Dual Exchange Markets and Intervention

Pertti Haaparanta

WP 6 August 1986

World Institute for Development Economics Research of the United Nations University
DUAL EXCHANGE MARKETS AND INTERVENTION

by

Pertti Haaparanta

World Institute for Development Economics Research (WIDER)
Annankatu 42 C
SF-00100 Helsinki
Finland

August 1986
Abstract

It is argued that the theoretical literature on dual exchange markets has completely neglected the form of central bank intervention emphasized by the "classics". They advocated neutral intervention where the central bank sells in the capital market all foreign exchange it acquires from the current transactions. Current literature concentrates on the non-sterilized intervention. In a choice-theoretic framework it is shown that the form of intervention matters very much for the transmission of changes in foreign rate of interest and in terms of trade. On normative side it is shown that one can always design the dual exchange system in such a way that it is superior to the uniform fixed rate system.
I. Introduction

J. Marcus Fleming emphasized that the dual exchange market does not work properly unless the following holds: "It is sometimes thought to be of the essence of the dual exchange market that the rate for capital transactions is allowed to float freely without official intervention. This is a misunderstanding of the possibilities of the system. There is no reasons why the authorities should not buy or sell foreign currency for domestic currency on the capital exchange market. Indeed, if they wish that market to make its maximum contributions to the equilibrium of the balance of payments as a whole they must (emphasis of JMF), selling in the capital transactions market the foreign exchange they are acquiring in the current transactions market and buying in the former the foreign exchange they are selling in the latter (emphasis mine)." (Fleming, 1974). Lanyi (1975) named this strategy as neutral intervention. But all of the the theoretical literature I am aware of has neglected this point raised by Fleming, See Flood (1978), Flood and Marion (1982), Marion (1981), Adams and Greenwood (1985), Aizenman (1985), Obstfeld (1986). Indeed, they have taken the lead given by Flood (1978): "As other writers have pointed out, the type of dual exchange market system outlined above chokes off all net capital movements into or out of a country adopting the regime. ... If a domestic resident wants to purchase internationally traded securities he must either buy them from another domestic resident (no net capital flow) or purchase them from a foreign resident. But to purchase from a foreign resident the domestic resident must first obtain foreign exchange which is eligible for use in the financial market, and this foreign exchange can only arise from the sale by
another domestic resident of traded securities to a foreigner. ... The result is no net capital movements once dual markets have been implemented."

Hence, in the Flood framework the central bank does not sell any foreign exchange in the capital market. This strategy can be called non-sterilized intervention since a change in central bank foreign exchange holding is automatically reflected as a change in money supply.

I indicated above that all of the theoretical discussion has assumed that the central bank in a dual exchange regime adopts the policy of non-sterilized intervention. In this paper I shall analyze the dual exchange system with neutral intervention and contrast it with the dual exchange system and non-sterilized intervention. To get out the basics I shall employ a choice theoretic framework quite similar to the one used by Adams and Greenwood (1985). These authors come closest to recognizing the point raised by Fleming since they do not assume that net private capital flows are nil. Instead they assume that the net private capital flows are equal to some exogenous amount dictated by policy makers. The rule advocated by Fleming requires that net private capital flows are endogenous. One can as implying that, that if non-sterilized intervention is used, then the authorities regard the amount of private borrowing without controls as excessive, and want to restrict it. With neutral intervention the authorities want to improve the net foreign asset position of the central bank.

II. The Model

Consider an economy which is going to live two periods, period 1 and period 2. The economy produces an exportable good in every period. This good is not consumed in the home country, however; only a single importable good
is consumed. This is the most simple assumption to allow a meaningful incorporation of terms-of-trade changes. The utility function of the representative consumer is \( u(c_1, c_2) \), where \( c_1 \) = consumption of the importable in period 1, \( c_2 \) = consumption of the importable in period 2. Utility, maximization is subject to the budget constraint.

\[
(1) \quad p_1 c_1 + \left[ \frac{p_2}{(1+i)} \right] c_2 = I
\]

where \( p_j \) = price of the importable in period \( j \), \( j = 1,2 \), \( i \) = domestic nominal interest rate, and \( I \) = the present value of consumer's income. The optimum choices are \( c_j = c_j(p_1, \frac{p_2}{(1+i)}, I), j = 1,2 \). They give the indirect utility \( u = u(p_1, \frac{p_2}{(1+i)}, I) \). Income \( I \) is given by

\[
(2) \quad I = \left[ (1-v(m_1/y_1))y_1 + t_1 - m_1 \right] + \\
+ \left[ (1-v(m_2/y_2))y_2 + t_2 - m_2 \right] + \\
+ \left[ \frac{p_1}{(1+i)} \right] m_1
\]

Here \( y_j \) = value of domestic export production in terms of the importable, i.e., \( y_j = \beta_j q_j \), where \( \beta_j \) = terms of trade in period \( j \) and \( q_j \) = production of the exportable in period \( j \), \( j = 1,2 \). I assume that \( q_j \) is exogenous, \( j = 1,2 \). Economic transactions create costs in terms of domestic production. These costs are (in each period) proportional to the production, \( v(*) p_j y_j \). These costs can be reduced by increasing the holdings of domestic currency; \( m_j \) = value of cash balances (in terms of the importable). Alternatively, one could assume that real money balances enter the utility function, e.g., because transactions create costs is terms of utility, and these costs vary with real money balances. This specification is used, e.g., in Obstfeld (1986). I do not see any reason to prefer either alternative, the choice
must be based on the convenience of the specification. Of the transactions costs I assume that \( v' < 0 \) \( v'' > 0 \). Finally, \( t_j = \) net transfers from the public sector, \( j = 1, 2 \).

Welfare optimum requires obviously that the consumer holds money balances to maximize the value of income. Hence

\[
(3) \quad -v'(m_1/y_1) = i/(1 + i)
\]

\[
(4) \quad -v'(m_1/y_2) = 1
\]
or

\[
(5) \quad m_1 = k_1 (i/(1 + i)) y_1, \quad k_1' \leq 0
\]

\[
(6) \quad m_2 = k_2 y_2, \quad k_2 = \text{constant}
\]

where \( k_j \) is the Cambridge \( k \) for period \( j, j = 1, 2 \). Demand for money in period 1 declines when the domestic interest rate increases.

In the dual exchange system different exchange rates apply to current and capital transactions. Both of these rates can be variable (see Flood and Marios (1982)), but I assume that the commercial rate, \( e \), is pegged in all periods, \( e_1 = e = e_2 \). The financial exchange rate \( f \), on the other hand, is flexible. Consider now the return on investing in foreign assets. The foreign nominal interest rate is \( i^* \). One unit of domestic currency buys \( 1/f_1 \) units of foreign currency which can be used to buy foreign assets. In period 2 the investor has \((1 + i^*)/f_1 \) units of foreign assets, which must be repatriated at the prevailing financial exchange rate \( f_2 \) to give \((1 + i^*)f_2/f_1 \) units of domestic currency. Hence, if domestic and foreign interest bearing assets are perfect substitutes, the domestic interest rate must satisfy the relation
where \( h = (f_2 - f_1)/f_1 = \) rate of depreciation of the financial exchange rate. (Note my assumption that interest revenue \( i*/f_1 \) is repatriated also at the financial exchange rate, even though it is a current account item. This assumption is made for simplicity only (following Adams and Greenwood (1985). For the completely "correct" treatment see e.g., Flood and Mario (1982).)

I assume that the world market prices of all goods are parametric. Hence \( p_j = e p_j^* \), \( p_{xj} = e p_{xj}^* \), and \( b_j = p_{xj}^*/p_j^* \), \( j = 1,2 \). Here \( p_{xj} \) = price of the exportable in period \( j \).

To close the system we must still specify the money supply and transfer policies. Money supply in period 1 is

\[
(8) \quad m_1 = \mu_1 + b
\]

where \( \mu_1 \) = domestic credit and \( b = \) central bank acquisition of foreign assets (i.e., accumulation of reserves). In the second period the money supply is

\[
p_2 m_2 = p_1 m_1 + \mu_2 - p_1 (1 + i*)b
\]

or

\[
(9) \quad \mu_2 = m_2 - (p_1/p_2)m_1 + (p_1/p_2)(1 + i*)b.
\]

In addition the central bank runs the dual exchange market. This creates revenues or losses which have to be transferred to the consumer. In the first period the consumer wants to acquire
(1 - v(k_1))y_1 + t_1 - c_1 - m_1

foreign bonds. Her/his net revenue from exports in domestic currency is 
\[(1 - v(k_1))y_1 - c_1\]p_1 which is augmented by the transfer p_1 t_1. The surplus
is then allocated to money and foreign assets. Hence she/he has at her/his
disposal \[(1 - v(k_1))y_1 + t_1 - c_1 - m_1\]p_1 = sp_1 units of domestic currency to
purchase foreign assets. So she can purchase sp_1/f_1 units of foreign
currency. Hence, the profits of the central bank form this operation are
\((f_1 - e)sp_1/f_1\), since it has acquired the foreign exchange at the rate e and
sells it at the rate f_1. So

(10) \quad n_1 = (f_1 - e)s/f_1 = period 1 real profit.

The total value of private investment in the second period is \((1 + i^*)p_1s/f_1\). They sell it to the central bank at the rate f_2 but have to buy
it back at the rate e to be able to buy the importable. Hence central bank
profit from this operation is \((e - f_2)(1 + i^*)sp_1/f_1\) or

(11) \quad n_2 = (e - f_2)(1 + h)sp_1/p_2 = period 2 real profit.

n_1 and n_2 are transferred to the private sector. Total transfers are
then t_1 = \mu_1 + n_1, t_2 = \mu_2 + n_2.

Finally by way of definition, it is clear from above that the capital
account (inflow of capital), ca_1 is

(12) \quad ca = -es/f_1
In equilibrium money demands are equal to money supplies. Hence

\[ k_1 \frac{(i/(l + i))}{y_1} = \mu_1 + b \]

\[ \mu_2 = k_2 y_2 - (p_1/p_2)(\mu_1 + b) + (p_1/p_2)(l + i^*) b \]

Using the expressions for the transfers \( t_1 \) and \( t_2 \) and central bank budget constraints (8) and (9) it is straightforward to show that private income \( I \) is

\[ I = p_1[(1 - v(k_1))y_1 - b + ca] + \]
\[ + (p_2/(1 + i))[(1 - v(k_2))y_2 + (p_1/p_2)(l + i^*)(b - ca)] \]

The overall balance of payments must be in equilibrium

\[ b = (1 - v(k_1))y_1 - c_1(p_1, p_2/(1 + i), I) + ca \]

Finally, the welfare of the consumer is

\[ u = u(p_1, p_2/(1 + i), I) \]

With non-sterilized intervention the endogenous variables in this economy are \( i, b, \mu_2, I \) and \( u \). Among the exogenous variables is the capital account \( ca \); in fact in most models \( ca = 0 \) (see the introduction). Adams and Greenwood allow \( ca = 0 \) even though they treat it as an exogenous variable. With neutral intervention advocated by Fleming (see the introductory
section) the endogenous variables are $i$, $ca$, $\mu_2$, $I$ and $u$. Among the exogenous variables is $b$. In the following I shall assume that $b = 0$. We can now go on to analyse the differences created by the intervention policies.

III. Dual Exchange Markets with Neutral Intervention

The equilibrium conditions are (13)-(17). Among them equations (13), (15) and (16) form an independent system which determines the values of $i$, $ca$ and $I$, and after substitution of (15) in (16), (13) and (16) form a system which can be solved for $i$ and $ca$. It is also seen that equation (13) alone determines the domestic interest rate (and hence the rate of depreciation of the financial exchange rate $h$). Equation (16) then determines the amount of private borrowing $ca$.

Consider first the domestic interest rate which is determined by equation (13) alone. It gives

\begin{equation}
(18) \quad i = i(\beta_1 q_1, \mu_1, i^*), \quad i_{13} > 0, i_2 < 0
\end{equation}

or when translated

\begin{equation}
(18a) \quad h = h(\beta_1 q_1, \mu_1, i^*), \quad h_{13} > 0, h_2 < 0
\end{equation}

An improvement of current terms of trade or an increase in production increases the current demand for money. With money supply constant the domestic rate of interest must increase to keep the demand for money unchanged. This means that the depreciation of the financial exchange rate over time increases i.e., the current rate appreciates relative to the future rate. In a similar fashion, an increase in current supply of money
makes the current financial exchange rate depreciate relative to the future rate. An increase in foreign rate of interest does not have any effect on the domestic interest rate: the dual system with neutral intervention "insulates" the domestic money market from foreign monetary disturbances. Yet, it has an effect on the financial exchange rate: current rate depreciates relative to the future rate. Finally, for discussion in the next section it is worthwhile to notice that changes in future terms of trade do not have any impact on the domestic interest rate and on the financial exchange rate:

$$\frac{\partial^{1}l}{\partial \beta_{2}} = \frac{\partial h}{\partial \beta_{2}} = 0$$

Consider next the effects on the capital account:

$$ca = c_{1}(p_{1}, p_{2}/(1 + i), I) - [1 - v(k_{1})]b_{1}$$

where I is given by equation (15)

Hence:

$$[1 - c_{1I}(\partial I/\partial ca)](\partial ca/\partial z) =$$

$$-c_{12}(p_{2}/(1 + i)^{2})(i/z) + c_{1I}(I/i)(i/z) +$$

$$c_{1I}(\partial I/\partial z) - \partial[(1 - v(k_{1}))y_{1}]/\partial z + (1/(1 + i)^{2})v'k_{1}^{*}(a/\partial z)$$

where

$$z = \beta_{1}q_{1}, \mu_{1}, \mu^{*}, \beta_{2}$$

and

$$\partial I/\partial a = p_{1}[1 - ((1 + i^{*})/(1 + i))]$$

$$\partial I/\partial i = (1/(1 + i)^{2})(-p_{1}v'y_{1} - p_{2}c_{2})$$

Clearly $$\partial I/\partial i < 0$$. If $$i < i^{*}$$ then $$\partial I/\partial ca < 0$$, and $$\partial I/\partial ca > 0$$ if $$i = i^{*}$$. If the purpose of imposing capital controls is to allow the central bank to undertake expansionary monetary policies (relatively to policies within a
uniform fixed rate system) then $i^* i^*$ is the relevant case and the coefficient of $dca$ in (19) is larger than unity. Since $c_{11} p_1 < 1$ the coefficient is always positive. $c_{12} > 0$ because the consumptions in each period are substitutes and normal.

Let first $z = \beta_1 q_1$. Then $\frac{\partial z}{z} = p_1 (1 - v(k_1))$. Since $\frac{\partial z}{z} > 0$ the sum of the first 4 terms in the RHS of (19) is negative. The last term is positive: the increase in the rate of interest leads to economizing in money holdings which reduces the period 1 income. Hence, most likely the response is normal, $\frac{\partial c_{a}}{\partial i^*} < 0$, i.e., private borrowing declines. Consider next monetary expansion, $z = \mu_1$. Then $\frac{\partial z}{\partial \mu_1} = 0$, $\frac{\partial z}{\partial \mu_1} < 0$. Again the response is in principle ambiguous, though most likely the response is normal, i.e., $\frac{\partial c_{a}}{\partial \mu_1} > 0$. The source of the ambiguity is the same as just above: the effect on income through the transactions cost works in other direction than the other effects, i.e., the substitution effects and income effects created by the decline in the domestic rate of interest.

Since $\frac{\partial z}{\partial i^*} = 0$, the sign of $\frac{\partial c_{a}}{\partial i^*}$ is the same as the sign of $\frac{\partial z}{\partial i^*}$. But $\frac{\partial z}{\partial i^*} = \frac{\partial z}{\partial i^*} = -p_1 c_{a}/(1 + i)$.

If the private sector is a net debtor vis a vis the rest of the world then $\frac{\partial z}{\partial i^*} < 0$ and $\frac{\partial c_{a}}{\partial i^*} < 0$. The reverse occurs if the private sector is a net creditor.

Finally, consider the effects of changes in future terms of trade. Since $\frac{\partial z}{\partial \theta_2} = 0$ the sign of $\frac{\partial c_{a}}{\partial \theta_2}$ is the sign of $\frac{\partial z}{\partial \theta_2}$, and $\frac{\partial z}{\partial \theta_2} > 0$. Hence private sector borrowing increases. All in all the results were

$$ca = c_{a} (\beta_1 q_1, \mu_1, i^*, \beta_2)$$

with

$ca = ? (\leq 0$ most likely), $c_{a2} = ? (> 0$ most likely)

sign of $ca_3 < 0$ sign of $-ca$, $c_{a4} > 0$. 


To complete the discussion the welfare effects of the disturbances must be evaluated. From (17):

\[
(20) \quad (\frac{\partial u}{\partial z}) = -u_2(p_2/(1+i)^2)(\partial i/\partial z) + \\
+ u_1[\partial I/\partial z + (\partial I/\partial i)(\partial i/\partial z) + \\
+ (\partial I/\partial ca)(\partial ca/\partial z)]
\]

\[ u_1 > 0 \quad \text{and} \quad -u_2/u_1 = c_2 \quad \text{by Roy's identity.} \]

In what follows I assume that i ≤ i* to avoid too much of a taxonomic discussion. Then \( \partial I/\partial ca \) 0. Consider again first the case \( z = \beta_1x_1 \). Then from (20)

\[
(\frac{\partial u}{\partial \beta_1 q_1}) = \\
\quad u_1 \left[ p_1(1 - v(k_1)) - p_1v'k_1'y_1 (\partial I/\partial \beta_1 q_1)/(1+i)^2 + \\
+ p_1[1 - ((1+i^*)/(1+i))(\partial ca/\partial \beta_1)]\right]
\]

If the response of the capital account is normal, \( \partial ca/\partial \beta_1 q_1 < 0 \), then the welfare increases when the first period income increases, since then the second term within \( \{ \} \) is "small".

Turn now to the case \( z = \mu_1 \). Then

\[
(\frac{\partial u}{\partial \mu_1}) = \\
\quad u_1 \left[ - p_1v'k_1'y_1 (\partial I/\partial \mu_1)/(1+i)^2 + \\
+ p_1[1 - (1+i^*)/(1+i)](\partial ca/\partial \mu_1)\right]
\]

If \( \partial ca/\partial \mu_1 > 0 \) (the normal response) the sign of this expression is ambiguous. If the domestic interest rate is close the foreign rate of interest, then the second term is negligible, and since \( \partial I/\partial \mu_1 < 0 \) welfare is increased by the monetary expansion. On the other hand, if i and i*
differ very much and substitution effects in consumption are strong (i.e., $c_{12}$ is large) then welfare may decline.

Next come the welfare effects of changes in the foreign rate of interest. Since $\Delta i/\Delta i^* = 0$, (20) yields:

\[
\Delta u/\Delta i^* =
\frac{1}{u_1} \left\{ - p_1 ca/(1 + i^*) - p_1 \left[ 1 - ((1 + i^*)/(1 + i)) \right] \frac{c_{11} p_1 ca/(1 + M)(1 + i)}{c_{11} p_1 ca/(1 + M)(1 + i)} \right\}
\]

where

\[
M = -c_{11} p_1 \left[ 1 - ((1 + i^*)/(1 + i)) \right]
\]

Hence $\Delta u/\Delta i^* \geq 0$ when $ca \geq 0$. If the private sector is a net debtor then the welfare declines.

Finally, on the basis of (20) and (19) it is straightforward to calculate that

\[\Delta u/\Delta \beta_2 > 0\]

So a future improvement in terms of trade increases welfare.

IV. Dual Exchange Markets with Non-sterilized Intervention

With non-sterilized intervention central bank borrowing is endogenous and private foreign borrowing exogenously restricted. In fact most models assume that $ca = 0$. Equations (13) and (16) again form a subsystem from which $i$ and $b$ can be solved.
The Jacobian of the system (13) and (16) (after (15) has been substituted in) is

\[
\begin{vmatrix}
  k_1 \beta_1 q_1 & -1 \\
  v'k_1 \beta_1 q_1 - \left( \frac{c_{12} p_2}{(1 + i)^2} \right) + c_{11} (I/1) \left( 1 + c_{11} (I/b) \right) & = A
\end{vmatrix}
\]

\[
\frac{\partial I}{\partial b} = -p_1 \left( 1 - (1+i^*)/(1 + i) \right)
\]

\[
\frac{\partial I}{\partial \beta_1 q_1} = -p_2 c_2 / (1 + i)^2
\]

Hence, if again \( v'k_1 \beta_1 q_1 \) is not "too large" then \( A < 0 \). I assume this to be the case. Then the comparative statics with respect to \( \beta_1 q_1 \) gives.

\[
\frac{\partial b}{\partial \beta_1 q_1} > 0, \frac{\partial I}{\partial \beta_1 q_1} = ?
\]

Where does the ambiguity in the behaviour of the interest rate come from? The increase in income raises the demand for money, but it also increases the private sector saving and, hence, the foreign exchange reserves of the central bank. This latter effect increases the supply of money. The net effect on the excess demand for money is unclear and, hence, the interest rate may increase or decrease. This implies that the behaviour of the financial exchange rate is also ambiguous, \( h/\beta_1 q_1 = ? \).

Domestic monetary expansion on the other hand, has the expected effects:

\[
\frac{\partial I}{\partial \mu_1} < 0, \frac{\partial b}{\partial \mu_1} < 0
\]
Interest rate declines and the current financial exchange rate depreciates relative to the future rate Central bank loses reserves. Note, however, that $-1<\frac{\partial b}{\partial \mu_1}$, i.e., that the reserve loss is not one-to-one to the monetary expansion, since the decline in interest rate increases the demand for money. This validates the thesis that the dual exchange system allows an independent monetary policy (at least in comparison to the system with uniform fixed exchange rate). (With neutral intervention the policy independence holds trivially.)

Consider next the impact of the foreign rate of interest. It is easily calculated that the effect depends on whether the country as a whole is a net creditor or debtor.

$$\frac{\alpha_i}{\alpha_i^*} \leq 0 \text{ as } b - ca \leq 0$$

$$\frac{\alpha b}{\alpha_i^*} \geq 0 \text{ as } b - ca \geq 0$$

If the country is net debtor, $b - ca \leq 0$ then the domestic rate of interest decreases (the current financial exchange rate depreciates relative to the future rate). Also, the borrowing by the central bank decreases. This is easily explained: an increase in $i^*$ reduces income which reduces current consumption and hence, decreases the current account deficit. Thus, dual exchange system with non-sterilized intervention does not insulate domestic monetary conditions from foreign monetary disturbances. Note also that in the case studied most frequently in the literature $ca = 0$. Then the impacts of changes in $i^*$ depend only on the net asset position of the central bank.

Finally, consider the effects of an improvement in future terms of trade. The results are:
\[ \frac{\partial i}{\partial \beta_2} > 0, \frac{\partial b}{\partial \beta_2} < 0 \]

The increase in future income is partly spread to current consumption making the current account deficit increase. Hence, central bank borrowing increases and money supply declines.

The results above for the non-sterilized intervention can be collected to give

\[ i = i(\beta_1 q_1, \mu_1, i^*, \beta_2), \text{ with} \]
\[ i_1 = ?, i_2 < 0, \text{sign of } i_3 = \text{sign of } (b - ca), i_4 > 0 \]

For the neutral intervention they were

\[ i = i(\beta_1 q_1, \mu, i^*, \beta_2) \text{ with} \]
\[ i_1 > 0, i_2 < 0, i_3 = i_4 = 0 \]

Hence, only in case of domestic monetary expansion do the policies produce similar qualitative results. For the behaviour of the financial exchange rate the results are almost equally diversified. If the country is net debtor within the non-sterilized intervention then \( \frac{\partial h}{\partial i^*} < 0 \) under both systems, i.e., the current financial exchange rate depreciates relative to the future rate. But even then quantitative difference exists: \( \frac{\partial h}{\partial i^*} < -1 \) under non-sterilized intervention, and hence, the change in financial exchange rate is smaller when neutral intervention is used. This may be important for the working of the dual system, since large deviations of the financial exchange rate from the commercial rate create incentives to shift transactions between the two accounts.
Before going to welfare effects under non-sterilized intervention let me consider the effects of a change in the foreign borrowing allowed by the private sector. The results are:

\[ \frac{\partial i}{\partial c_\alpha} < 0, \quad \frac{\partial b}{\partial c_\alpha} > 0 \]

Increased private borrowing increases central bank reserves and thereby the supply of money. Note, however, that

\[ \frac{\partial b}{\partial c_\alpha} < 1 \]

Turn now to the welfare effects when non-sterilized intervention is used. The expression equivalent to (20) is

\[ (21) \quad \frac{\partial u}{\partial z} = - u_2 \left( \frac{p}{1 + i} \right)^2 \left( \frac{\partial i}{\partial z} \right) + u_1 \left[ \frac{\partial i}{\partial z} + (\partial i/\partial i)(\partial i/\partial z) + (\partial i/\partial b)(\partial b/\partial z) \right] \]

Here

\[ \frac{\partial i}{\partial b} = - p_1 (1 - ((1 + i^*)/(1 + i))) \]
\[ \frac{\partial i}{\partial i} = - p_1 (v'k_1'y_1 + p_2c_2)/(1 + i)^2 \]

I assume \( i \leq i^* \). Hence \( \partial i/\partial b \geq 0, \quad \partial i/\partial i \leq 0 \) definitely.

Take again first \( z = \beta_1 q_1 \). Then if \( \partial i/\partial \beta_1 q_1 > 0 \) all other terms except \( (\partial i/\partial i)/(\partial i/\partial z) \) are positive. But since I have assumed that \( v'k_1' \) is "small" an improvement in welfare occurs since (21) can be written as
(21') \( \frac{\partial u}{\partial z} = u_1 [3I/\partial z - p_1 v'k_1'y_1 (\partial i/\partial z) + (\partial I/\partial b)(\partial b/\partial z)] \)

If \( \partial i/\partial b q_1 < 0 \), then welfare unambiguously increases. Hence, \( \partial u/\partial b q_1 = ? \) but > 0 most likely.

Turn next to the welfare effects of a monetary expansion. Since \( \partial i/\partial q_1 < 0, \partial b/\partial q_1 < 0, \) (21') tells that the net effect is ambiguous (note that \( 3I/\partial q_1 = 0 \)). But if \( i \) is close to \( i^* \) the adverse effect on income which works through the central bank borrowing disappears. Hence

\[ \partial u/\partial q_1 = ? \]

but \( \partial u/\partial q_1 > 0 \) if the capital market distortion created by the dual markets is small, i.e., if \( i^* - i \) is "small".

The effect of the foreign rate of interest on income is

\[ 3I/3i^* = p_1 (b - ca)/(1 + i). \]

For definiteness consider only the case \( b - ca < 0 \), i.e., the country is a net debtor. Then the net effect on welfare appears to be ambiguous since \( 3I/3i^* < 0 \) and \( \partial b/\partial i^* > 0 \). It is easy to calculate, however, that \( 3I/3i^* = (3I/3b)(\partial b/\partial i^*) < 0 \). Then, since I have assumed that \( v'k_1'y_1 \) is "small", the net effect most likely is that welfare declines. Thus:

\[ \text{sign of } \partial u/\partial i^* = \text{sign of } 3I/3i^* \text{ most likely} \]
Finally come the welfare effects of an improvement in the future terms of trade. Since $\frac{\delta I/\delta B_2}{\delta I/\delta B_2} > 0$ and $\frac{\delta I/\delta B_2}{\delta I/\delta B_2} > 0$, the sign of the welfare effect seems to be ambiguous. Again, however, the term $\frac{\delta I/\delta B_2}{\delta I/\delta B_2} + (\delta I/\delta a)(\delta a/\delta B_2) > 0$. Hence, if $v'k_1'y_1$ is "sufficiently small", a welfare improvement is guaranteed. Hence:

$\Delta u/\Delta B_2 = ?$ but $> 0$ most likely.

In terms of qualitative welfare effects the type of intervention does not seem to create any large differences. The only real differences arise in connection with changes in the foreign rate of interest and in the future terms of trade. With neutral intervention the welfare effects had in most cases a definite sign whereas with non-sterilized intervention the effects were, in principle at least, ambiguous. Even in these cases, however, the effects most likely are similar. So there exist differences in welfare effects but to smaller extent than in effects on the domestic interest rate.

The effects on welfare, and on the domestic interest rate under both systems of intervention are summarized in table 1:

<table>
<thead>
<tr>
<th></th>
<th>$B_1q_1$</th>
<th>$\mu_1$</th>
<th>$i^*$</th>
<th>$B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$i^n$</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>n-s</td>
<td>?</td>
<td></td>
<td></td>
<td>sign of $b-ca$</td>
</tr>
<tr>
<td>u</td>
<td>$u^n$</td>
<td>?(+)</td>
<td>?</td>
<td>sign of $-ca$</td>
</tr>
<tr>
<td>n-s</td>
<td>?(+)</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
V. Welfare and Intervention Regime

In the previous sections I analyzed the welfare implications of changes in exogenous data. But one may also ask which of the regimes provides the highest welfare.

Consider first the economy under a regime of uniform fixed exchange rate. In this regime $i = i^*$ and the endogenous variables in the system (13), (16) are $b$ and $c_a$. The fixed rate system is not optimal, however, since the quantity of money is below the optimal quantity (see Adams and Greenwood (1985) for a closer argument in a similar framework). With fixed rate system the private opportunity cost of holding money is the interest foregone, $i/(1 + i)$, whereas the social opportunity cost of money is zero. Hence a system with uniform managed floating exchange rates is socially optimal. The optimal rate of float reduces the domestic rate of interest to zero. Hence, one may compare the dual exchange rate system with the uniform fixed rate system.

Let the equilibrium values of $b$ and $c_a$ under fixed rate system be $b_f$ and $c_a_f$. Then the supply of money under fixed rate system is $\mu_1 + b_f$. (Note that under the fixed rate system money is completely neutral: an increase in $\mu_1$ causes changes in $b$ and $c_a$ so that $db = -d\mu_1$, $dca = db$, and $du = 0$). In section III it was shown that in the dual exchange system with neutral intervention a small monetary expansion increases welfare if initially $i = i^*$ (or more generally if $i$ is "close" to $i^*$). Consider now
the dual system where the domestic credit $\mu_{ln}$ has initially been set so that (recall $b = 0$ in the dual system):

$$\mu_{ln} = \mu_1 + b_f$$

Equation (13) then implies that under the dual system $i = i^*$. At that point the level of welfare is the same under both regimes, if the commercial exchange rate in the dual exchange system has been set equal to the exchange rate in the fixed rate system. This is since $i = i^*$ and

$$I = p_1(1 - v(k_1))y_1 + (p_2/(1 + i))(1 - v(k_2))y_2$$

in both systems. Consequently, an infinitesimal monetary expansion in the dual system above $\mu_1 + b_f$ will increase welfare above that achieved in the fixed rate system. Hence, it is always possible to design the dual exchange system using neutral intervention so that welfare is increased above the welfare achieved in the fixed rate system. It is easy to see that this conclusion extends to the comparison between the fixed rate system and the dual exchange system with non-sterilized intervention. These results provide also the rationale for why I in Sections III and IV concentrated on the cases where $i < i^*$. The dual exchange system gives the possibility to conduct an independent monetary policy. This possibility should be used to conduct expansionary policies. The intuition for the result is obviously that if initially $i = i^*$ the monetary expansion drives the opportunity cost of holding money towards its optimal value without creating large distortions in consumption.
The comparison between the two systems of intervention within the dual system is more complicated, since one must agree on a common basis of comparison. One natural (?) starting point is the point where both systems give equal welfare, i.e., where the money supplies have been so adjusted that \( i = i^* \). Let the equilibrium private borrowing with neutral intervention be \( c_a^n \) (with \( b = 0 \)). Assume \( c_a^n > 0 \), i.e., that the private sector is a net borrower. The system with non-sterilized intervention produces the equilibrium \( i = i^* \), \( b = 0 \) if \( c_a \) is set equal to \( c_a^n \). But one may argue that since \( c_a \) is a policy variable when non-sterilized intervention is used it must be the case that policy makers want to reduce private borrowing. Hence, consider a small reduction of \( c_a \) from \( c_a^n \), \( dca < 0 \).

In section IV it was shown that \( \partial i / \partial c_a < 0 \), \( \partial b / \partial c_a > 0 \) under non-sterilized intervention. Hence, since \( \partial b / \partial c_a = \partial b / \partial b = 0 \) when \( i = i^* \), welfare unambiguously declines when \( dca < 0 \). This is seen from equation (21,).

Welfare with non-sterilized intervention is thus below the welfare with neutral intervention. So, in this very weak sense one can judge that dual exchange system with neutral intervention out performs the dual exchange system with non-sterilized intervention.

VI. Concluding Comments

It has been shown that the form of intervention under dual exchange markets matters very much for the behaviour of the domestic rate of interest and the financial exchange rate. Especially striking are the responses for changes in foreign monetary conditions. With neutral intervention the domestic rate of interest is not at all affected whereas with non-sterilized intervention an increase in the foreign rate of
interest leads to a reduction in the domestic interest rate (if the country is a net debtor). Similar difference is observed in case of changes in future terms of trade.

The differences in welfare responses caused by changes in exogenous data are not so striking. The most crucial difference appears to be that with neutral intervention the effects are more unambiguous than with non-sterilized intervention.

In normative analysis the most important conclusion is that the dual exchange markets can always be planned in such a way that they lead to a higher level of welfare than the system with uniform fixed exchange rate. The comparison between the two intervention systems did not provide any clearcut solution, though a very weak case for the superiority of the neutral intervention was provided.

Many of the results can be model specific. But they clearly provide a basis for the importance of the issue brought up by J. Marcus Fleming quoted in the introduction but which has been neglected in the literature.

To check the robustness of the conclusions one can utilize the more ad hoc type macromodels. Especially in the portfolio balance models one can add a third type of intervention policy to be considered. It is the sterilized intervention in the dual exchange markets. They provide also some other possibilities to check whether the intervention policies are feasible, e.g., whether they provide unique solutions for the financial exchange rate. I shall take up these issues in some of my future work.
References


