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On Measuring Undernutrition

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1. INTRODUCTION

Despite a steady rise in the average per capita production of food in the world, the problem of hunger and undernutrition has become intensified. This paper is concerned with problems of quantifying undernutrition so that appropriate policies and programmes for alleviating it can be planned efficiently.

According to the United Nations Food and Agricultural Organisation (FAO) Third World Food Survey, "undernutrition is defined in terms of inadequacy of diet, that is, in calorie intake which, continued over a long period, results in either loss of normal bodyweight or reduction in physical activity or both". This definition is not strictly appropriate for children because allowance must be made for their satisfactory growth and high degree of activity characteristic of healthy children (Sukhatme 1961). Malnutrition on the other hand is a broader term defined as "a pathological state", general or specific, resulting from a relative or absolute deficiency or an excess in diet of one or more essential nutrients" (FAO 1982). Undernutrition is primarily due to inadequate intake of calories whereas malnutrition is caused by inadequacy of particular (or several) essential nutrients. Thus, a person who is undernoureshed is also malnourished, though the converse may not hold.

The most widespread form of malnutrition is protein-calorie deficiency. But it is now believed that in the case of most diets there is a positive association between calorie intake and intake of protein which means that if one gets enough calories then one should be getting enough protein. This issue has been thoroughly researched by Sukhatme (1974) who concluded that energy (food) and not protein was the limiting factor in our diet. This view has been widely accepted, although it is argued that the deficiency of some vitamins can not always be eliminated

just by consuming sufficient calories - "there is no reason to expect the same degree of correlation between calorie intake and intake of vitamin A (caroten), because the food sources of carotene and vitamin A are not generally the major sources of calories", Gopalan (1983). The focus of the present paper is on undernutrition which results from inadequate intake of calories.

One of the major problem in estimating the extent of undernutrition in a population is the identification of undernourished persons. FAO has been concerned with the issue of determining the dietary energy requirements of individuals in different age and sex groups which will allow them to maintain the required physical efficiency. An expert consultation group representing three major UN organisation, viz., FAO, WHO and UNU that met in 1981 defined an individual's energy requirements as "that level of energy intake which will balance energy expenditure when the individual has a body, size and composition and level of physical acitivity consistent with long-term good health, and which will allow for the maintainance of economically necessary and socially desirable physical activity".[1]

If the energy requirements and intake of each individual in the population are known, one can estimate the proportion of population which is undernourished. This can be used as an aggregate measure of undernutrition and may be called head-count measure (the term used in the measurement of poverty literature). Most of the recent debate on undernutrition is centered around the head-count measure. The proportion of individuals suffering from undernutrition, as such, does not reflect the intensity of undernutrition suffered by the population because it does not make distinction between the mild and severe forms of undernutrition suffered by an individual. The measurement of degree of undernutrition must take into account the gap between the calorie

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1. WHO "Energy and Protein Requirements", report of a joint FAO/WHO/UNU meeting, Geneva.

requirements and intake for each individual. In this paper, we derive a class of aggregate measures of undernutrition which takes into account the proportion of undernourished individuals as well as their calorie short fall.

The main difficulty in identifying an undernourished person lies in the fact that the energy requirement of an individual is not fixed. There are both the inter- and intra-individual variations in calorie requirements. It can be assumed that the calorie requirement follows a probability distribution. Unfortunately, this distribution is not known. Some attempts have been made to measure undernutrition by using the average calorie requirements norms for a reference man or woman which are periodically published and evaluated by FAO. This approach which classifies a person as undernourished if his or her calorie intake is below the required norm was first followed by Ojha (1970) and Dandekar and Rath (1971) for India and later by Reutlinger and Selowsky (1976) and FAO (1977) in its Fourth Food Survey at global level.[2] These studies led to a heated debate among economists and statisticians, the important among them being Dandekar (1981, 1982), Sukhatme (1981, 1981a, 1982) and Srinivasan (1981).

The most severe criticism of the average calorie norm approach was put forward by Sukhatme (1978, 1981, 1982) who argues that it leads to considerable over-estimation of the degree of undernutrition in the population. The main justification of his criticism is based on the empirical studies conducted by Widdowson (1947) and Edholm, Adam, Healy and Wolff (1970) which report considerable variation in calorie intake of apparently similar individuals maintaining constant body weight and the same activity level. On the basis of analysis of variance of the data provided by these studies. Sukhatme concluded that the variation

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2. FAO has used the minimum requirement norm equal to 1.2 BMR (The Basal Metabolic Rate) which is lower than the average calorie requirement norm used by Reutlinger and Selovsky and Dandekar and Rath.

in the requirement of intra-individuals over time is considerably larger than the variation in the requirement between people. Further Sukhatme and Margen (1978), analysing the model of protein deficiency concluded that the variation in energy balance (i.e.,intake minus expenditure) of a healthy individual maintaining body weight and performing the same activity follows a first order auto-regressive process with zero mean and stationary variance. These observations led them to hypothesize the existence of a regulatory mechanism in the human body which controls the efficient energy utilization. Under this hypothesis the body can within a certain range accommodate different intake levels without changing either body weight or activity. If, however, the energy intake lies outside the homoeostic range, the regulatory mechanism may break down and the person suffers from stress leading to change in either activity level or weight. Following this reasoning Sukhatme arrived at a minimum energy requirement for a healthy individual as $\bar{R} - 2\,\sigma_w$, $\bar{R}$ and σ_w being the mean and variance of the distribution of individual energy requirements, respectivily. Sukhatme extended this model to the entire population in order to estimate the degree of undernutrition at aggregated level. It is this extension which we argue has serious problem of empirical applicability which he completely ignores.

In this paper we argue that it is not possible to determine a single thershold point (below which everyone in the population is considered to be undernourished) without bringing in some set of value judgements.[3] In order to compute the degree of undernutrition, it is considered appropriate to utilize the entire distribution of calorie requirement. Since this distribution is unkown (except its mean and probably variance) we present numerical estimates on the basis of both a uniform and a normal distribution of calorie requirement.

In this paper we also derive the upper and lower bounds on the

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3. See Dasgupta and Ray (1986) who make a similar point.

aggregate measure of undernutrition which do not require the knowledge of the calorie requirement distribution except its mean. The lower bound is particularly useful in judging the validity of other approximate methods of measuring undernutrition (such as one proposed by Sukhatme). If for instance, Sukhatme's procedure (based on mean minus twice the standard deviation) provides estimates of undernutrition lower than the lower bound, one may doubt the validity of such a procedure on the ground that the lower bound is derived distribution free (requiring only the available distribution of calorie intakes).

Finally, we ask the question that if the distribution of calorie requirement is completely unknown (including its mean), is it possible to say on the basis of distributions of calorie intake only whether one population has greater or less degree of undernutrition than the other population? In this paper we derive a criterion of ranking any two populations with respect to the degree of undernutrition provided we can assume that the both populations have the identical distributions of calorie requirements. This criterion will be particularly useful in comparing the degree of undernutrition of a population at different time periods given the fact that the distribution of individual requirements does not change that much during a short period. However, the main limitation of this approach is that it provides only the partial ranking of the two populations.

The methodology developed in this paper is applied to the Indian data (National Sample Survey 1971-72) which formed the basis of earlier computations carried out by Sukhatme (1978, 1981, 1982) and Dandekar (1981). In this paper, the estimates of undernutrition have been obtained using alternative procedures which produce quite conflicting results. An attempt is made here to settle these conflicts. A numerical method of computing undernutrition from grouped data is also provided.

The out-line of the paper is as follows. Section 2 discusses the aggregate measure of undernutrition which are derived from the given

distributions of calorie intakes and calorie requirements. It has been shown that the average calorie norm approach overestimates the degree of undernutrition but the extent of overestimation may be small. Section 3 provides a critical evaluation of Sukhatme's approach to measuring undernutrition. Section 4 derives the upper and lower bounds on the aggregate measure of undernutrition which do not require the knowledge of the distribution of calorie requirements except its mean. Section 5 develops a new class of undernutrition measures which take into account the proportion of undernourished individuals as well as the extent of their sufferings. Section 6 discusses the criterion for ranking the distributions of calorie intakes without using any knowledge of the distribution of calorie requirements (including its mean). Sections 7 and 8 discusses the estimation of alternative measures of undernutrition using the uniform and normal distributions of calorie requirements. The estimation of undernutrition from grouped observations is also discussed in this section. Finally, section 9 provides a critical evaluation of numerical estimates of undernutrition obtained earlier for India. It also presents the new estimates of undernutrition using the appropriate methodology.

2. AVERAGE CALORIE NORM APPROACH

Suppose that the calorie intake x of an individual is a random variable with mean μ and the probability density function f(x). If the calorie requirement of an individual is a given number R and his calorie intake is x, then the person is said to be suffering from undernutrition if $x < R$.

The main difficulty in identifying an undernourished individual lies in the fact that the energy requirement is not fixed. It not only varies across individuals but also for the same individual during different periods. FAO publishes the average calorie requirements for a group of reference type individuals with given age, sex, body size and physical activity. Since different individuals differ with respect to many known

and unknown factors producing variations in requirements, so there is a distribution of requirements. It can, therefore, be assumed that calorie requirement of an individual follows a probability distribution with density function g(R) with mean calorie requirement $\bar{R}$.

Let P(x) be the probability that a person with calorie intake x suffers from undernutrition. This probability must depend on g(R) and is given by

$$P(x) = \int_{x}^{b} g(R)\, dR = 1 - G(x),$$

where $a \leqslant R \leqslant b$ and G(x) is the probability distribution function of calorie requirement. An index of undernutrition is then given by

$$M = \int_{0}^{\infty} [1 - G(x)]\, f(x)\, dx \qquad (1)$$

which is interpreted as the probability that a randomly selected person in the population suffers from undernutrition. This index is referred to as the head-count of measure of undernutrition.

An approximate procedure to estimate the extent of undernutrition is to calculate the proportion of population with energy intake less than $\bar{R}$, i.e.,

$$F(\bar{R}) = \int_{0}^{\bar{R}} f(x)\, dx \qquad (2)$$

where F(x) is the probability distribution function of calorie intake. This approach, which may be referred as average calorie norm (ACN) approach was first followed by Ojha (1970) and Dandekar and Rath (1971) for India and later by Reutlinger and Selowsky (1976) and FAO (1977) at global level.

If the distribution g(R) collapses at the mean $\bar{R}$, then we must have

$$G(R) = 0 \cdot \quad \text{if} \quad R < \bar{R}$$

$$= 1 \quad \text{if} \quad R \geqslant \bar{R}$$

then it can be seen that the measure M leads to ACN measure.

The average calorie norm approach has been criticized on the ground that it does not take into inter and intra individual variations in the calorie requirements. It assumes that requirement distribution collapses at the mean value implying that all individuals below the mean value are undernourished. Because of these criticisms it has been suggested that this approach may not measure undernutrition but still it can provide a useful indicator of poverty. Sen (1980) makes this point as "Malnutrition can provide basis for a standard of poverty without povery being identified as the extent of malnutrition. The level of income at which an average person will be able to meet his nutritional requirements has a claim to being considered as an appropriate poverty line even when it is explicitly recognized that nutritional requirements vary interpersonally around that mean".[4]

Sen goes on to make distinction between poverty and undernutrition as "20 per cent of the population failed to have imcomes adequate in buying enough food to meet the average nutritional requirements for that community is a statement about poverty of some interest of its own, even though it is not at all equivalent to saying that 20 per cent of the population failed to meet their nutritional requirements. Two statements are of interest for rather different reasons: the first enlightens us on an income deprivation related to some average standard (paying no attention to the fact that some are luckier than others in terms of need

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4. Sen does not make distinction between undernutrition and malnutrition. We believe that by malnutrition he means undernutrition.

for income in meeting nutritional requirements), while the second throws light on the prevalence of actual malnourishment".

Although we are in agreement with the main thrust of Sen's argument, it will be useful to point out that Sen is implicitly assuming a monotonic relationship between calorie intake and income. Because of differences in tastes and habits many individuals with adequate income may be unable to meet the average nutritional requirements for that community. The numerical example given in section 9 clearly demonstrates that the rankings of households (adjusted for size and composition) according to calorie intake and total expenditure may be quite different. Thus, we can not assume a monotonic relationship between calorie intake and total expenditure (or income).

In this paper we argue that the average calorie norm approach can provide a reasonably close approximation to the extent of undernutrition. As already pointed out, Sukhatme (1978) and also Srinivasan (1983) have questioned the ACN approach on the grounds that it leads to considerable over-estimation of the degree of undernutrition in the population. In order to assess the extent of over-estimation, let us intergrate (1) by parts:

$$M = \int_0^\infty F(x)\, g(x) d(x) \tag{2}$$

then using Taylor's expansion gives

$$F(x) = F(\bar{R}) + (x - \bar{R}) f(\bar{R}) + \frac{1}{2} (x - \bar{R})^2 f'(\bar{R}) \tag{3}$$

where $f'(x)$ is the first derivation of $f(x)$ and the terms of higher order of smallness have been omitted. Hence (3) is an approximate relationship. Combining (2) and (3) gives an approximate relationship

$$M = F(\bar{R}) + \frac{1}{2} \sigma_R^2 f'(\bar{R}) \tag{4}$$

where σ_R^2 is the variance of the requirement distribution.

Like any income distribution, we may assume that the distribution of calorie intake is a skewed distribution with a single mode. One charasteristic of such distributions is that the mean is greater than the mode. Since $\bar{R}$ (the avera calorie requirement) is generally greater than μ (the average calorie intake), $f'(\bar{R})$ (the slope of the calorie intake density function) will be negative. Thus, equation (4) implies that $F(\bar{R}) > M$, i.e., the average calorie norm approach tends to over-estimate the extent of undernutrition. This conclusion will hold even if $\bar{R} < \mu$, provided it is not less than the mode of the calorie intake distribution. The extent of such over-estimation depends on the second term in the right-hand side of (4). It is expected that $f'(\bar{R})$ will be of smaller order of magnitude than σ_R^2. Thus, we conjucture that the degree of over-estimation is not large. The validity of this conjucture is examined with the help of a simulation exercise on the Indian data in section 9.

3. SUKHATME'S APPROACH TO MEASURING UNDERNUTRITION

The calorie requirement of an individual depends on several factors including age, sex, activity level and environmental conditions. It will, indeed, be a difficult task to quantify all these factors. Even if all these factors can be controlled by observing individuals with the same age, sex and activity level who are living in the same environmental conditions, requirements can still vary among them because some individuals are apparently more efficient metabolic machines than others. These are called inter-individual differences in requirement arising from differences in energy utilization of different individuals (Widdowson 1947).

In order to take into account both age and sex of individual in measuring undernutrition, a commonly employed method is to measure daily calorie intake and requirements in terms of per consumer unit. The consumer units scales which have been constructed for this purpose give different weights to individuals differing with respect to age and sex. Although the construction of such scales involve serious methodological problems, we assume here that they have been constructed accurately.

Having taken into account age and sex, there still remain other factors which can vary substantially between individuals. For instance, Sukhatme points out that "energy expenditure rates vary from 2-5 calories per minute for light work to over 10 calories per minute for heavy work". Because of the variations in other factors, it can be assumed that requirements of individuals follow a probability distribution with mean $\bar{R}$, $\bar{R}$ being the average calorie norm per consumer unit, and variance σ_R^2.

Let us suppose that x now represents calorie intake per consumer unit and R calorie requirement per consumer unit. Although in a healthy active population of refereence age, sex and weight the intake is expected to be equal on the average to the energy expenditure, the available evidence indicates that the correlation between the two is small.[5]

Following Sukhatme (1961), we assume that x and R are independently distributed for practical evaluation, then the appropriate index of undernutrition will be given by (1). Since G(x) is not known, he proposed a new index given by

$$S = \int f(x)\ d(x) \qquad (5)$$

the intergral being evaluated over the range $x < \bar{R} - 3\ \sigma_R$, where σ_R is the standard deviation of R on a consumer unit basis - reflecting the inter-individual variation due to components other than age and sex. He gives the justification of this approach as following: "Ordinarily in a healthy active population with no one underfed one would expect, assuming normal distribution, no more that 1 per cent of the households to have calorie intake on a consumer unit basis below $\bar{R} - 3\ \sigma_R$. Consequently, in any observed intake distribution, the proportion of households with calorie intake per consumer unit following below $\bar{R} - 3\ \sigma_R$ can be considered to provide an estimate of the underfed in the population".

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5. See Sukhatme (1961) who quotes Edholm, Widdowson and McCance (1955) and Garry, Passmore, Warnock and Durnin (1955) for this available evidence.

It is clear from the above reasoning that all the households (or persons) whose calorie intake per consumer unit is below $\bar{R} - 3\sigma_R$ are undernourished. But the probability that a person is classified as undernourished when he is not is less than .005 which is obviously very small. So, only a very small fraction of healthy households will be classified as undernourished. This may be called type I error. What about the households who are not classified as undernourished (households whose calorie intake per consumer unit is above $R - 3\sigma_R$)? Are they all healthy receiving calories greater than an equal to their requirements?[6] The answer is of course not. In fact the probability that a person is classified as not undernourished when he actually is undernourished is less than .955. This may be called type II error. It can be seen that smaller the type I error, larger will be the type II error and vice-verca. It is then a question of value judgement as to which one of the two types of errors should be smaller. Sukhatme has preference for smaller type I error. His null hypothesis is that there is no undernutrition. It means that as Dandekar (1981) points out "we shall not acccept the existence of undernutrition unless the evidence is overwhelming". A more acceptable alternative may be that the two types of errors are of the same magnitude which leads to the average calorie approach discussed in the previous section.[7] It may be interesting to point out that Sukhatme in his later writings (beginning from 1978) advocated to use the mean minus two standard deviation to compute undernutrition. It means that he increased the type I error from .005 to .025, a five-fold increase and consequently reduced the type II error by the same magnitude. This alternative still shows Sukhatme's strong bias against the existence of undernutrition. Dasgupta and Ray (1986) emphasize this point as "it will not reject the hypothesis of a well-nourished individual unless told that it is untrue with greater than 95 % probability. In a less-developed country where equity and poverty alleviation are presumably primary aims, this is unwarranted."

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6. It may be noted that persons who are overfed (consuming more calories than they should) may also be unhealthy. But for the purpose of measuring undernutrition such persons may be regarded as healthy even if they are not.

7. This point has also been made by Dasgupta and Ray (1986).

Despite above limitations it must be admitted that Sukhatme's (1961) paper made a pioneering contribution in high-lighting the problem of world's hunger. He presented this paper at the joint meeting of the Royal Statistical and Nutritional Societies where his method of estimating the incidence of undernutrition received wide acceptance from the scientists who participated in the meeting. In the paper he presented the justification of his method purely in terms of inter-individual variation in calorie requirement. But in his later writing in the seventies he completely changed the justification of the same method by arguing that inter-individual variation is in fact negligible compared to the intra-individual variation. This change in his thinking was probably brought about by the publication of a study by Edholm, Adam, Healy and Wolff (1970) which made extensive measurements of calorie intake and expenditure on 35 young army recruits at six depots during the second, fifth and eight weeks. This study showed that there was wide inter- and intra-individual variation in the daily calorie intake and as Sukhatme (1982) points out that it "provided the first opportunity to examine the size, nature, source and significence of intra-individual process governing nutritional status".

Using the analysis of variance technique on Edholm's data Sukhatme (1974) concluded that "almost all the variation in requirement was intra-individual and not inter-individual, as wrongly concluded earlier" (Scrimshaw, Hussein, Murray, Rand and Young (1972)). Further, he investigated whether intra-individual variations were random arising from errors of mesurement and arrived at the conslusion that observations on energy balance (i.e., intake minus expenditure) on successive days were not independent but correlated.

In order to understand the phenomenon of undernutrition, Sukhatme postulated that the day-to-day variation in requiremetns of an individual follows a first order auto-regressive process

$$w_t = \rho w_{t-1} + \varepsilon_t \qquad (6)$$

where w_t is the energy balance on the t´ th day, ρ is the serial correlation of order unity and ϵ_t is the random variable distributed around zero mean with variance σ^2_ϵ. This model led Sukhatme to hypothesize the existence of a regulatory mechanism in a human body which controls the efficient energy utilization as a result the body can accommodate different intake levels in the homeostatic range without changing either body weight or activity. ρ is interpreted as an index of the power with which regulation at any given level of intake is controlled. Outside the limits of homeostasis, ρ is zero indicating that the body is under stress from undernutrition. Sukhatme asserts that the lower limit of this homeostatic range is $\bar{R} - 2\ \sigma_w$, σ^2_w being the intra-individual variance of requirements. It is implicitly assumed that the requirement distribution is normal which means that the probability that a healthy individual is classified as undernourished is .025. According to Sukhatme an individual who has calorie intake higher than $\bar{R} - 2\ \sigma_w$ is not undernourished because he can regulate his requirement to intake by varying his efficiency of utilization. Thus, this model implies that type I error is only .025 whereas type II error is zero. Sukhatme extended this model to the entire population in order to estimate the degree of undernutrition at aggregate level. Following are some of the criticisms of his approach.

First, the existence of a first ordrer auto-regressive process in energy balance is not scientifically established. Sukhatme (1978, 1981) himself admits that since the energy balance study reported by Edholm et al. (1970) is limited to 3 non-continuos weeks, it does not, therefore, permit a direct study of auto-correlations.[8] True that Sukhatme and Morgan (1978) derived an indirect evidence by analysing the nitrogen balance series for five individuals over a long period and found an auto-regressive process in nitrogen requirements. But it is yet to be established that the process generating energy balance is the same or even similar to that of generating nitrogen balance.

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8. See Dandekar (1982) who argues that Sukhatme´s analysis of Edholm´s data to arrive at the conclusion of auto-regressive process in energy balance is not adequate.

Secondly, the existence of auto-correlation even if established scientifically does not imply necessarily that energy requirement is self-regulated over a range of intakes. Auto-correlation only means that a low intake is followed by a low intake and a high intake by a high intake. In this regard Mehta (1982) concludes that "in the long run an individual with daily intake equal to the mean requirement would be malnourished according to the auto-regressive model".

Thirdly, Sukhatme's argument that all individuals whose calorie intake is higher than $\bar{R} - 2\sigma_w$ possess self-regulatory power to adopt requirement to intake by varying the efficiency of utilization is rather unrealistic given the fact that cut off point suggested by him has been fixed arbitrarily. Since individuals differ with respect to their energy utilization, it is, therefore, to be expected that their regulatory power will also be different. This is quite evident from the study by Paranjke (1980) which observes that weekly average intakes are serially correlated with $\rho = .5$ for herself and $\rho = .6$ for her husband. The lower limit of the homeostatic range naturally will vary among individuals. Using a single cut off point to measure undernutrition as suggested by Sukhatme is inappropriate.

Sukhatme's theory of auto-regulation has serious problems of empirical applicability which he completely ignored. He develops his methodology on the basis of intra-individual variation which he says is the fundamental source of variation but when he derives his empirical results, he uses the threshold point $\bar{R} - 2\sigma$ where

$$\sigma^2 = \sigma_w^2 + \sigma_b^2,$$

σ_b^2 being the variance of inter-individual variation. This procedure will, of course, be consistent with his methodology if it can be assumed that $\sigma_b^2 = 0$. With the help of analysis of variance on Edholm's data he, of course, demonstrated that σ_b^2 is small compared to σ_w^2, but the inter-individual variation he speaks of is one which arises among apparently similar individuals (with the same age, sex, activity level

and weight) due to differences in their efficiency of energy utilization. Since their differences in age and sex can be taken into account by using the consumer unit scale, the variation among individuals due to differences in activity level and weight would still remain. These variations, as Sukhatme (1961) points out, are not insignificant.

Above discussion suggests that it is not possible to determine a single point below which everyone in the population is considered to be undernourished without bringing in some set of arbitrary judgements. Sukhatme's approach based on single threshold appears to be of questionable validity. In order to estimate the degree of undernutrition, it is appropriate to utilize entire distribution of calorie requirement which makes allowances for both inter- and intra-individual variations in requirements. This approach is expected to take into account the inter-individual variation arising from, say, differences in activity levels as well as the intra-individual variation of Sukhatme's type, the lower limit of which varies with individuals.

4. UPPER AND LOWER BOUNDS ON THE HEAD-COUNT MEASURE OF UNDERNUTRITION

Since the density function g(R) of calorie requirement is not known, it may be useful to derive bounds on the measure of undernutrition without requiring the knowledge of g(R) except its mean $\bar{R}$.

Let us write M as

$$M = \int_0^{\bar{R}} [1 - G(x)]\, f(x)d(x) + \int_{\bar{R}}^{\infty} [1 - G(x)]\, f(x)d(x)$$

It is obvious that

$$\int_0^{\bar{R}} [1 - G(x)]\, f(x)dx \geqslant [1 - G(\bar{R})] \int_0^{R} f(x)dx = [1 - G(\bar{R})]\ F(\bar{R}) \qquad (7)$$

and

$$\int_{\bar{R}}^{\infty} [1 - G(x)]\ f(x)d(x) \geqslant 0 \qquad (8)$$

which are derived from the fact that the distribution function G(x) is a non-decreasing function in its domain.

Combining (7) and (8), we obtain

$$M \geqslant [1 - G(\bar{R})]\ F(\bar{R})$$

and if we assume g(R) is symmetrically distributed around its mean $\bar{R}$, $G(\bar{R}) = \frac{1}{2}$, which gives

$$M \geqslant \frac{1}{2}\ F(\bar{R})$$

which provides a lower bound on M and can be obtained by knowing $\bar{R}$ and the distribution of calorie intake.

Similarly, it can be seen that

$$\int_{\bar{R}}^{\infty} [1 - G(x)]\, f(x)dx \leqslant [1 - G(\bar{R})]\ [1 - F(\bar{R})] \tag{9}$$

and

$$\int_{0}^{\bar{R}} [1 - G(x)]\, f(x)d(x) \leqslant F(\bar{R}) \tag{10}$$

which gives

$$M \leqslant F(\bar{R}) + [1 - G(\bar{R})]\ [1 - F(\bar{R})]$$

and if g(R) is symmetric around its mean, we obtain an upper bound on M as

$$M \leqslant \frac{1}{2} + \frac{1}{2}\ F(\bar{R})$$

This leads to the following proposition

PROPOSITION 1: If the distribution of calorie requirement is symmetric around its mean, then

$$\frac{1}{2} F(\bar{R}) \leqslant M \leqslant \frac{1}{2} + \frac{1}{2} F(\bar{R}).$$

In section 8 we have computed the numerical values of $F(\bar{R})$ for rural and urban areas of India (1971-72) as .524 and .675, respectively. Then lower and upper bounds on M will be given by

$.262 \leqslant M \leqslant .762$ rural India

$.337 \leqslant M \leqslant .837$ urban India

Thus, in the rural areas of India at least 26.2 per cent of the population suffers from undernutrition whereas the similar figure for urban areas is 33.7 per cent.

5. A NEW CLASS OF UNDERNUTRITION MEASURES

The aggregate measure of undernutrition M given in (1) is interpreted as the probability that a randomly selected individual in the population suffers from undernutrition. This measure provides an estimate of the proportion of population which is undernourished. Thus, it may be called a head-count measure of undernutrition (the term used in the measurement of poverty literature).[9]

Most of the recent debate on undernutrition is entirely centered around the head-count measure. The proportion of individuals suffering from undernutrition, as such, does not reflect the intensity of undernutrition suffered by those who are undernourished because it does not make distinction between the mild and severe forms of undernutrition

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9. Sen (1976) calls the head-count ratio as a very crude index of poverty because "an unchanged number of people below the "povery line" may go with a sharp rise in the extent of the short-fall of income from the poverty line".

suffered by individuals. The measurement of degree of undernutrition must take into account the gap between the calorie requirement and intake for each individual. It if does not. It can lead to perverse results.

Suppose the distribution of requirements is given by a sector R = (1, 1.5, 10) which is fixed for a population. Further suppose that there are three individuals in the population whose calorie intakes are given by the vector x = (2, 2.5, 3). It can be easily verified that $M = \frac{1}{3}$,i.e., the proportion of population suffering from undernutrition is 33.3 per cent.

Let us now suppose that individuals are given food which can increase their calorie intake by three-fold. It can be verified that M is still equal to $\frac{1}{3}$ despite the fact the intensity of hunger (or undernutrition) is considerably reduced. In order to rectify such a defect, we develope below a class of undernutrition measures which take into account the proportion of undernourished individuals as well as the extent of their sufferings.

Let h(x, R) be the degree of undernutrition suffered by an individual with calorie intake x and requirement R. Since R follows a probability distribution, the expected undernutrition suffered by an individual with intake x is given by

$$E\ [U/x\,] = \int_0^\infty h(x,\ R)\ g(R)dR \tag{11}$$

Since the individual suffers from undernutrion only if R > x, we must have

$$\begin{aligned} h(x,R) &= 0 \qquad \text{if} \quad x > R \\ &> 0 \qquad \text{if} \quad x < R \end{aligned}$$

then (11) should be written as

$$E(U/x) = \int_x^\infty h(x,\ R)\ g(R)dR$$

In order to make this idea empirically operational, it is necessary to specify the function h(x, R). One simple specification in terms of one parameter is given by

$$h(x, R) = \left(\frac{R - x}{\bar{R}}\right)^{\alpha}$$

$\bar{R}$ being the average calorie norm, which gives

$$E(U/x) = \int_{x}^{\infty} \left(\frac{R - x}{\bar{R}}\right)^{\alpha} g(R)dR$$

The average undernutrition suffered by the population is then given by

$$K(\alpha) = \int_{0}^{\infty} \left[\int_{x}^{\infty} \left(\frac{R - x}{\bar{R}}\right)^{\alpha} g(R)dR \right] f(x)dx \qquad (12)$$

where α is the parameter to be specified. If $\alpha = 0$, $K(\alpha)$ is equal to measure M when $\alpha = 1.0$, $K(\alpha)$ becomes

$$K = \frac{(\bar{R} - \mu)}{\bar{R}} + \frac{1}{\bar{R}} \int_{0}^{\infty} x\, G(x)\, f(x)dx - \int_{0}^{\infty} G_1(x)f(x)dx \qquad (13)$$

where

$$G_1(x) = \frac{1}{\bar{R}} \int_{0}^{x} R\, g(R)dR$$

It can be proved that the sum of the two integral in the right-hand side of (13) is non-negative which implies that $K \geqslant (\bar{R} - \mu)/\bar{R}$.

A common procedure to determine undernutrition at aggregate level is to compare the average per capita availability of energy with the per capita energy needs. When the requirement exceeded the availability, the country or region is classified as inadequately nourished. Such a measure may be given by $(\bar{R} - \mu)/\bar{R}$ which, of course, has many wellknown limitations (mainly because populations are not homogeneous with respect to calorie intake and requirements of indivivuals belonging to them).

Since the measure K derived above takes into account the

distribution of calorie intake among different individuals as well as the distribution of calorie requirements, it may be considered to be a suitable measue of undernutrition at aggregate level. It can be seen that the commonly used measure, i.e., $(\bar{R} - \mu)/\bar{R}$ underestimates the degree of undernutrition, which implies that even if the average per capita calorie intake of a country is exactly equal to its per capita calorie requirement, the undernutrition will still exist.

This observation was also made at the United Nations World Food Conference held in Rome in 1974 where it was considered that energy supplies in the developing regions should be at least 10 per cent above aggregate requirements to allow for maldistribution. The figure of 10 per cent was arrived at on the ad hoc basis but now equation (13) can be used to estimate the magnitude of under estimation. We performed these calculation on the Indian data in section 9 of this paper and found that, the average energy supply for the rural areas should be about 20.6 % above the average energy requirement and similar figure for the urban areas was found to be 11 %. It means that in the rural areas the average calorie intake per consumer unit must increase from the value of 2952 to 3353, an increase of 13.6 per cent, in order that the undernutrition (as measured by K) is completely eliminated. Similarly in the urban areas the calorie intake per consumer unit must increase by 19.2 % from the value of 2588 to 3086. These calculations are, of course, based on the assumption that the distribution of calorie intake does not change when the average calorie intake is increased in the population.

Further, note that when $\alpha = 1.0$, it means that the degree of undernutrition suffered by an individual is given by the exact amount of his calorie short-fall. It would be more appropriate to give higher weight to the larger calorie short-fall which implies that α should be greater than unity. How much greater α should be is a matter of value judgement.

If the distribution of g(R) collapses at the mean $\bar{R}$, it can be prove that $K(\alpha)$ becomes

$$K(\alpha) = \int_0^\infty \left(\frac{\bar{R} - x}{\bar{R}}\right)^\alpha f(x)dx$$

which is an expression for the class of decomposable poverty measures proposed by Foster, Greer and Thorbeeke (1984) with poverty line $\bar{R}$. Thus, $K(\alpha)$ provides a generalization of their poverty measure when the poverty line is not a fixed number but follows a probability distribution.

6. PARTIAL RANKING OF POPULATIONS ACCORDING TO THE DEGREE OF UNDERNUTRITION

In this section we deal with a question that if the distribution of calorie requirement is completely unknown (including its mean), is it possible to say on the basis of distribution of calorie intake only whether one population has greater or less degree of undernutrition than the other population? This issue is of considerable practical impartance because the average calorie norms published by FAO are derived from studies on healthy young men in the United States and are of questionable applicability for other populations (FAO 1978). We derive below a criterion for ranking any two populations with respect to the degree of undernutrition provided we can assume that the both populations have identical distributions of requirements. This criterion will be particularly useful in comparing the degree of undernutrition of a population at different time periods given the fact that the distribution of individual requirements does not change that much during a short period.

Suppose we are interested in comparing undernutrition in populations I & II which have density functions of calorie intake as $f_1(x)$ and $f_2(x)$, respectively. Let $g(R)$ be the common density function of the requirement distribution, then the indices of undernutrition for the two pupulations are

$$M_1 = \int_0^\infty 1 - G(x)\ f_1(x)dx$$

and

$$M_2 = \int_0^\infty [1 - G(x)] f_2(x)dx$$

which on integration by parts become

$$M_1 = \int_0^\infty F_1(x)g(x)dx$$

and

$$M_2 = \int_0^\infty F_2(x)g(x)dx$$

respectively. It can be seen that if $F_1(x) \geqslant F_2(x)$ for all x, $M_1 \geqslant M_2$. This leads to the following proposition.

PROPOSITION 2: <u>If $F_1(x) \geqslant F_2(x)$ for all x, then the undernutrition in population I will always be greater than or equal to that in population II.</u>

This proposition provides a criterion of ranking the two population according to the head-count measure of undernutrition provided the two curves $F_1(x)$ and $F_2(x)$ do not cross. If, however, these curves cross, one can not say whether one population has greater or less degree of undernutrition than the other population. Thus, this criterion provides only the partial ranking of populations.

Next we consider the ranking of populations according the undernutrition measured by a class of measures derived in (12) of the previous section. These measure for the two populations I & II may be written as

$$K_1(\alpha) = \int_0^\infty E(U/x)\ f_1(x)dx \qquad (14)$$

and

$$K_2(\alpha) = \int_0^\infty E(U/x)\ f_2(x)dx \qquad (15)$$

respectively, where

$$E(U/x) = \int_x^{\infty} \left(\frac{R - x}{\bar{R}}\right)^{\alpha} g(R)dR \tag{16}$$

The first and second derivations of E(U/x) with respect to x are derived as

$$\frac{\partial E(U/x)}{\partial x} = \frac{-\alpha}{\bar{R}} \int_x^{\infty} \left(\frac{R - x}{\bar{R}}\right)^{\alpha-1} g(R)dR < 0 \tag{17}$$

$$\frac{\partial^2 E(U/x)}{\partial x^2} = \frac{\alpha(\alpha-1)}{\bar{R}^2} \int_x^{\infty} \left(\frac{R - x}{\bar{R}}\right)^{\alpha-2} g(R)dR > 0 \tag{18}$$

for $\alpha \geq 1$, respectively. Integrating (14) and (15) twice by parts gives

$$K_1(\alpha) = \int_0^{\infty} \phi_1(x) \frac{\partial^2 E(U/x)}{\partial x^2} dx \tag{19}$$

and

$$K_2(\alpha) = \int_0^{\infty} \phi_2(x) \frac{\partial^2 E(U/x)}{\partial x^2} \tag{20}$$

where

$$\phi_1(x) = \int_0^x f_1(x)dx \quad \text{and} \quad \phi_2(x) = \int_0^x f_2(x)dx$$

It can be readily seen that since $\frac{\partial^2 E(U/x)}{\partial x^2} > 0$ for $\alpha \geq 1$, if $\phi_1(x) \geq \phi_2(x)$ for all x, then $K_1(\alpha) > K_2(\alpha)$ which leads to the following proposition.

PROPOSITION 3: <u>If $\phi_1(x) \geq \phi_2(x)$ for all x then undernutrition in population I will be greater than or equal to that in population II when undernutrition is measure by the entire class of undernutrition indices $K(\alpha)$</u> (except $\alpha = 0$).

Note that $F_1(x) > F_2(x)$ and $\phi_1(x) > \phi_2(x)$ for all x are the first and second order dominance conditions in the field of decision making under uncertainty, respectively.[10] The first order dominance condition always implies the second order dominance condition but converse is not true.

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10. See Rothschild and Stiglitz (1969), and Madar and Russell (1969).

The graphs for $F_1(x)$ and $F_2(x)$ can be drawn without any diffifulty from the data on the distribution of calorie intakes. But, however, values of $\Phi_1(x)$ and $\Phi_2(x)$ are not directly obtainable from these data. In order to tackle this problem we utilize the idea of Lorenz curve L(p) (which is interpreted as the proportion of calories consumed by the bottom 100 x p percent of the population; where p lies in the range $0 \leqslant p \leqslant 1$).

The relationship between the Lorenz curve ranking and the ranking implied by $\phi(x)$ is given by the following lemma (Atkinson 1970):

LEMMA: The following statements are equivalent:

A. $\phi_1(x) \geqslant \phi_2(x)$ for all x

B. $\mu_1 L_1(p) \leqslant \mu_2 L_2(p)$ for all p

where μ_1 and μ_2 are the average calories consumed by population I and II, and $L_1(p)$ and $L_2(p)$ are their Lorenz curves, respectively.

Following Kakwani (1984), the product of mean calorie intake and the Lorenz curve may be called the generalized Lorenz curve which is directly observable from the data the distribution of calorie intake. Thus the following proposition which follows immediately from proposition 3 and the above lemma provides an empirically operational criterion to rank any two distributions of calorie intakes.

PROPOSITION 4: If the Generalized Lorenz curve for population II is higher than that for population I at all points, then undernutrition in population I will be higher than that for population II when the undernutrition is measured by the entire class of indices $K(\alpha)$ (except $\alpha = 0$).

Note that we can rank the populations only if the Generalized Lorenz curves for the two distributions do not intersect. If they do intersect then it is not possible to say which of the two distributions has greater degree of undernutrition. In order to obtain the complete ranking we have to specify a distribution of calorie requirements. This is attempted in the remaining sections of this paper.

7. EMPIRICAL ESTIMATION OF UNDERNUTRITION MEASURES

In order to estimate the measures of undernutrition discussed in the previous sections, we need to specify the distribution of calorie requirement. We performed the computations on the basis of two distritutions, viz. uniform and normal.

UNIFORM DISTRIBUTION

First we assumed that g(R) is uniformly distributed with mean $\bar{R}$ and standard deviation σ_R. It can be shown that

$$1 - G(x) = 1 \text{ if } \quad x < \bar{R} - \sqrt{3}\,\sigma_R$$

$$= \frac{\bar{R} + \sqrt{3}\,\sigma_R - x}{2\sqrt{3}\,\sigma_R} \text{ if } \bar{R} - \sqrt{3}\,\sigma_R \leqslant x \leqslant \bar{R} + \sqrt{3}\,\sigma_R$$

$$= 0 \qquad \text{if } \quad x > \bar{R} + \sqrt{3}\,\sigma_R$$

which on substituting in (1) gives

$$M = F(\bar{R} - \sqrt{3}\,\sigma_R) + \frac{(\bar{R} + \sqrt{3}\,\sigma_R)}{2\sqrt{3}\,\sigma_R}\,[F(\bar{R} + \sqrt{3}\,\sigma_R) - F(\bar{R} - \sqrt{3}\,\sigma_R)]$$

$$- \frac{\mu}{2\sqrt{3}\,\sigma_R}[F_1(\bar{R} + \sqrt{3}\,\sigma_R) - F_1(\bar{R} - \sqrt{3}\,\sigma_R) \qquad (21)$$

where F(x) is the probability distribution function of the distribution of calorie intake and

$$F_1(x) = \frac{1}{\mu}\int_0^x xf(x)dx$$

is the first moment distribution function, which is interpreted as the proportion of calories consumed by the people who have calorie consumption less than or equal to x.

Further, it can be verified that the class of undernutrition measures $K(\alpha)$ derived in (12) is given by

$$K(\alpha) = \frac{1}{2\sqrt{3}\,\sigma_R\,(\alpha+1)} \int_0^{\bar{R}+\sqrt{3}\,\sigma_R} (\bar{R}+\sqrt{3}\,\sigma_R - x)^{\alpha+1} f(x)dx$$

$$- \frac{1}{2\sqrt{3}\,\sigma_R\,(\alpha+1)} \int_0^{\bar{R}-\sqrt{3}\,\sigma_R} (\bar{R}-\sqrt{3}\,\sigma_R - x)^{\alpha+1} f(x)dx \qquad (22)$$

which leads to M when we substitute $\alpha = 0$.

NORMAL DISTRIBUTION

Assuming that $g(R)$ is normally distributed with mean $\bar{R}$ and standard deviation σ_R, then

$$G(x) = Q\left(\frac{x-\bar{R}}{\sigma_R}\right) \qquad (23)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$$

Thus, M will be given by

$$M = \int_0^{\infty} \left[1 - Q\left(\frac{x-\bar{R}}{\sigma_R}\right)\right] f(x)dx$$

which can be readily computed given the distribution of calorie intakes.

K given in (13) can be computed if we know $G(x)$ and $G_1(x)$. $G_1(x)$ for

a normal distribution is given by

$$G_1(x) = \frac{1}{\bar{R}\ \sqrt{2\pi}\ \sigma_R} \int_0^x R\ e^{-\frac{1}{2}\left(\frac{R-\bar{R}}{\sigma_R}\right)^2} dR$$

and G(x) is derived in (23). So substituting (23) and this equation in (13), we can obtain an estimate of K given the distribution of calorie intake.

Data on the distribution of calorie intake are provided in the group form, giving (a) the number of persons in a calorie intake range and (b) the average calorie intake in each range. From these basic data we derive the data on ps and L(p)s for each calorie intake range, L(p) being the Lorenz curve for the distribution of calorie intake (interpreted as the proportion of total calorie consumption of the bottom 100 x p percent of persons who are arranged in ascending order of their calorie intake). The following equation of the Lorenz curve was estimated by the ordinary least-squares method after applying the logarithmic transformation:

$$L(p) = p - ap^{\alpha}(1 - p)^{\beta} \tag{24}$$

where a, α and β are the parameters and are assumed to be greater than zero. Note that L(p) = 0 for both p = 0 and p = 1. The sufficient condition for L(p) to be convex to the p-axis are $0 < \alpha < 1$ and $0 < \beta < 1$. This new functional form of the Lorenz curve was introduced by Kakwani (1981) in connection with the estimation of a class of welfare measures. His empirical results on 62 countries indicated that the density function underlying this Lorenz curve provides an extremly good fit to the entire income range of the observed income distributions. In the present study this new functional form has provided an extremely good fit to the distribution of calorie intake.

From the property of the Lorenz curve we observe that (see Kakwani 1980):

$$x = \mu L'(p)$$

which gives the value of $p = F(x)$ for a given value of x. Substituting this value of p in L(p) gives $F_1(x)$ for any given x. Using these properties in conjunction with the fact that $dp = f(x)dx$, all the measures of undernutrition discussed above can be readily computed. The numerical results are presented in the next section.

8. MEASUREMENT OF UNDERNUTRITION WHEN CALORIE INTAKE AND REQUIREMENT ARE CORRELATED

All the measures of undernutrition presented so far are based on the assumption that the calorie intake x and the calorie requirement R, both mesured in terms of per consumer unit are independently distributed. This was the suggestion given by Sukhatme (1961) who argued that since the available evidence indicates small correlation between the two variables, x and R can be assumed to be independently distributed for practical evaluation. Despite this evidence it may be useful to see how the value of correlation coefficient affects the estimates of undernutrition. So, we consider a bi-variate density between intake x and requirement R, f(x, R). Then the head-count measure of undernutrition, which is the probability that a randomly selected person in the population suffers from undernutrition is given by

$$M^* = \int\int_{R < x} f(x, R)\, dx\, dR \tag{25}$$

In practice the bi-variate density function f(x, R) is not known. One common procedure is to assume that it is a bi-variate normal density. This approach has two major limitations. First, the distribution of calorie intake is expected to be skewed whereas the bi-variate normal distribution implies that it is symmetric. Secondly, the entire distribution of calorie intake is characterized by only two parameters viz., μ and σ^2, therefore, it can not provide good fit to the entire distribution of calorie intake.

In order to solve this difficulty, let us write

$$f(x, R) = g(R/x)\ f(x)$$

where $g(R/x)$ is the conditional density of R given x and $f(x)$ the marginal density of x. Then M* can be written as

$$M^* = \int_0^\infty \int_x^b g(R/x)\ f(x)\ dR\ dx$$

where the density function $f(x)$ can be obtained from the given data on calorie intakes. To compute M*, it will be necessary to specify the density function $g(R/x)$. So, we assume that $g(R/x)$ follows a univariate normal distribution with mean and variance as

$$E(R/x) = \bar{R} + \rho \frac{\sigma_R}{\sigma} (x - \mu)$$

and

$$\text{var}\ (R/x) = \sigma_R^2\ (1 - \rho^2),$$

respectively, where ρ is the correlation coefficient between x and R. Then M* will be given by

$$M^* = 1 - \int_0^\infty Q\ \left[\frac{x - \bar{R} - \rho \frac{\sigma_R}{\sigma} (x - \mu)}{\sigma_R \sqrt{1 - \rho^2}}\right]\ f(x)dx \qquad (26)$$

where $Q(x)$ is defined in (23) and $f(x)$ is derived from the Lorenz curve (given in (24)) fitted to the actual data on calorie intakes.

The numerical estimates of M* for alternative values of ρ are presented in the next section.

9. NUMERICAL ESTIMATES OF UNDERNUTRITION

This section presents the numerical estimates of undernutrition based on the Indian data (National Sample Survey 1971-72) which formed the basis of earlier computations carried out by Dandekar (1981) and Sukhatme (1977, 1981, 1982). The numerical estimates presented by them generated heated debate between them leading to considerable confusion regarding the extent of undernutrition in India. The purpose of this section is two-fold. First, it points out the methodological errors in their estimates and secondly it presents the estimates of alternative measures of undernutrition (discussed in the previous sections) using the appropriate methodology.

First of all we outline the computing procedures adopted by Dandekar and Sukhatme. Their estimates were derived from the data given in Table 1 which gives the distribution of calorie intake by households. The households have been arranged in ascending order by their per consumer unit monthly expenditure. The average calorie requirement for India as developed by FAO/WHO is 2780 Kcal at the retail level. Dandekar used the average calorie norm approach to estimate undernutrition, i.e., the proportion of population which has calorie intake less than 2780.

Instead of estimating the proportion of individuals suffering from undernutrition, he estimated the proportion of consumer units who have calorie intake less than 2780. By the method of linear interpolation in the 5th expenditure group he arrived at a figure of 4.6 + 11.8 + 8.0 + 11.0 + 11.0, i.e., 46.4 percent of consumer units suffering from undernutrition.

Sukhatme's (1978) estimation procedure is based on the threshold point of $\bar{\bar{R}} - 2\ \sigma_R$, $\bar{\bar{R}}$ being the average calorie requirement and σ_R the standard deviation of the requirement distribution. He placed the coefficient of variation of requirement at 15 percent which puts the standard deviation of an individual at 417. Since he estimated the undernutrition on the basis of households which are group of individuals, the standard deviation of household will be smaller than

TABLE 1: NATIONAL SAMPLE SURVEY, 26th ROUND (1971-72): RURAL HOUSEHOLDS .

Monthly expenditure per consumer unit (RS)	Number of households	Average number of consumer units per consumer	Average calorie intake per day per consumer unit	Distribution of percent of consumer units
0-15	444	4.99	1493	4.6
15-21	1207	4.74	1957	11.8
21-24	813	4.78	2287	8.0
24-28	1174	4.51	2431	11.0
28-34	1748	4.44	2734	16.0
34-43	2028	4.20	3127	17.6
43-55	1655	4.08	3513	14.0
55-75	1319	3.70	4016	10.1
95-100	598	3.31	4574	4.1
100+	482	2.84	6181	2.8
All classes	11468	4.29	2724	100.0

that of individuals. The average size of the household is 4.29 consumer units, which he assumes to be approximately 4. This means that the standard deviation of the household requirement per consumer unit will be half of that of a consumer unit. Thus, per consumer unit energy requirement of a household will be distributed with mean 2780 calories and standard deviation 208.5 calories. This gives a threshold point of 2363 calories which by the method of linear interpolation implies that 28.5 per cent of consumer units suffer from undernutrition in the rural areas of India , not 20 percent as calculated by Sukhatme (1978) by his own method.

Both Dandekar and Sukhatme derived their estimates of undernutrition on the basis of grouped data which were obtained by ranking households according to their per consumer unit monthly expenditure. Their estimates will be correct only if there is a monotonic relationship between calorie intake and total expenditure. Such a relationship may not exist due to differences in tastes and habits of different individuals. In order to investigate this empirically we computed the average calorie intake of various household deciles when households are ranked by both energy intake and monthly expenditure per consumer unit. The numerical estimates are presented in Table 2.

It can be seen from these results that average calorie intake in lower deciles is higher when households are ranked by monthly expenditure than when they are ranked by energy intake. This clearly indicates that the number of undernourished will be underestimated when calculated from the data based on expenditure ranking. This is an important observation because it casts doubt on the estimates of global undernutrition obtained by FAO (1977) and the World Bank (Reutlinger and Selowsky (1976) based on the income or expenditure distribution data. To determine the extent of underestimation we used the data given in Tables 3 and 4 to compute the incidence of undernutrition in the rural and urban areas of India, respectively. These data were, of course, obtained by ranking households by their energy intake per consumer unit.

Applying the linear interpolation on these data, the average calorie norm approach yielded the percentage of consumer units suffering from undernutriton as 52.3 for the rural areas and 66.5 for the urban areas. Similarly Sukhatme´s approach with a threshold of 2363 calories gave the incidence of undernutriton as 35.17 % and 45.59 % for the rural and urban areas, respectively. These results clearly suggest that the substantial underestimation occurs when the estimates of undernutrition are derived from the data based on expenditure or income ranking instead of the ranking by calorie intakes. This conclusion will hold if even the Engel curve, relating calorie intake with total expenditure is estimated

TABLE 2: AVERAGE CALORIE INTAKE OF VARIOUS HOUSEHOLD DECILES WHEN HOUSEHOLDS ARE RANKED ACCORDING TO ENERGY INTAKE PER DIEM PER CONSUMER UNIT AND MONTHLY EXPENDITURE PER CONSUMER UNIT ALL INDIA RURAL AREAS 1971-72

Household Deciles	AVERAGE CALORIE INTAKE	
	Ranking by energy intake per diem per consumer unit	Ranking by monthly expenditure per consumer unit
1	1333	1675
2	1818	2094
3	2093	2331
4	2341	2541
5	2586	2745
6	2849	2960
7	3145	3197
8	3508	3482
9	4040	3880
10	5843	5058
Total population	2956	2996

with high value of R^2 (the coefficient of determination). The occurence of high R^2 is a common phenomenon when Engel curves are estimated from the grouped data which may lead to a belief that there is a one to one correspondance between the calorie intake and the total expenditure. This is clearly shown to be untrue.

TABLE 3: CALORIE INTAKES: RURAL AREAS OF INDIA 1971-72

Energy intake per consumer unit Kcals	No of sample households	Average households size	No of consumer units per households	Average calorie intake per consumer unit	Distribution of per cent of consumer units	Standard deviation of requirement distribution
1	2	3	4	5	6	7
Up to--1500	651	6.61	5.01	1221	6.67	186
1501--1700	455	5.94	4.77	1604	4.44	191
1701--1900	576	5.93	4.75	1801	5.60	191
1901--2100	762	5.86	4.71	2002	7.34	192
2101--2300	854	5.95	4.79	2203	8.37	191
2301--2500	947	5.65	4.51	2399	8.74	196
2501--2700	882	5.73	4.55	2599	8.21	195
2701--3000	1234	5.43	4.31	2842	10.88	201
3001--3500	1774	5.21	4.13	3227	14.99	205
3501--4000	1174	4.94	3.92	3717	9.42	211
4001 & Above	2159	4.35	3.47	5248	15.33	224
All Groups	11468	5.39	4.26	2724	100.00	-

Source: Government of India, National Sample Survey Organization, National Sample Survey Report No. 238, Volume I, Appendix III, Table (O.OR)

TABLE 4: CALORIE INTAKES: URBAN AREAS OF INDIA 1971-72

Energy intake per consumer unit Kcals	No of sample households	Average households size	No of consumer units per households	Average calorie intake per consumer unit	Distribution of per cent of consumer units	Standard deviation of requirement distribution
1	2	3	4	5	6	7
Up to--1500	1028	6.20	5.03	1238	7.05	186
1501--1700	808	6.20	5.01	1604	5.52	186
1701--1900	1201	6.18	5.01	1800	8.20	186
1901--2100	1569	5.93	4.80	2001	10.26	190
2101--2300	1744	5.73	4.64	2201	11.03	194
2301--2500	2054	4.92	4.00	2400	11.20	208
2501--2700	2075	4.37	3.55	2598	10.04	221
2701--3000	2571	4.29	3.48	2799	12.19	223
3001--3500	2830	3.92	3.17	3184	12.22	234
3501--4000	1525	3.60	2.90	3720	6.03	245
4001 & Above	2054	2.84	2.24	5217	6.27	279
All Groups	19459	4.72	3.81	2699	100.00	-

Source: Government of India, National Sample Survey Organization, National Sample Survey Report No. 238, Volume II, Table (0.OU).

It is interesting to note that both Sukhatme and Dandekar measure the incidence of undernutrition by estimating the percentage of consumer units suffering from undernutrition. This approach runs into serious conceptual problem because it measures the sufferings of hypothetical consumer units, but not the sufferings of individuals within the households. Consider for instance two households with the same number of consumer units but they differ with respect to the number of individuals belonging to them - one household having more children than the other. The above approach will give equal weight to the sufferings of individuals in both households despite the fact that one household has larger number of individuals than the other. This is clearly unwarranted because it implies that the suffering of children and female adults is less import than that of adult males. Any two individuals suffering from the same degree of undernourishment must be given exactly equal weight - there exists no justification for discriminating one individual from another with respect to their sufferings. Since our utmost concern is with the individuals, the aggregate index of undernutrition should be defined in terms of the distribution of individual sufferings only.

The data on individual energy intakes are seldom available. The household expenditure surveys which provide information on calorie intakes for households have been used to measure undernutrition. Since the distribution of calorie intakes within households is not available, the estimates of undernutrition can not be obtained accurately. It will be interesting to estimate the bias involved in using such data but this exercise is beyond the scope of the present paper.

A household may be said to be suffering from undernutrition if its calorie intake per consumer unit is less than its calorie requirement per consumer unit. If we make an assumption that if a household is undernourished, then all its memebers are also undernourished, then one may estimate the number of undernourished individuals from the number of undernourished households whose size is given. This approach uses the household as the unit of observation and will require the distribution of calorie requirement for households. The mean of this distribution can

TABLE 5: HEAD-COUNT MEASURE OF UNDERNUTRITION USING HOUSEHOLD AS UNIT OF OBSERVATION
INDIA 1971-72

Head Count Measures	Average Calorie Norm Approach	Sukhatme's Approach
	Rural Areas	
% of undernourished households	47.8	31.6
% of undernourished individuals	52.5	35.5
% of undernourished households*	41.7	22.4
	Urban Areas	
% of undernourished households	58.3	37.5
% of undernourished individuals	67.2	47.1

* Households are ranked according monthly expenditure per consumer unit.

be taken to be the average calorie norm per consumer unit which in the case of India is 2780. Sukhatme derived the standard deviation of the household distribution of requirements as equal to half of the standard deviation of the distribution of consumer units. Estimates of undernutrition following this approach are presented in Table 5.

As can be seen from the table, the incidence of undernutrition for individuals is much higher than that for households. This reflects the higher incidence of undernutrition among large households. This is an important finding from the policy point of view, suggesting that the government should pay more attention to the nutritional status of large households.

Secondly, it can be seen that the incidence of undernutrition is considerably higher in urban areas than in rural areas. It is difficult to give definitive explanation for this phenomenon because of non-availability of further information. One possible reason may be that the distribution of requirement in the two sectors may be different because of higher activity levels of rural population. But in this paper the calculations have been performed on the assumption that the two sectors have identical distributions of energy requirements.

As can be seen from the table that according to Sukhatme's approach the incidence of undernutrition comes to 47.5 percent for the urban areas and 35.5 percent for the rural areas. In 1978, Sukhatme reported these estimates to be 25 percent and 20 percent for the urban and rural areas respectively. Since both these estimates are based on the same data and the same assumptions regarding the distribution of energy requirements, the large divergence between two sets of results needs to be explained. Three possible explanations may be offered.

The most important reason for the divergence is the fact that Sukhatme based his estimates on the grouped data obtained by ranking households according their monthly expenditure per consumer unit. As argued earlier this procedure will almost certainly lead to considerable underestimation of the incidence of undernutrition. Secondly, Sukhatme measured undernutrition in terms of consumer units, and not individuals. Thirdly, although Sukhatme placed the coefficient variation at 15 percent, according to which the standard deviation should be 417 but he uses the figure of 480 (2780 - 2300 = 480), which, of course, will make considerable difference to his results.

The estimates of undernutrition based on the household as a unit of observation suffer from one serious methodological problem. The problem relates to the standard deviation of the distribution of household calorie requirements. Sukhatme assumed that this standard deviation is constant for all households. This is an unrealistic assumption because the number of consumer units vary widely from one household to another.

It can be seen from tables 3 and 4 column 7 that if the individual standard deviation is fixed at 417 (15 % coefficient variation), the standard deviation of household requirement varies from 186 to 224 calories for the rural areas and from 186 to 279 for the urban areas. The variation would be considerably larger if we observed the individual households. It is difficult to see how the degree of undernutrition can be estimated at all when the variance of households calorie requirement vary so widely.

The main difficulty in the above procedure lies in the determination of the distribution of calorie requirements by households. Since our objective is to measure undernutrition among individuals, the best approach seems to be to use individual as a unit of observation. Under this approach each individual in a household is assigned a value equal to the calorie intake per consumer unit for that household which yields the distribution of calorie intake by individuals. Since the distribution of calorie requirement is given at individual level, one can compute the measure of undernutrition from the distribution of calorie intake so derived at the individual level. This approach will be appropriate if we assume that every member in the household suffers from exactly the same degree of undernutrition. Putting it in other words it means that the food within the household is distributed such a way that every member gets calories proportional to its requirement. The validity of this assumption is difficult to assess because of the limited knowledge available on the internal structure of the household. If, however, we can assume that the household members care about each other, then it may be reasonable to say that the household will allocate its food resources so that every household member gets calories proportional to its needs. If this assumption is not satisfied, however, the undernutrition measures presented in this section will not be accurate.

The data on the distribution of calorie intakes are provided in grouped form, so we need to fit some distribution function to calculate measures of undernutrition. FAO used the two-parameter log-normal and Beta distributions and concluded that in all cases, the log-normal

TABLE 6: ESTIMATES OF LORENZ CURVE AND GOODNESS OF FIT: INDIA 1971-72

Frequency used	a	α	β	R^2
		Rural Areas		
Households	.434	.870	.608	.999
Individuals	.424	.860	.602	.999
Households*	.345	.863	.661	.999
		Urban Areas		
Households	.354	.831	.565	.999
Individuals	.343	.823	.584	.999

* Households are ranked according to monthly expenditure per consumer unit.

distribution proved to be by far the best fitting. But it is well-known that the log-normal distribution fits poorly toward the tails; it tends to overcorrect for the positive skewness of income distribution (Kakwani 1980). In this paper we fitted the equation of Lorenz curve given in (24) which captures the skewness of the distribution of intakes quite well. The numerical estimates of the parameters of the proposed Lorenz curve are presented in Table 6 along with the square of coefficient determination R^2 in the last column. It can be seen that R^2 is consistently very high. In fact, this function fitted so well that it was difficult to provide the visual picture of the actual and estimated values of the Lorenz curve.

TABLE 7: HEAD COUNT MEASURE OF UNDERNUTRITION BASED ON DISTRIBUTION OF INDIVIDUALS

INDIA 1971-72

Head-count Measure	Average Calorie Norm Approach	Sukhatme Approach	Uniform Distribution	Normal Distribution
Rural Areas	52.4	18.8	51.3	51.4
Urban Areas	67.5	23.0	64.2	64.5
Ratio of Rural Urban Undernutrition	.776	.817	.799	.797

The various estimates of head-count measure of undernutrition based on the distribution of individuals are presented in Table 7. These estimates are derived by assuming the average calorie normal of 2780 calories per consumer unit with coefficient of variation of 15 % around this value. It is interesting to note that the average calorie norm approach provides estimates quite close to estimates obtained by using entire distribution of calorie requiremets. Sukhatme´s approach provides very low values of undernutrition. Further, it should be noted that estimates of undernutrition obtained by uniform and normal distribution are almost identical implying that it makes little difference as to what form of the calorie requirement distribution is chosen.

TABLE 8: HEAD COUNT MEASURE OF UNDERNUTRITION FOR DIFFERENT VALUES OF COEFFICIENT OF VARIATION

INDIA 1971-72

Coefficient of Variation	Rural		Urban	
	Uniform Distribution	Normal Distribution	Uniform Distribution	Normal Distribution
0.0	52.4	52.4	67.5	67.5
2.5	52.4	52.4	67.4	67.4
5.0	52.3	52.3	67.1	67.1
1.5	52.1	52.1	66.6	66.6
10.0	51.9	51.9	66.0	66.0
12.5	51.6	51.6	65.2	65.3
15.0	51.3	51.4	64.2	64.5
17.5	50.9	51.1	63.3	63.7
20.0	50.5	50.8	62.2	62.9
22.5	50.2	50.5	61.2	62.1
25.0	49.8	50.3	60.3	61.3
27.5	49.5	50.0	59.4	60.6
30.0	49.2	49.8	58.6	59.7

Since the coefficient of variation of the distribution of requirements is not exactly known, it may be useful to see the sensitivity of estimates of undernutrition with respect to the coefficient of variation. Table 8 presents these estimates for different values of the coefficient variation varying from 0 to 30 per cent. It is remarkable that estimates do not vary as much as one would expect particularly in the rural areas. This is an important observation because of the uncertain knowledge about the exact value of coefficient variation in the distribution of calorie requirements.

TABLE 9: CLASS OF POVERTY MEASURES
INDIA 1971-72

	Uniform Distribution k(α)			Normal Distribution		
	0.0	1.0	2.0	0.0	1.0	2.0
Rural	51.3	14.4	6.07	51.4	14.3	-
Urban	64.2	17.9	7.30	64.5	17.9	-
Rural-Urban Ratio	.799	.804	.831	.797	.800	

Table 9 presents the estimates of class of undernutrition measures proposed in the paper. These estimates show quite similar pattern as the head-count measures. Although the ratio of the degree of undernutrition in the rural-urban areas increases monotonically with α,but the differences are not all that large.

The estimates of undernutrition presented so far were computed on the assumption that the calorie intake x and the calorie requirement R, both measured in terms of per capita consumer unit are independently distributed. This is an unrealistic asssumption because in a healthy active population of reference age, sex and weight, the intake is expected to be equal to the average of the energy expenditure. Since the population under consideration consists of both healthy and unhealthy (undernourished) correlation between x and R will be less than unity. Sukhatme (1961) argued that this correlation will be small and, therefore, for all practical purposed can be assumed to be zero. This

TABLE 10: HEAD-COUNT MEASURES OF UNDERNUTRITION WHEN THE CALORIE INTAKE AND REQUIREMENT ARE CORRELATED - INDIA 1971-72

Coefficient of correlation	Rural Areas	Urban Areas
0.00	51.40	64.50
0.10	51.07	64.64
0.20	50.79	64.86
0.30	50.50	65.16
0.40	50.21	65.56
0.50	49.93	66.09
0.60	49.65	66.79
0.70	49.37	67.73
0.80	49.12	69.02
0.90	48.90	70.88

procedure will provide only an approximate value of the extent of undernutrition. It is interesting to see how good this approximation will be when the correlation coefficient between x and R is not small. Table 10 presents the values of the head-count measure of undernutrition for alternative values of R the correlation coefficient ρ.

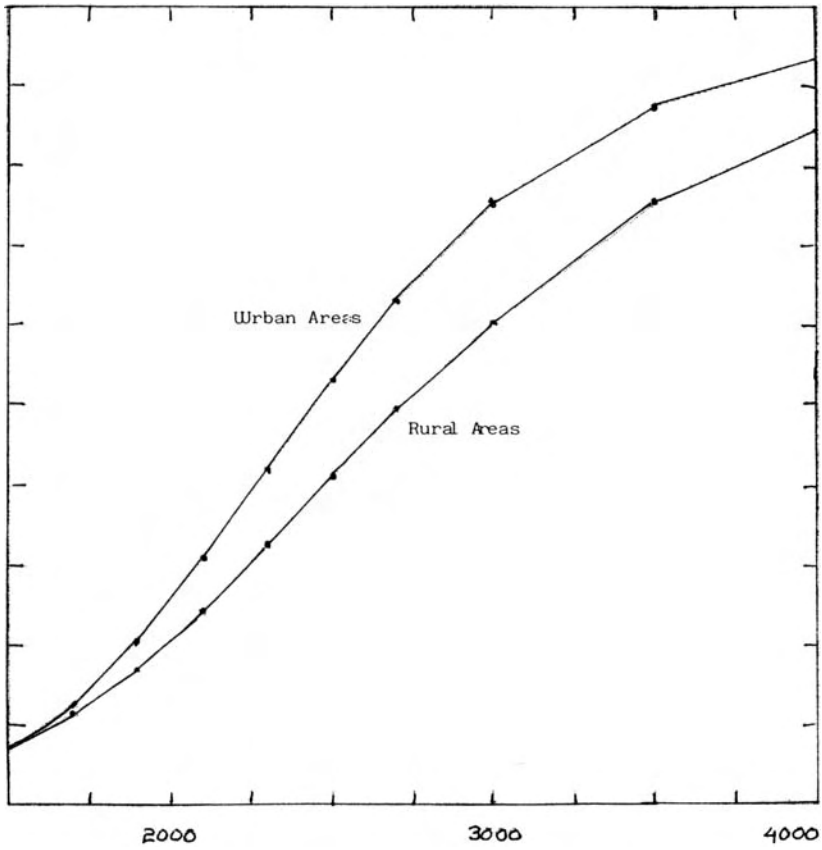
As can be seen that estimates of undernutrition are not as sensitive as one would expect as ρ varies. It is interesting to know that the degree of undernutrition decreases monotonically with ρ in the rural areas but in urban areas it shows a monotonic increase. We did some further simulations and arrived at the conclusion that if $\mu > \bar{R}$, the degree of undernutrition decreases monotonically with ρ and reverse is the case when $\bar{R} > \mu$. Anyway these results indicate that the estimates of undernutrition will not be too much biased if ρ is assumed to be zero.

TABLE 11: GENERALIZED LORENZ CURVE FOR RURAL AND URBAN AREAS: INDIA 1971-72

	Rural Areas		Urban Areas	
p	L(p)	μ L(p)	L(p)	μ L(p)
.10	.0451	133.3	.0516	133.3
.20	.1072	316.8	.1200	309.9
.30	.1786	527.9	.1967	508.0
.40	.2583	163.4	.2803	123.9
.50	.3462	1023.2	.3707	957.4
.60	.4427	1308.5	.4681	1208.9
.70	.5490	1622.7	.5734	1480.9
.80	.6673	1972.3	.6885	1778.2
.90	.8033	2374.3	.8180	2112.6
1.00	1.0000	2955.7	1.0000	2582.7

Finally, Table 11 presents the estimates of generalized Lorenz curve μ L(p) for different values of ρ. It can be seen that the generalized Lorenz curve for calorie intakes for rural areas is higher than that for urban areas for all values of ρ. From proposition 4, it follows that the undernutrition in rural areas will be lower than that in the urban areas for all distributions of calorie requirements when undernutrition is measured by the entire class of indices $K(\alpha)$ (except $\alpha = 0$). But if the undernutrition is measured by head-count measure, we require the stronger condition of first dominance, viz., $F_1(x) \geqslant F_2(x)$ for all x. It can be seen from figure 1 that the probability distribution function for urban areas is higher than that for rural areas for all values of calorie intakes. Thus, proposition 2 leads to the conclusion that the undernutrition in rural areas is unambiquosly lower than that in urban areas.

FIGURE 1: CUMULATIVE DISTRIBUTION FUNCTION OF POPULATION SHARE VERSUS CALORIE INTAKE LEVEL. CONSECUTIVE DATA POINTS CONNECTED BY STRAIGHT LINE SEGMENTS.



10. CONCLUDING REMARKS

The issue of quantifying undernutrition is extremly complex. It involves a number of conceptual and practical difficulties. This issue which has both political and policy implications has generated heated debate among economists, statisticians and nutritionists. One group argues that the degree of undernutrition in India is of the order 50 to 55 per cent of the population while the other group puts this figure at 15 to 20 per cent. If the latter estimate is accepted it will support the view that India's economic development plans are working well and should be continued. On the other hand if the high figure is established, the government may be compelled to follow an alternative food strategy.

The entire debate on undernutrition has been rather confusing. Many of the calculations on the degree of undernutrition are based on disputable assumptions and lead to conflicting results. An attempt has been made in this paper to settle these conflicts. The paper also develops a new class of undernutrition measures which take into account the proportion of undernourished individuals as well as the extent of their sufferings.

The main actor in the debate has been a well-known statistician Professor P.V. Sukhatme who suggested that the human body possesses a regulatory mechanism which allows individuals to adapt to much lower levels of energy and protein intake than of a reference figure at no fucntional cost. Despite the fact that Sukhatme's arguments have not been scientifically established, he has been strongly supported by leading economists such as Srinivasan (1981) and Lipton (1983). Srinivasan (1981) believes that there is no point in exaggerating the degree of undernutrition because such overestimation will only make the problem appear even more intractable than it already is. He argues in favour of allocating more resources to health, clean water, sanitation and other economic programmes which might ultimately prove more

effective than food and nutrition programmes in improving nutrition. No body would deny the importance of such measures, but if undernutrition is really as high as the evidence presented above suggest, then the human suffering is so widespread that a more direct approach of providing food to people is critical.

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