive for the Production of Oil (CEPO) which sets production quotas for each firm. The total quantity-restrained supply so determined, in conjunction with the demand conditions in the market, will then determine the equilibrium price for crude oil; and the equilibrium position so determined will be the one relevant for Walras' Law. But this will not be an equilibrium in the full sense of the term, for it co-exists with market pressures to disturb it. In particular, the monopolistic price resulting from CEPO's policy is necessarily higher than the marginal cost of any individual member of the cartel. Hence it is to the interest of every firm in the cartel that all other firms adhere to their respective quotas and thus "hold an umbrella" over the price, while it itself surreptitiously exceeds the quantity restraint imposed by its quota and thus moves closer to its notional supply curve as represented by its marginal-cost curve. And since in the course of time there will be some firms who will succumb to this temptation, a temptation that increases inversely with the ratio of its quota to the total set by CEPO, actual industry output will exceed this total, with a consequent decline in the price of oil. Indeed, if such violations of cartel discipline should become widespread, its very existence would be threatened (see Patinkin, 1947, pp. 110-11).

This analogy is, of course, not perfect. First of all, unlike workers in the labour market, the member-firms of CEPO have themselves had

a voice in determining the quantity restraints. Second, and more important, any individual firm knows that by "chiseling" and offering to sell even slightly below the cartel price, it can readily increase its sales. But analogies are never perfect: that is why they remain only analogies.

A final observation: the discussion until now has implicitly dealt with models with discrete time periods. In models with continuous time, there are two Walras' Laws: one for stocks and one for flows: one for the instantaneous planned (or constrained) purchases and sales of assets (primarily financial assets) and one for the planned (or constrained) purchases and sales of commodity flows (cf. May, 1970; Foley and Sidrauski, 1971, pp.89-91; Sargent, 1979, pp.67-69; Buiter, 1980). On the other hand, in a discrete-time intertemporal model, in which there exists a market for each period, there is only one Walras' Law: for in such a model, all variables have the time-dimension of a stock (see Patinkin, 1972, chap. 1).

## BIBLIOGRAPHY

Reprinted or translated works are cited in the text by year of original publication; the page references to such works in the text are, however, to the pages of the reprint or translation in question.

Arrow, K.J. and Hahn, F.H. (1971). General Competitive Analysis. San Francisco: Holden-Day and Edinburgh: Oliver and Boyd.

Barro, R. and Grossman, H. (1971). "A General Disequilibrium Model of Income and Employment." American Economic Review 61 (March) 82-93.

Barro, R. and Grossman, H. (1976). Money, Employment and Inflation. Cambridge: Cambridge University Press.

Buiter, W.H. (1980). "Walras' Law and All That: Budget Constraints and Balance Sheet Constraints in Period Models and Continuous Time Models." International Economic Review 21 (Feb.): 1-16.

Clower, Robert (1965). "The Keynesian Counterrevolution: A Theoretical Appraisal." In The Theory of Interest Rates, edited by F.H. Hahn and F.P.R. Brechling (London: Macmillan), pp.103-25.

Drazen, A. (1980). "Recent Developments in Macroeconomic Disequilibrium Theory." Econometrica 48 (March): 283-306.

Fitoussi, J.P. (1983). "Modern Macroeconomic Theory: An Overview." In J.P. Fitoussi, ed. Modern Macroeconomic Theory (Oxford: Basil Blackwell), pp. 1-46.

Gale, D. Money: in Disequilibrium. Cambridge: Cambridge University Press.

Grandmont, J.M. (1977). "Temporary General Equilibrium Theory." Econometrica 45 (April): 535-72.

- Grossman, H.I. (1971). "Money, Interest, and Prices in Market Disequilibrium." Journal of Political Economy 79 (Sept/Oct): 943-61.
- Hicks, J.R. (1939). Value and Capital. Oxford: Clarendon Press.
- Keynes, John Maynard (1936). The General Theory of Employment Interest and Money. London: Macmillan.
- Lange, Oscar (1942). "Say's Law: A Restatement and Criticism." In Studies in Mathematical Economics and Econometrics, edited by Oscar Lange et al. (Chicago: University of Chicago Press), pp.49-68.
- Leijonhufvud, Axel (1968). On Keynesian Economics and the Economics of Keynes. New York: Oxford University Press.
- Leijonhufvud, Axel (1981). Information and Coordination: Essays in Macroeconomic Theory. Oxford: Oxford University Press.
- May, J. (1970). "Period Analysis and Continuous Analysis on Patinkin's Macroeconomic Model." Journal of Economic Theory 2: 1-9.
- Modigliani, Franco (1944). "Liquidity Preference and the Theory of Interest and Money." Econometrica 12 (Jan.): 45-88. As reprinted in Readings in Monetary Theory, selected by a committee of the American Economic Association (Philadelphia: Blakiston, for the American Economic Association, 1951), pp. 186-240.
- Patinkin, Don (1947). "Multiple-Plant Firms, Cartels, and Imperfect Competition." Quarterly Journal of Economics 61 (Feb.): 173-205. As reprinted in Patinkin, Essays On and In the Chicago Tradition (Durham, N.C.: Duke University Press, 1981), pp. 91-114.
- Patinkin, Don (1956). Money, Interest, and Prices. Evanston, Ill.: Row, Peterson. (The material referred to here appears unchanged in the second, 1965 edition.)
- Patinkin, Don (1958). "Liquidity Preference and Loanable Funds: Stock and Flow Analysis." Economica 25 (Nov.): 300-318.

- Patinkin, Don (1972). Studies in Monetary Economics. New York: Harper and Row.
- Ricardo, D. (1821). On the Principles of Political Economy and Taxation, 3rd ed. As reprinted in The Works and Correspondence of David Ricardo, vol.I, ed. P. Sraffa. (Cambridge: Cambridge Univ. Press, 1951).
- Sargent, T.J. (1979). Macroeconomic Theory. New York: Academic Press.
- Say, J.B. (1821). Traité d'économie politique, 4th edition. Paris: Deterville. As translated by C.R. Prinsep under the title A Treatise on Political Economy (Philadelphia, Grigg R. Elliot: 1834).
- Say, J.B. (1821). Letters to Thomas Robert Malthus on Political Economy and Stagnation of Commerce London. As reprinted with an Historical Preface by H.J. Laski (London: George Harding's Bookshop, 1936; Wheeler Economic and Historical Reprints No.2).
- Walras, Léon (1874). Eléments d'économie politique pure. Paris: Guillaumin & Cie. (sections I-III of the work).
- Walras, Léon (1900). Eléments d'économie politique pure. 4th edition. Paris: F. Pichon.
- Walras, Léon (1926). Eléments d'économie politique pure. Edition définitive. Paris: F. Pichon (for our purposes, identical with 4th edition). As translated by William Jaffé under the title Elements of Pure Economics (London: George Allen and Unwin, 1954).