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Distributing Health: The Allocation of Resources by an International Agency

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by

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1. <u>The problem</u>

An agency of the United Nations has at its disposal an endowment of resources that it will distribute to various countries, with the aim of lowering their rates of infant mortality. The rate of infant survival, abbreviated RIS (one minus the rate of infant mortality), is taken to be an important indicator of the general level of physical well-being of a population, for a high rate is only achieved by good nutrition for women of child-bearing age, clean water supplies, and programs providing pre-natal care to women in the rural population. If infant mortality is low, the factors responsible impact upon others (than infants and mothers) in the population generally. The level of a population's physical well-being is, of course, an essential ingredient of its quality of life.

When the UN agency looks at the various countries, the data that are most important to it are the rates of infant survival in each country, and the 'technology' that the country will use to convert the resources it is granted into a higher RIS. Technology must be broadly interpreted. One country may be particularly efficient at using such resources because it already has a well-organized network of rural clinics in place. Another country might make less effective use of resources granted, not because it has a less adequate distribution system for bringing the resources to the population, but because the regime in power channels too large a fraction of the resources, from the agency's point of view, to the urban middle class, who comprise a small fraction of the population. The agency is powerless to affect this kind of political decision, in part because it is an international and not a supranational agency.

The technology for increasing the RIS, from the agency's viewpoint, is a function $u(x)$, where x is the vector of resources <u>per capita</u> that the agency grants to the country, and $u(x)$ is the consequent RIS achieved. Thus $u(0)$ is the RIS before UN intervention. I take $u(x)$ to have the usual features of a production function: it is non-decreasing in each component of x, it is continuous and concave. If, for example, a country siphons off, and uses for another purpose, some of the resources that it is allocated to increase its RIS, then 'x' in $u(x)$ stands for the vector of resources <u>per capita</u> allocated by the agency, not the vector of resources actually used effectively by the country. The UN has no control of the siphoning: this is what it means to say that u is the technology from the UN's viewpoint.

A first pass at formulating the resource allocation problem that the agency faces is to represent the relevant information by an ordered set $E=\langle M,n,\Omega,(u^1,N^1),(u^2,N^2),\ldots,(u^r,N^r)\rangle$, where M is the budget of the agency, n is the number of resources the agency decides are relevant, Ω is the set of all n-dimensional vectors of resources that the agency can purchase, at going prices, with budget M, u^i is the technology the i^{th} country uses, from the agency's viewpoint, to increase its RIS, and N^i is the population of country i. Each u^i is a function of the n resources, and it expresses the country's RIS as a function of the resources per capita allocated to it by the agency. There are r countries, the i^{th} one of which has a pre-intervention RIS of $u^i(0)$.

Other information, however, may be relevant to the allocation decision. Perhaps the agency will take into consideration the various endowments of the countries, which affect their RIS, although they are not specifically represented in E: their climates, their population densities, the degree of organization of their health services. These things appear in E only implicitly, as they affect the technologies u^i. Perhaps the agency should make use of its knowledge of this information directly. For example, suppose two countries have the same technologies $u(x)$ (in particular, they have the same pre-intervention RIS, $u(0)$), and the same population size. In the case of the first country, there is a good water supply, favorable climate, and a corrupt government, which siphons resources away from their intended use. In the case of the second, there is an unfavorable climate, dirty water, but a conscientious bureaucracy, which uses the resources allocated effectively. The upshot is that the two countries have the same effective technology u. Should the UN allocate the same bundle of resources to both countries? If not, then one believes it should make use of this other information.

Represent the ancillary information, for a country i, by a set Φ^i, which summarizes all kinds of political, social, geographical, and cultural information about a country, which may be relevant to decisions involving resource allocation for the purpose of raising the RIS. Ancillary information is taken to include only facts that are known to the agency. (One such fact might be that, for a certain country, the agency knows that it knows very little ancillary information.) A more complete representation of the problem the agency faces is $E=\langle M,n,\Omega,(u^1,N^1,\Phi^1),(u^2,N^2,\Phi^2),\ldots,(u^r,N^r,\Phi^r)\rangle$.

I have argued that the information summarized in E should suffice for the agency to decide how resources should be allocated among countries to achieve its goal. But how, precisely, should the agency proceed? Given the problem E, what distribution of resources $(x^1,x^2,\ldots,x^r)$ of some feasible resource bundle $\overline{x}\varepsilon\Omega$ should the agency choose, where x^i is the vector of resources allocated to country i, $\sum x^i=\overline{x}$, and $\overline{x}\varepsilon\Omega$?

2. Necessary conditions for resource allocation

What the agency must discuss is the budget allocation rule, which will associate to every reasonable problem E that the agency might face, an allocation of resources that the agency should implement. The rule, F, thusfar unknown, can be viewed as a function that maps possible problems E into feasible allocations for those problems: using the above notation, $F(E)=(x^1,x^2,\dots,x^r)$. In fact, we can dispense with the information on the agency's budget, and consider problems of the type $\xi=\langle n,\Omega,(u^1,N^1,\Phi^1),\dots,(u^r,N^r,\Phi^r)\rangle$. The feasible set upon which the agency concentrates is the set Ω of possible resource bundles that it can purchase with its budget. The budget is only needed to determine the set Ω. From now on, it is only necessary to consider problems of the form ξ.

I will propose that the agency proceed by discussing general principles that should apply to any resource allocation rule that it might adopt. This is a piecemeal approach, less ambitious that trying to come up with a complete allocation rule all at once. Deciding upon these principles can considerably narrow down the class of acceptable rules. I will propose five general principles, and particular axioms that follow from these principles. The perhaps surprising conclusion is that, haveing adopted restrictions on the class of acceptable allocation rules that are suggested by the five principles, the problem of choosing an allocation rule will have been completely solved.

Let $F(\xi)=(x^1,\dots,x^r)$ specify the allocation rule; define $F^i(\xi)=x^i$. Thus $u^i(F^i(\xi)/N^i)=u^i(x^i/N^i)$ is the RIS in country i after it receives and puts into use the resources it has been allocated[1]. The notation $u(F(\xi)/N)$ is the r-vector whose i^{th} component is $u^i(F^i(\xi)/N^i)$.

1. Efficiency. The allocation of resources should be efficient in the sense of Pareto optimality (PO). That is, no other allocation of any vector of resources in Ω can raise the RIS of some countries above what it is at $F(\xi)$ without lowering the RIS of some other country. This will be called axiom PO.

2. Fairness. Suppose that two problems ξ^1 and ξ^2 differ only in that, in the first, the agency faces a feasible set of resources Ω^1 which includes the set of resources, Ω^2, available in the second. The principle of monotonicity (MON) states that every country in the first problem should end up, after the allocation, with at least as high a RIS as in the second. Formally, let $\xi^1=\langle n,\Omega^1,(u^1,N^1,\Phi^1),\ldots,(u^r,N^r,\Phi^r)\rangle$ and $\xi=\langle n,\Omega^2,(u^1,N^1,\Phi^1),\ldots,(u^r,N^r,\Phi^r\rangle$ be possible worlds with $\Omega^1\supset\Omega^2$. Then $u(F(\xi^1)/N)\geq u(F(\xi^2)/N)$.

In particular, MON implies that if the budget increases, but nothing else changes, then each country should end up at least as well off (in terms of its RIS).

Pareto optimality is probably not a contentious principle, in the context of a resource allocation problem. Monotonicity certainly does not summarize all aspects of fairness that may be of import, but it is arguably a necessary condition of a fair allocation rule. Note that MON is a weaker principle than one that would require each country to receive more of every resource as the

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[1] It is more realistic to think of $u^i(x^i/N^i)$ as the RIS that the agency expects to attain after the country uses the resources x^i.

agency's resource bundle increases. It says that no country should suffer in terms of its RIS as resources become collectively more abundant. It also requires that if the agency's budget is reduced, each country should (weakly) share in the decrease of aggregate RIS that must ensue.

MON is not a principle that is patently required. For instance, if the agency were only concerned with increasing the world's aggregate RIS, taken as the population-weighted sum of the countries' individual rates, then it should not adopt MON as a principle. Population-weighted utilitarianism, the allocation rule that allocates resources in that way which maximizes the weighted sum of the countries rates of infant survival, violates MON[2]. MON is adopted only if the agency views its charge as reducing the rate of infant mortality in each country, not simply the rate of infant mortality internationally. This kind of principle might be adopted, for instance, if all countries contribute to the budget of the UN. If the allocation rule violated MON, some country could have reason to reduce its allocation to the UN, in order to thereby increase its effective allocation from the agency.

A second principle of fairness is Symmetry(S): if all the countries happen to be identical, with respect to both their technologies for processing resources and their sets of ancillary information, then the resources should be distributed in proportion to their populations. Formally, if $\xi = \langle n, \Omega, (u, N^1, \Phi), (u, N^2, \Phi), \ldots, (u, N^r, \Phi) \rangle$, then $F^i(\xi) = (N^i/N)\overline{x}$, for some $\overline{x} \varepsilon \Omega$, for all i.

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[2]That is, there are problems ξ having the property that, should resources increase, the international RIS (i.e., the population-weighted average of the country RISs) is increased by transferring some resources from a low RIS country to a high RIS country.

3. Neutrality. For a problem $\xi=\langle n,\Omega, (u^1,N^1,\Phi^1),\ldots,(u^r,N^r,\Phi^r)\rangle$, the distribution of resources should depend only on the technological information $u^1,\ldots,u^r$, the populations N^i, and the resource availabilities Ω. I call this the Irrelevance of Ancillary Information (IAI); it is formally stated as follows. Let ξ be as above, and let $\xi^*=\langle n,\Omega,(u^1,N^1,\Phi^{*1}),\ldots,(u^r,N^r,\Phi^{*r})\rangle$ be another problem with technologies, populations, and resource data exactly as in ξ, but in which the vectors of ancillary information (may) differ. Then $F(\xi)=F(\xi^*)$: resource allocation should be the same for the two problems.

IAI may be a contentious principle, as can be seen from the examples given in Section 1. It may be motivated by noting that the agency is directed by the general assembly[3] of the UN to distribute its endowment in a neutral fashion, that is, without regard to the internal politics and culture of the countries. Its task is one of engineering -- given the present state of resource-processing technologies as summarized in the $\{u^i\}$, to allocate resources in a fair manner in order to reduce infant mortality.

Because the Independence of Ancillary Information is assumed throughout, we may delete the symbols Φ^i and from now on represent a problem as $\xi=\langle n,\Omega,(u^1,N^1),\ldots,(u^r,N^r)\rangle$.

4. Consistency. There are many versions of consistency conditions in resource allocation problems. Generally, consistency means that if two problems are

[3] If the World Health Organization (WHO) is the agency, then the question is decided by its own general assembly. WHO operates as an independent affiliate of the UN.

related in a certain natural way, then their solutions should be related in a similarly natural way. I consider two consistency axioms.

Suppose the agency faces a problem in which there are n+m resources to distribute: $\xi=\langle n+m;\ \Omega;\ (u^1,N^1),\ldots,(u^r,N^r)\rangle$, where the set of feasible resources Ω is of dimension n+m. The technologies are defined as functions of all the resources, u(x,y), where x represents the first n resources per capita and y the last m resources per capita. Under the allocation rule F, the distribution of resources is $F(\xi)=((x^1,y^1),\ldots,(x^r,y^r))$, where (x^i,y^i) is the allocation to country i. Suppose the y resources are distributed first, as planned: $(y^1,y^2,\ldots,y^r)$. The agency can now be viewed as facing, temporarily, a new problem, as the countries are 'consuming' the y resources. The technologies of the countries, with the y resources fixed, are now $u^{*1}(x)=u^{*1}(x,y^1/N^1),\ldots,u^{*r}(x)=u^r(x,y^r/N^r)$, and there are n x-resources left to distribute. The new problem is $\xi^*=\langle n,\Omega^*,(u^{*1},N^1),\ldots,(u^{*r},N^r)\rangle$, where Ω^* describes the possible allocations of the x resources (it is the projection of Ω onto the n dimensional subspace of x-resources at the point $(y^1,y^2,\ldots,y^r)$). The principle of Consistent Resource Allocation across Dimension (CONRAD) states that the solution to ξ^* must be $(x^1,\ldots,x^r)$: that is, F must allocate the x-resources in ξ^* just as it allocated them in ξ.

Formally, CONRAD says that, for any problem ξ as described above, if $F(\xi)=((x^1,y^1),(x^2,y^2),\ldots,(x^r,y^r))$, and if ξ^* is defined as above with respect to $(y^1,\ldots,y^r)$, then $F(\xi^*)=(x^1,\ldots,x^r)$. If this principle of consistency holds, then, having decided upon the allocation of all the resources, the agency can distribute resources to countries as they become available, and it will never be faced with a need to revise its plan[4].

The second consistency axiom is called the Deletion of Irrelevant Countries. Suppose there is a problem $\xi=\langle n,\Omega,(u^1,N^1),\ldots,(u^r,N^r)\rangle$, and $F^1(\xi)=0$: the first country is allocated no resources. Consider the same problem, but without the first country: $\xi^*=\langle n,\Omega,(u^2,N^2),\ldots,(u^r,N^r)\rangle$. Then $F^i(\xi^*)=F^i(\xi)$, for $i=2,\ldots,r$. That is, if a country that is allocated nothing withdraws from the problem, the allocation of resources to the other countries should not change.[5]

5. Scope. The agency must adopt an allocation rule that will be applicable for a variety of problems it may encounter. It may, over the course of years, have many different budgets, face many different prices that change the resource availabilities, face variations in the number of resources, their identities, as well as their quantities. It can be expected that the technologies of the countries will change, and their rates of infant survival will change. The

[4]Indeed, the dimension of time is fabricated for this example, and may make CONRAD a less appealing axiom than it actually is. The axiom states that if the agency faces two problems that are related to each other in the manner of ξ and ξ^*, then it must allocate the x-goods in the same way in both problems -- there is no presumption that the agency faces ξ^* after it faces ξ. It should also be pointed out that the version of CONRAD stated here is stronger than what is actually required below for Theorem 1. CONRAD need only apply when the y-goods have a special property: that they are completely country-specific in their use, that is, that each y-good, $j=n+1,m+n$, is useful to only one country. Because these kinds of good hardly ever exist in practical problems, the CONRAD axiom can be viewed as a weak restriction on the behavior of resource allocation. For further discussion of CONRAD, see Roemer (1986, 1987).

[5]This axiom is a very weak version of an axiom called stability in bargaining theory, introduced by Lensberg(1987).

Domain (D) axiom states that the resource allocation rule the agency adopts should be capable of application to any possible problem ξ specified by arbitrary choice of n, for any convex set Ω in R_+^n, for any concave, monotonic, continuous functions $u^1, u^2, \ldots, u^r$ defined on R^n, and any distribution $N^1, \ldots, N^r$ of populations. The agency must also be able to solve problems for all r, such that $2 \le r \le \bar{r}$, for some integer $\bar{r}$.

Formally, let Δ be the domain of possible worlds on which the allocation rule is defined. Then for all $\{n, \Omega, (u^1, N^1), \ldots, (u^r, N^r)\}$ as specified, there exist $\Phi^1, \Phi^2, \ldots, \Phi^r$ such that $\xi \varepsilon \Delta$, where $\xi = \langle n, \Omega, (u^1, N^1, \Phi^1), \ldots, (u^r, N^r, \Phi^r) \rangle$[6]. This is a large class of possible worlds, but it consists of technologically reasonable ones. Some realism is imposed by requiring that the technologies be concave, monotonic functions of x.

3. Acceptable Allocation Rules

A resource allocation rule F is a function mapping any element ξ in the domain Δ into a feasible allocation. It is remarkable that the seven axioms discussed above suffice to determine a unique resource allocation mechanism on the domain $\tilde{\Delta}$, a sub-domain of problems of Δ defined precisely in the Appendix, available from the author.

Theorem 1 *Let a resource allocation rule, F, be defined on $\tilde{\Delta}$, and satisfy axioms PO, IAI, MON, S, CONRAD, DIC, and D. Then F must choose, for every problem ξ in $\tilde{\Delta}$, the distribution of resources that realizes the lexicographic*

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[6]The domain is more precisely defined in the unpublished Appendix, in which the theorem is proved, where the admissible technologies are discussed.

egalitarian distribution of the rates of infant survival; that is, F *is the leximin allocation rule.*[7]

Resources are allocated in the following way under the leximin rule. First, resources are allocated to the country with the lowest RIS until its RIS is raised to the RIS of the second lowest country. Then resources are used to raise the RISs of these two countries, until they become equal to the RIS of the third lowest country, etc. More generally, no resources are devcted to raise the RIS of a country until all countries with lower RISs have been raised either to its level, or if that is impossible, as high as they can be.

In a sense, the axioms S, MON, CONRAD, and DIC appear to be weak restrictions on the behavior of the allocation mechanism, because they each are concerned with situations that hardly ever occur. How often must the agency deal with a problem in which all the countries are identical, as postulated by S? How often must it deal with a pair of problems that are identical in their technological descriptions, except that there are more resources in one problem than in the other (MON)? (One might take the problems in consecutive years to be of this form, if the budget has increased, but, to be precise, this is not exactly the case, because the technology functions change at least slightly from year to year.) And how much of a restriction is the Deletion of Irrelevant Countries, since it hardly ever occurs that the agency faces a problem in which some country will be allocated no resources? Similarly, CONRAD refers to only a very small class of pairs of problems, which bear a certain intimate relation to each other.

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[7]The proof is available from the author, in the Appendix referred to above.

It would appear that the axioms are either quite weak, in the sense of the above paragraph, or are quite reasonable, or both. The theorem therefore claims to answer definitively the policy problem of the international agency. In the remainder of the paper, I discuss how salient this model and theorem are to the practice of one international agency.

4. The World Health Organization (WHO)

WHO is an international organization, with 166 member countries, which is affiliated with the United Nations, although it is a juridically separate organization. Its own World Health Assembly, which meets annually, is the supreme decision-making body. The budget of WHO comes from two sources, the first of which is the assessment of member countries. The United States, for example, is committed to supplying 25% of the biennial budget of the organization[8]. The planned income from member assessments in 1986-7 was $543 million. Secondly, WHO relies on 'extra-budgetary sources, contributions from private philanthropic organizations in various countries, and other national governmental sources, which contribute money to specific programs. These sources for the same budget period are estimated at $520 million. Thus, WHO operates on a biennial budget of approximately $1 billion.

WHO has a sequence of plans nested in time, which guide its operations. The World Health Assembly adopted a long-term strategy in 1981 for the 'attainment of health for all by the year 2000.'[9] The measures of health are

[8] At this writing (1988), the Reagan administration is delinquent with its payments.

[9] The twenty year strategy is summarized in World Health Organization (1981a).

defined with various degrees of precision by the organization. The most aggregate measures consist of 12 indicators, each defined as the number of countries in WHO that have achieved a certain degree of success with respect to a particular health indicator. For example, one such indicator is the number of countries in which at least 90% of newborn infants have a birth weight of at least 2500 grams. Others are: the number of countries in which the infant mortality rate is below 50 per 1000 live-births; ... in which life expectancy is over 60 years; ... in which the adult literacy rate for men and women exceeds 70%; ... in which the GNP per capita exceeds $500; ... in which safe water is available within 15 minutes' walking distance; ... in which there is complete immunization against diptheria, tetanus, whooping-cough, measles, polio, and tuberculosis; and so on[10]. The next level of planning consists of a six year plan. Finally, there are the biennial plans, which are specifically budgeted in the biennial budgets.

It is, therefore, a great oversimplification to model the problem of WHO as seeking to improve one indicator, such as the rate of infant mortality. Indeed, there are some 54 programs in WHO, administered under 20 divisions of the organization.[11] There is no attempt to form a single objective function to

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[10]These indicators may seem to be quite precise, but they are quite broad. An example of a more precise health indicator, taken from a list of over 100 such, is: the percentage of children in a country whose upper-arm circumference is no less than the value corresponding to the 5th percentile of the frequency distribution for well-nourished children. This physical measurement is apparently a sensitive indicator of malnutrition.

[11]There are divisions of environmental health, epidemiology, health education, communicable diseases, vector biology and control, mental health, health manpower development, noncommunicable diseases, and so on. There are programs in malaria control, parasitic diseases,

aggregate the many indicators, which measure success, into one welfare measure.

As well as the program or function dimension, there is an area dimension along which the operations and budget of WHO are disaggregated. The world is divided into six regions (Africa, the Americas, Southeast Asia, Europe, the Eastern Mediterranean, and the Western Pacific), as well as a Global and Interregional category, which handles interregional programs. Each region has a Regional Director (RD), appointed by the executive board in consultation with a regional committee. For each country, there is WHO Representative (WR), who represents the concerns of headquarters. Very briefly, the budget is negotiated as follows. First, only the regular budget -- the budget from country assessments -- is officially negotiated and allocated in this process. The secretariat proposes a division of the budget among the six regions and a global and interregional category. As will be seen below, this division is highly constrained by history. When the regular budget is virtually constant in real terms, as it has been during the 1980s, there is not much room for altering the division of the budget among regions from year to year. The regional allocation is followed by discussions between the regional committees of WHO and the governments in each region concerning the allocation of the regional budget among countries and among programs. Each region compiles a regional budget. Officially, the regions have control of these decisions. Countries must request specific programs. The important point is that, from an accounting point of view, the interregional division of the budget takes place

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immunization, diarrheal disease control, biomedical information, and so on.

first, in a centralized way, and the interprogram division is secondary and decentralized to the regional level.

In terms of our model, it is clear that the relevant units are not countries, but regions of the world: this is the level of disaggregation that is relevant for budget decision-making at WHO headquarters.

WHO has had surprisingly little discussion of the general principles that should guide budget allocation[12]. There is, however, a clearly enunciated principle of monotonicity: "...the Director-General has sought to effect necessary reallocations by means of selective application of increases in available resources, without reducing the current level of allocation to any one region (World Health Organization, 1979)." In the same document, the question is raised whether it is possible objectively to quantify health needs, and, if so, whether or to what extent the allocation of WHO resources between regions should be guided by these factors. "The definition of need is itself a subjective process, and it is not at all clear that criteria applicable to one population apply with equal force to all populations. The answer of the modern public health planner to the problem of allocation of resources would be to set up a mathematical model, using as objective, quantitative criteria as possible, but agreement on the parameters of such a model would be hard to reach." It is admitted, however, that "in view of the complexity of the matter and the great number of largely unquantifiable factors involved, it has been a matter of

[12]Officers of WHO whom I interviewed knew of only one document in which these principles were discussed, summarizing a meeting of the executive board held in 1979. The statements that follow are taken from that document.

'feeling one's way' over the years in arriving at the allocations of WHO resources between regions (p.7)"

In Table 1, the regular budget allocation among regions is presented for each biennial budget, beginning in 1978-9, calculated both in current prices and deflated prices, this last to make a real comparison with the previous biennial budget possible. It is important to note that only the regular budget is subject to this careful process, and the regularities that I discuss are observed only with respect to it. Note that the last period in which the regular budget increased in real terms was 1982-3. In the budgets of that and previous periods, there is monotonicity with respect to regions. The only deviation from monotonicity is in the treatment of the 'Global and Interregional' budget, from 1978-9 to 1980-81, when this allocation fell from $153 million to $142 million in real terms. This fall was the consequence of a World Health Assembly decision in 1978 to cut back on the operations at headquarters, and to direct a larger fraction of resources to country programs.

Beginning in 1984-5, the budget stagnated in real terms. In that period, when the real regular budget fell by $1.5 million, there were indeed violations of monotonicity. All regions should suffer a cut-back, according to the Monotonicity axiom, when the total budget is cut back[13]; but only the Western Pacific and the Americas region suffered, with the brunt being borne by the Americas. Upon further investigation, however, the apparent <u>large</u> fall in the

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[13]In fact, MON states that the <u>RIS</u> of no country should rise when the budget falls; throughout this discussion, however, I am taking the <u>budget allocation</u> to a region as the magnitude whose monotonicity is relevant. It is, of course, possible that the budget allocation to a region fall, while its RIS rises.

WHO Regular Budget Allocations by Region by Year

	Africa	Americas	S.E. Asia	Europe	E. Med.	W. Pacific	Global & Interregional	Total
1988-9 (p=86)+	98.9	57.9	68.8	33.6*	62.2	50.8	170.5	542.7
1986-7 (p=86)	98.9	57.9	68.9	32.2*	62.2	50.8	172.4	543.3
1986-7 (p=84)	90.1*	51.3	62.9	35.3	55.1	47.1	178.3	520.1
1984-5 (p=84)	94.3*	50.8	61.3	35.2	53.8	46.1	178.5	520.1
1984-5 (p=82)	82.9*	40.4	53.1*	32.6*	47.5*	38.3	172.6	467.4
1982-3 (p=82)	81.3	44.0	52.9	32.3	46.6	39.1	172.8	468.9
1982-3 (p=80)	70.7	37.6	46.8	24.0	41.1	34.4	182.3	436.9
1980-1 (p=80)	68.1	37.5	45.0	23.8	39.7	33.0	180.2	427.3
1980-1 (p=78)	61.3	31.7	40.3	21.2	35.3	29.8	141.9**	361.5
1978-9 (p=78)	55.4	30.3	36.2	20.2	32.4	26.6	153.2**	354.3

Compiled from Proposed Programms Budgets, WHO Geneva. Biennial budgets 1980-1981 to 1988-1989.

* Violation of monotonicity.
**Decreasing allocation to the secretariat, due to 1978 WHO resolution.
+ (p=86) means in 1986 prices.

Americas budget is due to an accounting procedure[14]. Nevertheless, some real fall must have been absorbed by the Americas region, since the total budget fell. The fall in the African real budget in 1986-7 is due to the same accounting practice. The only other violation of monotonicity occurs in 1988-9, when the European region is budgeted for a real increase, while other regions either experience no change or a slight decrease in their budgets, due to a small fall in the total real budget. But this turns out to be due to a re-classification of some 'global and interregional' programs to the European region.

Thus, the only clear violation of the monotonicity principle in these years is in the 1984-5 budget. Why does WHO seem to follow budget monotonicity in such strict fashion? From discussions with planners in the organization, it appears that this process is politically rather than ethically motivated. It would be difficult to cut the budget of any region, in an organization in which each region has political representation, and in which all regions contribute to the budget.

Although the motivation for budget monotonicity seems to be pragmatic, it is perhaps not coincidental that in many documents WHO expresses an egalitarian philosophy with regard to its project. "At present, health

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[14]Each region is asked to estimate the mark-up on its previous biennial allocation which is required due to changes in exchange rates alone. For the 1984-5 period, the Americas estimated a bigger mark-up than the secretariat was willing to grant. It would grant the mark-up only on the condition that the real budget allocation to the region would be proportionately reduced. Hence, the <u>nominal</u> allocation to the Americas was the same as it would have been with a slight increase in its real budget, had it not over-estimated the mark-up, from the secretariat's point of view.

resources are not shared equally by all the people: significant gaps still exist in many countries and health is the privilege of the few. Indicators should reflect progress toward correcting this imbalance and closing the gap between those who 'have health' and those who do not (World Health Organization, 1981b, p.12)." To be sure, this statement refers to inequality within a country, but the same egalitarian sentiment is expressed to apply across countries. As we have seen, monotonicity is closely linked with an egalitarian outcome, so the practice of monotonicity, if not ethically motivated, is serendipitous.

While monotonicity is observed at the regional level, it does not hold at the country level. There are many violations at the country level of disaggregation, which are mainly due to the lumpiness of programs. When an immunization program ends, for example, the allocation to the country may fall in the next period. These indivisibilities are not seen at the regional level, because there are, on the average, 28 countries per region.

How does the WHO budget allocation process conform to the other axioms? It is impossible to test for Pareto optimality, because we lack precise formulations of the functions which characterize the 'technologies' for the various countries. (Indeed, calculating such functions is not simply an engineering problem; it involves, as well, deciding upon a social welfare function that appropriately aggregates the many different measures of health achievement.) It also seems impossible to test the Symmetry axiom and the Deletion of Irrelevant Countries axiom, because the situations described in the hypotheses of these axioms do not occur in practice. But it seems uncontroversial to claim that the planners would follow these axioms if the occasion arose.

With regard to the Irrelevance of Ancillary Information, there is some evidence. Among the most important of considerations for planning a progam in a country is the 'probability of achieving successful and useful results.... a reasonable assurance from the government that the programme will be continued (World Health Organization, 1979, p.11).' Whether a country will, with reasonable assurance, continue a program is a characteristic not summarized in the technological information that describes it -- although the function u(x) does summarize how effectively the country uses resources. This must therefore count as a violation of IAI, although not, perhaps, an important one. I do not know how often this criterion come into play in deciding upon the allocation of resources among countries, or how well correlated the 'reasonable assurance' trait is with the effectiveness of the technology (u).

An apparent violation of IAI by WHO is that the Assembly voted, in 1947, to apply sanctions to South Africa, preventing it from voting in the World Health Assembly and from receiving assistance from WHO. This is tantamount *de facto* to excluding South Africa from membership in WHO; there is, however, no provision for explicit exclusion in the WHO Constitution. Here is a case where a country is refused assistance because of aspects of its society that are not clearly reflected in its technology -- i.e., because of ancillary information. It is noteworthy that this is, in fact, no violation of IAI, because South Africa does not enter the specification of the budget allocation problem[15]. *Among its effective members*, WHO claims to make budget decisions using only the 'technological' and resource information about a country.

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[15] I thank Joshua Cohen of the Massachusetts Institute of Technology for this fine point.

The allocation process conforms to the model in the respect that services and resources, not grants, are distributed to countries. Whether, however, resource allocation satisfies the Consistency axiom (CONRAD) is difficult to judge -- again, because it is difficult to imagine situations in which the axiom might actually act as a constraint on behavior. Suppose the agency decides to allocate resources to regions in a certain way, in a problem with 10 resource. Someone asks: if the agency faced a problem where the first five resources had already been allocated as the agency decided they should be in the 10 resource problem, should it reconsider how the remaining resources are to be allocated? If the answer is 'no,' then resource allocation is consistent in the sense that CONRAD requires.[16] I do not claim that CONRAD must be observed. It is certainly not a requirement of 'rational ' budget planning, although planning will be 'inconsistent' without it.[17]

Let us suppose that there is some technology function for each region that, although unknown to the secretariat, is being maximized subject to the

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[16]Recall the caveat that the CONRAD axiom does not actually have a time dimension. This is for heuristic purposes only.

[17] An example of an allocation rule that does not obey CONRAD is 'Equal Division Walrasian Equilibrium.' Divide the available resources among countries as they would be allocated according to the Walrasian equilibrium from equal initial endowments of the resources, where countries are assumed to take their technologies as utility functions. While it would be difficult to claim this allocation mechanism is irrational, it is inconsistent. Suppose, for example, we begin with a problem with two resources, and we compute the 'equal division Walrasian equilibrium' allocation. We now fix the first resource as it has been allocated, and ask, in the new one resource problem, how will the equal division Walrasian equilibrium mechanism allocate the remaining resource? In general, the allocation will not be the same as in the original two resource problem.

resources made available to the region. That is to say, although the planners are not able to articulate the functions $u^i(x)$, it is as if the problems they face are described in the form $\xi=\langle n,\Omega,(u^1,N^1),\ldots,(u^r,N^r)\rangle$. I have argued that it is reasonable to suppose that the axioms PO,S,MON,DIC, CONRAD, and IAI are being followed. Yet it is clear that the allocation rule is not the lexicographic egalitarian rule: for, even when there is a small increase in the total real budget, the resources allocated to all the areas are increased, while according to leximin, all the resources should be assigned to the region with the worst health status -- this is perhaps Africa -- until its health indicators are brought up to the level of the next worst region, perhaps South-east Asia. We can say this without knowledge of the particular technologies.

What may account for this apparent contradiction of the theorem is the domain axiom, which states that the allocation mechanism must be defined for 'all' possible problems. The planners of WHO only have to produce an allocation every two years. In the period of a generation, they will face only 12 'problems.' It is not difficult to allocate budgets, for 12 problems, which obey the six 'substantive' axioms listed above, but fail to conform to the leximin allocation rule. What we <u>càn</u> say is that it is impossible to extend the budget rule that WHO has been using to the class of all possible problems it might face, while not violating the six substantive axioms. Somewhere on the domain, the planners would be forced to violate MON or CONRAD or DIC. But this objection may seem pedantic, for the probability is almost zero that the organization will ever be forced into a violation of an axiom in any finite number of years. Discussion of this point is pursued in Section 5C.

5. Further Evaluation

Three questions will be discussed: (A) the tension between egalitarianism and utilitarianism in WHO; (B) the appropriateness of the specification of the technologies in the model; (C) the domain assumption of the model.

A. Egalitarianism versus Utilitarianism

A prominent competitor of the leximin allocation rule is the population-weighted utilitarian rule, which distributes resources among countries in that way that maximizes the population-weighted sum of the regional (or country) rates of infant survival. Indeed, it can be verified that the utilitarian rule satisfies all the axioms of Theorem 1, except MON, and because of this, our concentration on the observance of MON in the above discussion was not entirely innocent[18]. Note that population-weighted utilitarianism would, when faced with an allocation decision between two countries of the same population, assign the larger fraction of resources to the country whose health-status would gain the most. In particular, it is well-known that utilitarianism is insensitive to the initial statuses, $u^i(0)$; it takes into account only the rates at which the health indicators would improve under resource allocation.

While in modern ethical theory utilitarianism, as applied to the allocation of goods among persons, is the subject of much criticism[19], in the present

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[18] The population weighted symmetry axiom S is satisfied by population-weighted utilitarianism. If unweighted utilitarianism were the rule, the appropriate symmetry axiom would have to be unweighted symmetry, which is blind to the populations of countries. This is an indefensible axiom.

[19] For example, see the essays in Sen and Williams (1982).

context of health status among nations it is arguably quite an attractive alternative to leximin. To maximize the population-weighted sum of the country rates of infant survival is equivalent to maximizing the total number of infant lives saved internationally[20]. The difference between utilitarianism applied to persons and countries is this. Utilitarianism among persons treats each individual as a vessel for utility, but pays no attention to the boundaries, or rights, of the individual; utilitarianism with regard to countries treats each country as a vessel for health, but pays no particular attention to national boundaries, or the rights of countries. What in the first case violates conceptions that some of us hold about individual rights -- of the ethically relevant boundaries between individuals -- in the second ignores what some of us consider to be ethically irrelevant national boundaries.

The tension between population-weighted utilitarianism and lexicographic egalitarianism is observable in WHO. The stated goal of allocating resources to countries in which they will be most effectively used is utilitarian; the stated concern with egalitarianism suggests the leximin rule. In evaluating the achievement of various of the health indicators, stated in terms of the number of countries which have achieved certain levels, there is often a companion statement referring to the fraction of the world population that has achieved health: "It will be seen that 98 countries, representing 62% of the world population, have achieved a life expectancy of 60 years or more....On the other hand, more than a quarter of [the countries], representing 29% of the world

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[20] Actually, this is only strictly true if the intervention of WHO does not affect the total number of births. If, for example, education about and distribution of contraceptives is one program for reducing the rate of infant mortality, this will not necessarily be the case.

population, still have rates [of infant mortality] above the level of 100 per 1000 live births (World Health Organization, 1987, pp.70,73)."

The indicators that WHO has adopted, phrased in terms of the number of countries that have achieved specific levels of health status, are neither population-weighted utilitarian nor leximin. It will count more to lower the rates of infant mortality of several small countries over the threshold of 50 infant deaths per 1000 births than to lower the infant mortality rate of India from 100 to 80, although the second policy could save vastly more lives. By the same token, these indicators are not faithful to implementing leximin either. According to that objective, perhaps all the resources in the infant mortality program should go to Sierra Leone, whose rate of infant mortality is the highest in the world.

To maximize the number of countries whose rate of infant mortality is less than 50 per 1000, which is WHO's success indicator, one should proceed as follows. For each country i, calculate the cost, C_i, of bringing its RIS up to 950 per 1000. Arrange the countries in order of these costs, so that C_1 is the lowest cost. Let M be the budget and let j be the largest integer such that $\sum_1^j C_i \leq M$. Then the budget should be spent entirely on countries 1 thourgh j, to bring their rates of infant survival up to 950 per 1000. This procedure, in particular, would usually require not giving any resources to the worst off countries, so it is antithetical to leximin. On the other hand, it will tend to discriminate against large countries, because, other things being equal, they will require more resources to raise them up to the required rate-- so it is quite distant from population-weighted utilitarianism. It is closest to an 'unweighted country utilitarianism,' in the following sense. Define a new welfare indicator for each country, v^i. Let $v^i(x) = 1$ if, with resources x, country i has a rate of infant mortality of 50 or less, and $v^i(x)=0$ otherwise. Then maximizing the number of countries whose rates of infant mortality are 50 or less is equivalent to distributing resources to maximize $\sum v^i(x^i)$. I will therefore call the policy that follows from this procedure 'modified unweighted country utilitarianism.'

I asked planners at WHO to what extent the secretariat was guided by trying to maximize the 'numbers of countries' indicators, and was told that these were rules of thumb, but were not observed when their maximization clearly involved ignoring the severe problems of large countries. I was told that the indices were 'indicators,' not 'objectives.' Still, in a large and complex organization, where workers in bureaus at the lower levels may take seriously the precise indicators of performance set by higher authorities, it may be the case that such indicators guide policy more literally than is intended.

There have been some examples in the recent history of WHO where resource allocation has been guided by unmodified unweighted country utilitarianism, but these examples seem to be isolated cases. Several years ago, it was decided to allocate a more-than-usual amount of resources to certain countries-- one was Sri Lanka -- which were judged to be able to show fast and dramatic results. This move was a political one, whose intent was to demonstrate the potential impact of WHO programs. Apparently, the policy was quickly discontinued, however[21].

It is probably impossible to attain the WHO objective of Health for All by the Year 2000, by its own definitions of what constitutes health. Indeed, the slogan is put forth as a 'strategy.' That the organization follows in some

[21] I learned of this episode from Dr. Joshua Cohen of WHO.

cases a modified unweighted country utilitarian objective, sometimes a country-egalitarian one, and sometimes a population-weighted utilitarian one is in part due to political considerations (in the World Health Assembly, each of 166 countries has one vote), and in part due to having no clearly enunciated second-best policy. The most general policy statements from the Director General tend to propose objectives which are impossible (such as health for all); their flavor, however, is decidely egalitarian across people. For example: "All people in all countries should have a level of health that will permit them to lead a socially and economically productive life ...It [the policy] does mean that there will be an even distribution among the population of whatever resources for health are available (World Health Organization, 1981a, pp31-2)." If one tries to implement this policy by concentrating on the worst off country first, one gets leximin; if one tries to maximize the number of people who approach the goal, one gets population-weighted utilitarianism. If one tries to set a particulary simple indicator, which can be measured with some precision, and which can be easily communicated to and understood by politicians, potential donors, and the public at large, one has a 'modified unweighted country-utilitarian' policy.

B. Specification of the technologies in the model

WHO distiniguishes itself from the United Nations International Children's Emergency Fund (UNICEF) in that UNICEF provides materials and WHO provides technical assistance. (According to the organization, it 'engages in technical cooperation with its Member states.') WHO intends to build up the technical expertise and health infrastructure in the countries, rather than to supply them with materials. In an immunization campaign, for example, WHO is concerned with building up local clinics and educating health personnel so that immunizations will take place every year. UNICEF supplies the vaccines. Although the WHO allocation is just a tiny fraction of the health budget for each country, its importance is understated by this figure, because of the organizational nature of the service that it provides.

This suggests that the model I have studied may be seriously misspecified. It may be more accurate to model the WHO problem as the allocation of resources to most effectively <u>change</u> the technologies that the countries face. Let U be the class of all possible technologies. We can represent the technology of technical change by a mapping $T: U \times R_+ \to U$, where R_+ is the set of positive real numbers, interpreted as follows: $T(u,y)=v$ means that expending y dollars can transform technology u into techology v. Suppose that 'conventional' resources, such as vaccines, are available in amount x. Then the provision of 'technical assistance' in amount y by WHO has the effect of changing the RIS from $u(x)$ to $T(u,y)(x)=v(x)$. If we fix u, as it is fixed in a country, and recall that the set of available resources is Ω, then $T(u,y)(x)$ can be viewed as a mapping t_u from $\Omega \times R_+$ into R_+: $T(u,y)(x)=t_u(x,y)$. It may be appropriate to assume that t_u is convex and increasing in y and concave and increasing in x.[22] The convexity in y follows from the fact that investment in the development of infrastructure is best viewed as one of increasing returns to scale. The better the infrastructure is, the less costly it is to improve the 'technology' for transforming resources into a rate of infant survival. The function t_u is concave in x, for with fixed y, t_u is just a normal 'technology.'

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[22] By increasing in y, I mean that if $T(u,y)=v$ and $T(u,y')=v'$, for $y'>y$, then for all x, $v'(x) \geq v(x)$.

Suppose that the conventional resources to which a country has access are given -- from the viewpoint of WHO. The conventional resource distribution among countries is $(x^1,x^2,\ldots,x^r)$. WHO has budget M. The technical change transformation T is given, and the technologies $u^1,u^2,\ldots,u^r$ are given. Define, for any positive scalar y, $T(u^i,y)(x^i)\equiv v^i(y)$. By our assumptions, v^i is a convex, increasing function of y. WHO's problem would then be summarized as $\varepsilon=\langle M,(v^1,N^1),\ldots,(v^r,N^r)\rangle$: that is, how to distribute a budget M as $M=\sum y^i$ among the countries. Instead of the resource allocation problem with concave functions studied in Sections 1-3, we fact a budget allocation problem with convex functions.

The analysis of problems of the type ε will not be carried out here.[23] Is the criticism against the concave model, in regard to the specification of the WHO problem, apt? I am not sure. The technical assistance that WHO provides to a country takes the form of supporting specific programs. Within a program, the technology may be properly characterized as one of decreasing returns to scale (concave). If we take the case of the rate of infant survival, for example, it is surely the case that at some level the functions u^i become concave: for a doubling of resources will not forever bring a doubling of the RIS.

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[23]Preliminary work indicates that a characterization very close to Theorem 1 is true when the domain of technologies consists of all continuous, increasing functions that enjoy non-decreasing returns to scale in x.

C. The domain assumption

Even if the other axioms are followed by WHO in its resource allocation procedure, the domain axiom is not compelling, in the sense that the organization need only worry about efficiency, fairness, consistency, and so on, for a very small number of problems. Theorem 1 tells us that it is impossible to extend the resource allocation decisions that WHO has made over the past decade to a procedure which would be defined for every possible problem in the domain Δ, without violating at least one of the six substantive axioms. But is this not a foolish consistency to ask for?

The theory of resource allocation that I have presented depends, as do much of social choice theory and bargaining theory more generally, on the requirement that the allocation rule be defined for a large domain of possible problems. This axiom, in many circumstances, is justified not by the claim that, in the application at hand, all possible problems in the domain will eventually be encountered, but rather by the fact that one does not know beforehand which problems will be encountered, and so the allocation rule must be specified for all problems. But in WHO, and doubtless in most organizations, the allocation rule is not written down; the agency has the freedom to choose the allocation after the problem has been specified. With a history of a finite number of solved problems, it is almost always the case that when a new problem is introduced, the agency will have a great deal of latitude in proposing a solution for it, while not violating the substantive axioms that embody the agency's principles of resource allocation, within the set of problems that comprise recent history.

It is this difference in procedure, I think, that renders the formal theory of allocation mechanisms largely irrelevant for the study of practical policy. The domain axiom of the theory is most easily justified by the requirement -- an unstated axiom -- that the choice of mechanism must precede the specification of problems that are to be solved. In the real world, organizations have the freedom to specify the allocation after the problems are encountered. The use of mechanism theory to describe what resource-allocating agencies do must therefore be severely circumscribed.

My ambivalent thoughts are best phrased as a pair of questions: If WHO decides that it either should (e.g., consistency) or must (e.g., monotonicity) follow the substantive axioms, then, knowing that it will only encounter a small number of 'problems,' should it nevertheless follow a leximin policy? (Alternatively, an axiomatic characterization of population-weighted utilitarianism could be derived, and a similar question posed.) Or should the planners feel that they are following the spirit of the general principles, even if the leximin rule is not followed, knowing that they can in all likelihood avoid any overt axiom violation for the forseeable future? As a normative tool, at least, I think the axiomatic analysis is useful. Planners can perhaps gain insight about contrasting policies by understanding the axioms (such as Monotonicity) that distinguish them.

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Appendix: Theorem 1

The domain of technologies u^i is specified as follows. It is assumed that:

$$u^i(x) \text{ is a concave, weakly monotone increasing, continuous function defined on } \mathbb{R}^n_+ \quad \text{(A1)}$$

Because $u^i(x)$ is a rate of infant survival it is assumed that

$$0 < u^i(x) < 1 \quad \text{(A2)}$$

(It is never possible to eliminate all infant deaths; hence $u^i(x) < 1$.) It is furthermore assumed that:

$$(\forall\ \varepsilon > 0)(\exists x)(|\ u^i(x) - 1\ | < \varepsilon) \quad \text{(A3)}$$

That is, with a sufficiently large resource endowment, the RIS of a country can be made arbitrarily close to unity.[1]

The domain Δ of economic environments consists of all economic environments of the form $\varepsilon = \langle n,\Omega,u^1,N^1,\ldots,u^r,N^r\rangle$ where $n \geq 1$, Ω is a closed comprehensive convex set in $\mathbb{R}^n_+$, $N^i \geq 1$, u^i are functions defined on $\mathbb{R}^n_+$ satisfying (A1)-(A3), and $r \geq 2$. Axiom IAI implies that we may suppress the sets of ancillary information from the representation of an economic environment.

Let $A(\varepsilon)$ be the set of vectors of rates of infant survival that are feasible for environment ε, with all possible distributions of the resources.

Thus $A(\xi)$ is a convex set in $\mathbb{R}^r_+$ with its 'origin' at the point $(u^1(0),\ldots, u^r(0))$. Some possible 'RIS-possibility sets' are illustrated for r=2, in Figure 1. Let η be the ray of equal rates of infant survival in $\mathbb{R}^r_+$. We shall restrict ourselves to a sub-domain $\tilde{\Delta}$ of problems defined as follows: if $A(\xi) \cap \eta \neq \emptyset$, then there must be a point in that intersection which is Pareto optimal in ξ; if $A(\xi) \cap \eta = \emptyset$, then $A(\xi)$ must be 'strictly comprehensive,' i.e., there must be no horizontal or vertical segments on the frontier of $A(\xi)$. Thus, in Figure 1, $\xi^1,\xi^3,\xi^4,\xi^5 \in \tilde{\Delta}$, and $\xi^2,\xi^6 \notin \tilde{\Delta}$. In particular, $\xi^6 \notin \tilde{\Delta}$ because $A(\xi^6) \cap \eta \neq 0$, but the intersection contains no Pareto optimal point.

Let $\tilde{\Delta}^e$ consist of the sub-domain of $\tilde{\Delta}$ such that $A(\xi) \cap \eta \neq 0$. Thus $\xi^3,\xi^4 \cup \tilde{\Delta}^e$. Finally, let $\tilde{\Delta}^e_0$ be the sub-domain of $\tilde{\Delta}^e$ whose members have technologies for which $u^i(0) = 0$ for all i. In Figure 1, only $\xi^4 \in \tilde{\Delta}^e_0$.

We can state the theorem:

<u>Theorem 1 There is a unique allocation mechanism defined on $\tilde{\Delta}$, which satisfies PO, RMON, S, CONRAD and DIC</u>: F = L, <u>the lexicographic egalitarian (leximin) mechanism</u>.

First, we establish that L satisfies the stated axioms on $\tilde{\Delta}$. It is straightforward to observe that L satisfies PO, S and DIC. L satisfies CONRAD because L satisfies the stronger axiom Independence of Irrelevant Alternatives (IIA).

On the domain Δ, L does not satisfy RMON, but on the domain $\tilde{\Delta}$ it does, as we now prove. On $\tilde{\Delta}^e$, L = E, where E is the egalitarian allocation rule that equalize the rates of infant survival; so L satifies RMON on $\tilde{\Delta}^e$ because

E does. Next let, $\xi^1 \in \tilde{\Delta} \setminus \tilde{\Delta}^e$, $\xi^1 = \langle n, \Omega, u^1, N^1, \ldots, u^r, N^r \rangle$, $\xi^2 = \langle n, \hat{\Omega}, u^1, N^1, \ldots, u^r, N^r \rangle$, $\hat{\Omega} \supset \Omega$. Let $u(F(\xi^2)) = (\bar{u}^1, \bar{u}^2, \ldots, \bar{u}^r)$ and $u(F(\xi^1)) = (a^1, a^2, \ldots, a^r)$ and suppose $L(\xi^2)$ does not Pareto dominate $L(\xi^1)$. Then there is an i, such that $a^i > \bar{u}^i$. There must exist $j \neq i$ for which $a^j < \bar{u}^j$, or else $L(\xi^1)$ would Pareto dominate $L(\xi^2)$, contradicting Pareto optimality of $L(\xi^2)$. We must have $a^i > u^i(0)$; otherwise $u(F^i(\xi^2)) = \bar{u}^i < u^i(0)$, an impossibility. Say i = 1 and j = 2. It follows that $a^1 \leq a^k$, for $k = 2, \ldots, r$. For by the strict comprehensiveness of ξ^1, $(a^1 - \varepsilon, a^2, \ldots, a^k + \varepsilon, a^{k+1}, \ldots, a^r)$ is feasible for ξ^1, for any k and small ε, which contradicts the fact that $(a^1, \ldots, a^r) = L(\xi^1)$, if $a^k < a^1$. Therefore, we have

$$\bar{u}^1 < a^1 \leq a^2 < \bar{u}^2 \qquad (1)$$

But $(\bar{u}^1 + \varepsilon, \bar{u}^2 - \varepsilon, \bar{u}^3, \ldots, \bar{u}^r)$ is therefore feasible for ξ^2, which contradicts the fact that $L(\xi^2) = (\bar{u}^1, \ldots, \bar{u}^r)$, if ξ^2 is strictly comprehensive. (Note $\bar{u}^2 > a^2 \geq u^2(0)$.) On the other hand, if ξ^2 is not strictly comprehensive, then $\xi^2 \in \tilde{\Delta}^e$ and $L(\xi^2) = E(\xi^2)$, and so $\bar{u}^1 = \bar{u}^2 = \ldots = \bar{u}^r$, which contradicts (1). This proves that L satisfies RMON on $\tilde{\Delta}$.

The converse of <u>Theorem 1</u> is proved by establishing it successively on the domains $\tilde{\Delta}_0^e$, $\tilde{\Delta}^e$, and $\tilde{\Delta}$.

<u>Proposition 1 If F is defined on the domain $\tilde{\Delta}_0^e$, and satisfies PO, RMON, CONRAD, and S, then F is egalitarian: it assigns that allocation which equalizes the rates of infant survival across countries. That is,</u> F = E.

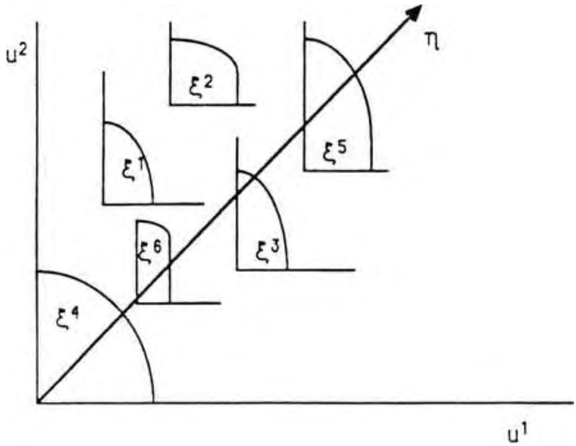

Figure 1

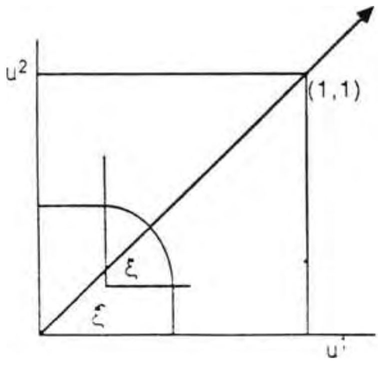

Figure 2

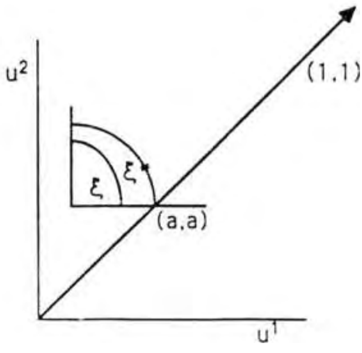

Figure 3

This proposition is a restatement of <u>Theorem 1*</u> in Roemer [1986], in the present context of the allocation problem of an international agency. Its proof is presented in that article. Two alterations are required here. We fix a population distribution $(N^1,\ldots,N^r)$ for the whole argument. Thus, we prove that F must be welfare egalitarian on each sub-domain $\tilde{\Delta}_0^e\ (N^1,\ldots,N^r)$ of $\tilde{\Delta}_0^e$, consisting of all problems in $\tilde{\Delta}_0^e$ with the particular population distribution $(N^1,\ldots,N^r)$. Secondly, we consider convex sets of resources Ω instead single resource vectors as in Roemer [1986]. This generalization readily follows from the axioms as stated here.

Note that <u>Propostion 1</u> demonstrates the <u>Theorem</u> on the domain $\tilde{\Delta}_0^e$, for the leximin rule <u>is</u> the egalitarian rule, when an egalitarian outcome exists which is Pareto optimal.

In the next step, we extend the <u>Theorem</u> to the sub-domain $\tilde{\Delta}^e$.

<u>Propostion 2. Let F be defined on $\tilde{\Delta}^e$ and satisfy PO, MON, CONRAD, and S. Then F is egalitarian.</u>

<u>Proof</u>:

Let $\xi \in \tilde{\Delta}^e$; $\xi = \langle n,\Omega,(u^1,N^1),\ldots,(u^r,N^r)\rangle$, and define $u^i(0) = u_0^i$.
Define function $U^i(x,t_1,t_2,\ldots,t_r)$ on $\mathbb{R}_+^{n+2}$ by

$$U^i(x,t_1,\ldots,t_r) = \begin{cases} t_i u^i\left(\frac{x}{t_i}\right) & \text{for } 0 < t_i < 1 \\ u^i(x) & \text{for } t_i > 1 \end{cases}$$

Note that, for any x,

$$\lim_{t\to 0} tu^i\left(\frac{x}{t}\right) = 0, \text{ from (A2).}$$

So define $U^i(x_1t_1,t_2,\ldots,t_{i-1},0,t_{i+1},\ldots,t_r) = 0$

It can now be verified that U^i are continuous, concave, monotone functions on $\mathbb{R}_+^{n+2}$, satisfying (A2) and (A3). Let e^i be the i^{th} unit vector in $\mathbb{R}^n$. Note that $U^2(x,e^i) = u^i(x)$. Consider the problem $\hat{\xi} = \langle n+r,\Omega x T_1,(U^1,N^1),\ldots,(U^r,N^r)\rangle$ where $T_1 = \{(x_1,\ldots,x_r) \in \mathbb{R}_+^r \mid x_i < 1, i=1,r\}$ is the unit cube in $\mathbb{R}_+^r$. By PO, the mechanism F must distribute the vector $(\overline{x}, 1,1,\ldots,1)$ for some $\overline{x} \in \Omega$. By PO again, the allocation must be of the form $((x^1,e^1),(x^2,e^2),\ldots,(x^r,e^r))$ since only country i makes any use of resource n+i.

From the definition of the functions U^i, it can be seen that the sets $A(\xi)$ and $A(\hat{\xi})$ are related as in Figure 2. That is $\hat{\xi} \in \tilde{\Delta}_0^e$, and the RIS-frontier of ξ coincides with a section of the frontier of $\hat{\xi}$. By Proposition 1, it follows that $F(\hat{\xi})$ is welfare egalitarian. But, since $U^i(x,e^i) = u^i(x)$, and because F allocates the one unit of resource n+i to agent i, CONRAD applies, and we conclude that $u(F(\xi)) = U(F(\hat{\xi}))$. This proves Proposition 2.

We next prove the theorem on the domain $\tilde{\Delta}$. Consider the sub-domain $\tilde{\Delta}^{(r)}$, for $r > 2$, where $\tilde{\Delta}^{(r)}$ is the sub-domain of problems in $\tilde{\Delta}$ with r countries. Thus $\tilde{\Delta} = \bigcup_{r=2}^{\infty} \tilde{\Delta}^{(r)}$. We prove the theorem on $\tilde{\Delta}$ by induction on r. First, we establish the theorem for r = 2.

Let $\xi \in \tilde{\Delta}^{(2)} \setminus \tilde{\Delta}^e$; for example, ξ in Figure 3. Let $\xi = \langle n,\Omega,(u^1,N^1),(u^2,N^2)\rangle$. By Property (A3), there is a set $\Omega^* \supset \Omega$ such that, for $\xi^* = \langle n,\Omega^*,(u^1,N^1),(u^2,N^2)\rangle$, $A(\xi^*)$ is strictly comprehensive, and $A(\xi^*) \cap \eta$ consists of exactly one point, (a,a), as illustrated in Figure 3. (This follows from (A2) and (A3).) Thus $\xi^* \in \tilde{\Delta}^e$, and so by Propostion 2, $u(F(\xi^*)) = (a,a)$. By

RMON applied to ξ^* and ξ, it follows that $u^2(F(\xi)) \leq a = u^2(0)$ and so $u^2(F(\xi)) = u^2(0)$. By PO, it follows that $u(F(\xi)) = Y$, in Figure 3. This proves that F is the leximin allocation mechanism on ξ. The argument is a general one for $r = 2$.

Assume that the theorem is true on $\tilde{\Delta}^{(r-1)}$; we establish it on $\tilde{\Delta}^{(r)}$. Let ξ $\tilde{\Delta}^{(r)} \setminus \tilde{\Delta}^e$, $\xi = \langle n,\Omega,(u^1,N^1),\ldots,(u^r,N^r)\rangle$. By property (A3), there is an admissable set of resources $\Omega^* \supset \Omega$ such that $A(\xi^*) \cap \eta = \{P\}$ consists of exactly one Pareto optimal point $P = (\bar{u},\bar{u},\ldots,\bar{u})$ of $A(\varepsilon^*)$, where $\xi^* = \langle n,\Omega^*,(u^1,N^1),\ldots,(u^r,N^r)\rangle$, $\varepsilon^* \in \tilde{\Delta}^e$. Let i_0 be an(the) index for which, for all i, $u^{i_0}(0) \geq u^i(0)$. We show that $\bar{u} = u^{i_0}(0)$. To the contrary, suppose that $\bar{u} > u^{i_0}(0)$. It follows that for all i $\bar{u} > u^i(0)$. (RMON tells as that $\bar{u} \geq u^i(0)$ for all i.) Then, by comprehensiveness of the set Ω^* and $A(\varepsilon^*)$, there exists an $\varepsilon > 0$ such that $(\bar{u}-\varepsilon, \bar{u}-\varepsilon,\ldots,\bar{u}-\varepsilon) \in A(\xi^*) \cap \eta$, which contradicts the supposition that $A(\xi^*) \cap \eta$ is a single point.

By Proposition 2, $u(F(\xi^*)) = P = (\bar{u},\ldots,\bar{u})$. In particular, $F^{i_0}(\varepsilon^*) = 0$ since $u^{i_0}(0) = \bar{u}$. By RMON, it follows that $F^{i_0}(\varepsilon) = 0$. Without loss of generality, take $i_0 = 1$. Now consider the (r-1)-country problem $\hat{\xi} = \langle n,\Omega,(u^2,N^2),\ldots,(u^r,N^r)\rangle$. By the induction hypothesis, $F(\hat{\xi}) = L(\hat{\varepsilon})$, where L is the leximin allocation rule. Applying DIC to the environment ξ, and deleting country 1, we deduce that $F(\xi) = (0,(F(\hat{\xi})) = (0,L(\hat{\xi}))$. If $u^1(0) \geq u^i(F(\xi))$ for all i, it will follow that $(0,L(\hat{\xi})) = L(\xi)$. By RMON, we have $\bar{u} \geq u^i(F(\xi))$ for all i. Since $\bar{u} = u^1(0)$, by the last paragraph, it follows that $u^1(0) \geq u^i(F(\xi))$, and the theorem is proved.

[1]This is perhaps not a realistic assumption. If it fails to hold, the theorem does not dissipate. It remains true on a more limited domain of possible problems.

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