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Inequality and Risk

Marcel Fafchamps*

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Abstract

In poor societies, asset accumulation serves as insurance. It also opens the door to wider inequality. Many societies prohibit certain types of accumulation, such as land sales or indenture contracts. This paper investigates the theoretical relationship between risk sharing, asset accumulation, and long-term inequality. Scenarios with and without growth are contrasted. The paper also examines how asymmetric risk sharing (patronage) interacts with wealth accumulation to generate unequal distribution of assets and consumption while providing insurance to the poor. The paper provides insights for policy related to poverty and insurance. It refers to empirical evidence without providing new empirical findings.

Keywords: inequality, poverty, precautionary savings, patronage, mutual insurance, risk sharing, wealth distribution

JEL classification: O16, D30

*University of Oxford, Department of Economics

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Inequality and Risk

Marcel Fafchamps

Department of Economics, Oxford University

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There has been a lot of interest in the risk coping strategies of the poor in the recent literature but little work on the relationship between these strategies and inequality (Fafchamps 1999). Some have begun to suspect that certain risk coping strategies further impoverish the poor (e.g. Dasgupta 1993, Sen 1981). Labor bonding and debt peonage are examples that have been discussed in the literature (e.g. Srinivasan 1989, de Janvry 1981). Patronage, that is, the protection of the poor by the rich in exchange for labor and services, is also suspected of perpetuating poverty (e.g. Platteau 1995, Platteau 1995, Fafchamps and Quisumbing 1999).

The purpose of this paper is to clarify the relationship between inequality and risk. Our objective is to understand how wealth accumulation and risk sharing affect the evolution of inequality over time. Instead of analyzing on a single model in detail, we provide a rapid overview of various modeling frameworks. This approach has the merit of identifying key trade-offs. Results presented here should be seen as preliminary and tentative. To keep things manageable, we focus on a two-agent economy and ignore incentive issues and asymmetric information. Agents are infinitely lived. Returns to wealth are taken to be deterministic but, unlike many models of long term inequality, income is stochastic. Five concepts of inequality are distinguished: in wealth, income, consumption, cash-in-hand, and welfare. We ignore possible feeback effects between inequality and social choices (Benabou 2000).

We begin by showing that when risk sharing is perfect, inequality in welfare is constant over time. This is hardly a novel result, but it implies that perfect risk sharing eliminates social mobility. This might be a 'good' thing if welfare is distributed equitably. But it is hardly equitable if the constant distribution of welfare is highly unequal. We also examine the constraints that voluntary participation to mutual insurance imposes on redistribution.

Next, we assume risk sharing away and examine how inequality evolves over time when agents accumulate an asset. We distinguish between three canonical situations. We first assume that asset accumulation is unbounded and the asset yields a positive return. In this case, wealth can be thought of as capital. We show that inequality converges to single value over time. If agents have different propensities to save, the shave of total wealth in the hands of the more thrifty agent converges to 1. If agents have identical savings functions, inequality converges to an arbitrary level that depends on the path of income realizations.

We then consider what happens when the asset yields a zero or negative return. Grain storage is an example of such asset. Here, the only motive for holding wealth is precautionary saving. In this case, there is no persistent inequality; agents switch rank as a function of income realizations. Inequality is nevertheless correlated over time. This correlation is higher when the asset return is higher (i.e., less negative).

Next we examine the situation when wealth yields a positive return but is in finite supply. Land is an example of such asset. Manpower can also, in principle, be accumulated via indenture contracts. We find that, in this case, persistent inequality naturally arises if one agent is more thrifty than the other. Persistent inequality may also arise even if agents have identical savings functions provided their savings rate increases with wealth. In this case multiple equilibria may obtain, especially if the return to the asset is high or the asset stock is large. Initial conditions or early realizations of income select which equilibrium the economy gets locked into. Societies might seek to prevent this kind of polarization by closing down markets in such assets. One possible example is the prohibition of land sales that prevails over most of the countryside in sub-Saharan Africa (e.g. Atwood 1990, Platteau 1992). Another example is slavery, which is now prohibited everywhere but was legal in many places a couple hundred years ago. Many European immigrants, for instance, financed their voyage to America through indenture contracts.

We then return to risk sharing in the presence of assets. With perfect risk sharing, welfare inequality is constant across time. But, as is well known, much indeterminacy arises in the definition (and packaging) of assets. For this reason, we focus on the accumulation of net wealth and ignore credit. We find that for continued participation to mutual insurance to be voluntary, welfare inequality must be consistent with agents' assets and incomes. A redistribution of assets may thus be necessary to support a particular level of inequality. This implies that asset inequality must remain 'close' to welfare inequality.

In the last section, we introduce imperfect commitment. The end result is a hybrid situation half-way between the risk sharing model and the pure accumulation model. One interesting result is that, if risk aversion is high for poor agents but low for rich ones, risk sharing with imperfect commitment is most likely to take the form of patronage. We find that, with imperfect commitment, patronage on average takes away from the poor. In this case, risk sharing becomes a factor of polarization as it makes inequality more likely and more persistent.[1] Patronage does, however, protects the poor from starvation so that for the poor it is preferable to less wealth inequality but no risk sharing. This might explain why patronage relations tend to arise endogenously in a variety of contexts, even after asset redistribution (de Janvry, Sadoulet and

[1] Here we do not use the term polarization to describe the distribution of income at a given point in time, as in Estaban and Ray (1994) and Zhang and Kanbur (2001), but as a differentiation process whereby persistent inequality endogenously arises among otherwise equal agents.

Platteau 2001).

1 The Economy

We consider an infinitely-lived, two-agent economy. Throughout the paper, we use the superscript $i \in \{1,2\}$ to denote the agent and the subscript t to denote the time period. Aggregate variables appear without i superscript. Utility is written $U_i(c_t^i)$. For simplicity, we assume that the common discount factor is unity. Each agent i derives a random stream of income $\{y_t^i\}$ on a finite support $(0, \bar{y}]$ with $\bar{y} < \infty$. We also assume that the probability of zero income is 0. Aggregate income y_t is defined $y_t = \sum y_t^i$.

Each agent is endowed with a vector of assets. Some of these assets are marketable, such as grain; others cannot be traded, such as entrepreneurship. The value of marketable assets is called wealth and denoted W_t^i. Wealth can be accumulated. In contrast, non-traded assets denoted Z^i cannot be accumulated and are constant over time. The institutional framework determines which assets are marketable and which are not. The prohibition of indenture contracts, for instance, means that manpower is not a marketable asset – although its product, work, can be traded. Wealth yields a return γ which, for simplicity, we assume constant over time and individuals. This return can be positive or negative. When $\gamma > 0$, we say that wealth is productive; when $\gamma \leq 0$, we say that wealth is storable but unproductive. Examples of productive wealth include land, manpower, and capital. Examples of storable but unproductive wealth include grain, water, and minerals.[2] Income is a function of marketable and non-marketable

[2]Storable wealth can, over a short period, yield a positive return. But, unlike productive assets, it has to be destroyed in order to produce something else.

assets:

$$y_t^i = \omega^i(Z^i, \theta_t) + \gamma W_t^i \tag{1}$$

where θ_t denotes the state of nature at time t. Income is random through the dependence of function $\omega^i(.)$ on θ_t. We assume here that θ_t is uncorrelated over time. In the remainder of this paper, we refer to $\omega^i(Z^i, \theta_t)$ as labor income and to γW_t^i as wealth income. Cash-in-hand X_t^i is defined as $X_t^i \equiv y_t^i + W_t^i$.

The object of the paper is to investigate how wealth accumulation and risk sharing affect inequality. We distinguish five concepts of inequality: in marginal utility, consumption, income, cash-in-hand, and wealth. Since there are only two agents, inequality can be represented as the ratio between the two agents. Inequality ratios are written as the letter N_t^z where z denotes the variable over which the ratio is computed. For instance, inequality in income is written:

$$N_t^y \equiv y_t^2 / y_t^1 \tag{2}$$

Similarly, inequality in consumption, cash-in-hand, and wealth are written $N_t^c \equiv c_t^2/c_t^1$, $N_t^x \equiv X_t^2/X_t^1$, and $N_t^w \equiv W_t^2/W_t^1$. For reasons that will become appararent when we discuss risk sharing, it is also useful to define a measure of welfare inequality as the ration of marginal utility:

$$N_t^u \equiv U_1'(c_t^1)/U_2'(c_t^2) \tag{3}$$

If $U(c) = \log(c)$, then $N_t^u = N_t^c$. In general N_t^c tends to track N_t^u and can thus be thought of as a money-metric measure of inequality in instantaneous welfare.[3] While four of the inequality measures are unproblematic, N_t^w is not defined when $W_t^1 = 0$.[4] This is taken into account in subsequent sections.

The purpose of this paper is to characterize what long-term inequality looks like and how it evolves over time. We also seek to relate the different measures of inequality to each other. As is clear from the notation, inequality measures vary over time and with the state of nature. Much of this paper is thus concerned with the probability distribution of inequality measures. We focus primarily on long-term – or steady state – inequality and thus seek to uncover the asymptotic (steady state) unconditional distribution of inequality measures. To reflect this fact, steady state inequality measures written without time subscript. For instance, $\Pr[N^y]$ denotes the probability distribution of steady state income distribution. The unconditional expected value of N^y is denoted $N_e^y \equiv E[N^y]$, and similarly for the other inequality measures.

We also wish to study the extent to which inequality endures over time. In particular we focus on the steady state correlation between N_t^y and N_{t+1}^y or, more precisely, on the relationship between N_t^y and $E[N_{t+1}^y|N_t^y]$. Suppose that

$$E[N_{t+1}^y|N_t^y] = \rho N_t^y + \delta N^y \tag{4}$$

If, for instance, $\delta = 0$ and $\rho = 1$,

$$E[N_{t+1}^y|N_t^y] = N_t^y \tag{5}$$

[3] To be a satisfactory measure of inequality, N_t^u needs to be normalized in some way. One possibility is to set $\frac{U_1'(c)}{U_2'(c)} = 1$ for some level of consumption c, e.g., average consumption. In practice, N_t^u is most useful when both agents have the same utility function.

[4] To ensure that income inequality is always defined, we assume that $\Pr[\omega^i(Z^i, \theta_t) = 0] = 0$. This also ensures that N_t^x is always defined. We also assume that $U(0) = -\infty$ so that zero consumption is never optimal. Since income is never 0, positive consumption is always feasible and N_t^c is always defined. Utility is assumed continuously differentiable so that N_t^u is always defined as well. In contrast, N_t^w need not be defined if $W_t^1 = 0$.

inequality follows a random walk: inequality today is likely to persist tomorrow. If, in contrast, $\rho = 0$ and $\delta = 1$, we have

$$E[N_{t+1}^y | N_t^y] = N^y \tag{6}$$

which implies that inequality is short-lived. In general, the closer to 1 ρ is, the more persistent inequality is.

We proceed as follows. We first examine a number of simple, limit cases. Then we introduce complications. Limited commitment is discussed in section 5. To characterize the distribution of inequality measures, we rely on a combination of algebraic and simulation methods. In all cases, we seek to relate different types of inequality with each other.

2 No Marketable Assets

We begin by assuming that marketable assets are absent and thus that accumulation is not feasible: $W_t^i = 0$ for all i and t.[5] Income is labor income only: $y_t^i = \omega^i(Z^i, \theta_t)$. It follows that $\Pr[N_t^y] = \Pr[N^y]$: the unconditional probability distribution of income inequality is at its steady state in all t; there is no transition period. It also follows that $N_t^x = N_t^y$ since $y_t^i = X_t^i$. The unconditional expectation of income inequality simply is:

$$E[N^y] = E\left[\frac{\omega^2(Z^2, \theta_t)}{\omega^1(Z^1, \theta_t)}\right] \tag{7}$$

This means that a more talented individual has a higher income on average. Since income is a function of never changing assets, there is no social mobility in incomes. We also see that, since we have assumed that the θ_t shocks are uncorrelated over time, conditional and unconditional

[5]This means that N_t^w is not defined. Consequently, it is ignored in this section.

income inequality are equal:

$$E[N_t^y | N_{t-1}^y] = E[N^y] \tag{8}$$

Any deviation from expected income inequality $E[N^y]$ is short-lived.

Inequality in consumption depends on whether income risk is shared or not. If risk sharing is not possible, then $c_t^i = y_t^i = \omega(Z^i, \theta_t)$. In this case, $N_t^c = N_t^y$ and shares all the properties of N_t^y. If, in addition, utility has the form $U(c) = \log c$, we also have $N_t^u = N_t^y$. In a world with no accumulable assets, no risk sharing, and relative risk aversion close to unity (i.e., log utility), income inequality summarizes all there is to know about welfare inequality.

Suppose, in contrast, mutual insurance contracts are perfectly and costlessly enforceable. Perfect competition thus reaches an efficient allocation. Pareto efficiency in the sharing of risk dictates that

$$\frac{U_1'(c_t^1)}{U_2'(c_t^2)} = \frac{U_1'(c_{t'}^1)}{U_2'(c_{t'}^2)} \equiv k \text{ for all } t \text{ and } t' \tag{9}$$

where k is a constant equal to the ratio of welfare weights (e.g. Mace 1991, Cochrane 1991, Altonji, Hayashi and Kotlikoff 1992). Since $\frac{U_1'(c_t^1)}{U_2'(c_t^2)} \equiv N_t^u$, it follows that welfare inequality N_t^u is constant over time with $N^u = k$. This is true even though N_t^y varies from period to period: welfare inequality is divorced from income inequality.

With perfect pooling of risk, individual consumption is only a function of aggregate income. This implies that, if aggregate income y_t is constant over time, consumption is constant as well. If the economy is subject to collective shocks, inequality in consumption varies in a deterministic fashion with aggregate income. But the distribution of welfare is unchanged, i.e., there exist monotonic functions of individual consumptions c_t^1 and c_t^2 (the marginal utility functions) such that the ratio of these functions is constant across time and states of nature. Thus, even though individual consumption and welfare might change over time (as aggregate resources expand or dwindle), inequality remains constant in some fundamental sense.

When agents have identical preferences with constant relative risk aversion, N_t^c remains constant over time even in the presence of aggregate shocks.[6] Consumption inequality $N_t^c = (N^u)^{\frac{1}{R}}$, with $N_t^c = N^u$ if $U(c) = \log(c)$. For other utility functions, N_t^c might change slightly over time and states of nature. But within this framework, N_t^c can be thought of as an approximation to N^u, which is the relevant welfare measure in our economy.

Although hardly novel, this result implies that perfect risk sharing, as could be achieved for instance via a perfect insurance market, would freeze welfare inequality to a permanent level. This might be socially acceptable if this level is relatively egalitarian. There are many Pareto efficient allocations in our stylized economy. But nothing guarantees that the Pareto efficient allocation selected by a competitive market equilibrium would be socially acceptable. This probably explains why there is a lot of public intervention in social insurance and why many insurance schemes pursue redistribution objectives. Some are even worded not as insurance but as anti-poverty programs. Because insurance also determines welfare inequality, it likely to be combined with redistribution so that it does not itself become the source of inequality.

This raises the issue of how much redistribution can be sustained if participation to risk sharing is voluntary. Not all risk sharing arrangements can be sustained in a decentralized market. The main constraint imposed by a decentralized market equilibrium is ex ante voluntary participation: agents must find it in their interest to participate to a risk sharing arrangement/to purchase an insurance contract. If this condition is not satisfied, agents would be better off by

[6] We have $U(c) = \frac{c^{1-R}}{1-R}$. Differentiating with respect to c and replacing in the definition of welfare inequality, we obtain $N_t^u = \left(\frac{c^2}{c^1}\right)^R$. When $U(c) = \log(c)$, $N_t^u = \frac{c^2}{c^1}$. Since $N_t^c \equiv \frac{c^2}{c^1}$, we see that whenever N_t^u is constant, so is N_t^c.

consuming their individual income.[7] This simple observation implies that agents who could guarantee themselves a higher utility in autarchy must in general have a higher level of utility with risk sharing.

Risk sharing does not preclude redistribution. Suppose we wish to achieve equality in expected consumption, i.e., we wish to attain $E[N^c] = 1$. If $E[N^y] = E[N^c]$, attaining equality is unproblematic since reversion to autarky – $c_t^i = y_t^i$ – would result in egality. We can thus define redistribution as the difference between $E[N^y]$ and $E[N^c]$. With taxation/forced participation in mutual insurance, any level of N^u is sustainable. This not true of decentralizable arrangements. Voluntary participation to any mutual insurance contract puts bounds on how far N^u (and thus $E[N^c]$) can stray from $E[N^y]$ and thus on the level of equality that can be achieved.

For instance, if all agents have the same utility function, agents with higher expected earnings must be ensured higher consumption. This puts a bound on $E[N^y] - E[N^c]$. Consumption inequality is thus, in general, a function of unalienable assets Z^i. The only case in which voluntary participation might induce agents with higher expected earnings to accept less consumption than agents with lower expected earnings is if the former are much more risk averse than the latter. By the same reasoning, we see that the more risk averse agents are, the more they are willing to accept a redistribution of average consumption in exchange for better insurance. We revisit these issues in Section 5.

These results are summarized in the following proposition:

Proposition 1 *In the absence of accumulable assets:*

(1) The distribution of income inequality is at its steady state in all periods: $\Pr[N_t^y] = \Pr[N^y]$ *for all* t*.*

(2) If income shocks are uncorrelated, conditinal expected inequality in income is constant:

[7] The allocation must be in the core (Hildenbrand 1974).

$E[N_t^y|N_{t-1}^y] = E[N_t^y] = E[N^y]$.

Proposition 2 *Without risk sharing:*

(1) Inequality in consumption is equal to inequality in income: $N_t^c = N_t^y$.

(2) Expected long-term inequality in consumption is constant: $E[N^c] = E[N^y]$

(3) With log *utility, inequality in welfare is equal to inequality in income:* $E[N^u] = E[N^y]$.

Proposition 3 *With perfect risk sharing:*

(1) $N_t^u = N^u$*: welfare inequality is constant; there is no social mobility.*

(2) $N_t^c = G(y_t)$ *– inequality in consumption is a deterministic function of aggregate income.*

(3) $N_t^c = N^c$ *if* $y_t = y$ *or if* $U_i(c) = \log(c)$ *of if* $U(c) = \frac{c^{1-R}}{1-R}$ *for* $i = \{1, 2\}$.

Proposition 4 *With voluntary participation in risk sharing:*

(1) $E[N^c]$ *is a non-decreasing function of* $E[N^y]$ *if all agents share the same utility function (up to an affine transformation).*

(2) The more risk averse agents are, the more redistribution $E[N^y] - E[N^c]$ *can voluntarily be achieved, and the closer* $E[N^c]$ *can be brought to unity (equality in expected ratio of consumption).*

3 Accumulation with No Risk Sharing

We now allow for wealth accumulation by assuming the existence of marketable assets. In this section, we focus on the case where explicit risk sharing is not possible – perhaps because a market for insurance does not exist. Agents potentially have two motives for saving: precautionary saving and growth in consumption – prudence and patience (Kimball 1990). If the return on marketable assets is negative, prudence is only motive for saving (Deaton 1991). Throughout we assume that there is no borrowing.

3.1 Unbounded accumulation

We first examine the case where $\gamma > 0$. Provide agents save enough, they can accumulate indefinitely. At the limit, their wealth becomes so large that it is entirely dominated by the return to wealth γW_t^i which, by assumption, is non-stochastic. Consequently, as wealth becomes large, income shocks have less and less effect on consumption. This implies that inequality is asymptotically deterministic. We now show this more rigorously

We first must establish conditions under which people save enough for wealth to grow indefinitely. We need $E[W_{t+1}^i|W_t^i] > W_t^i$. Suppose, for instance, that agents save a constant proportion s of their income $y_t^i = \omega^i + \gamma W_t^i$. We have:

$$\begin{aligned} E[W_{t+1}^i|W_t^i] &= E[s(\omega^i + \gamma W_t^i) + W_t^i] \\ &= sE[\omega^i] + s\gamma W_t^i + W_t^i > W_t^i \end{aligned} \tag{10}$$

which is satisfied for any $s > 0$. If each agent chooses its level of saving by maximizing its expected discounted utility subject to a budget constraint, we obtain an Euler equation of the form:

$$U_i'(c_t^i) = (1+\gamma)E[U_i'(c_{t+1}^i)] \tag{11}$$

where the intertemporal discount factor drops out since, by assumptions, agents do not discount the future.[8] We see from the above equation that the marginal utility of income must fall, on average, over time – which implies that consumption must rise and thus that assets must be accumulated (Deaton 1991). This establishes that when $\gamma > 0$ and agents do not discount the

[8] If agents discount the future, indefinite accumulation obtains if the return on the asset is higher than the rate of intertemporal preference.

future, indefinite accumulation obtains.

Having established that both agents accumulate, we now turn to the characterization of long-term inequality in wealth N_t^w. Since agents always hold positive assets, at least after a while, the distribution of N_t^w is well defined. [9] We note that, as both agents become wealthier, labor income becomes a vanishingly small prorportion of their cash-in-hand. Savings thus becomes almost deterministic and the path of individual wealth converges to a deterministic path. This is illustrated in Figure 1 which plots three possible paths of N_t^w for a fully symmetrical model with a constant saving rate. The only difference between the paths is the sequence of labor income shocks. To facilitate interpreation, we graph wealth shares $\frac{W_t^2}{W_t^1+W_t^2}$ instead of N_t^w. We see that all paths eventually converge to a single share.

How inequality evolves over time depends on the relative savings rate of both agents. If one agent saves faster than the other, wealth inequality N_t^w diverges permanently, either tending to infinity (if agent 2 saves more) or to 0 (if agent 1 saves more). If agents save at the same rate, their wealth asymptotically grows at the same rate: the ratio of their wealths tends to a constant. Wealth inequality tends to a constant (see Figure 1). This is true even if their labor income processes are different because, when wealth is large enough, labor income does not matter anymore. This finding is reminiscent of Polya urn processes (Arthur, Ermoliev and Kaniovski 1994). Wealth nequality N_t^w follows a random walk with smaller and smaller increments. Initial realizations of income determine the speed with which wealth is initially accumulated, and thus the process of initial differentiation. As time passes, however, changes in wealth inequality (that is, in the ratio of wealth levels) become smaller and smaller as income – and thus savings – become progressively dominated by wealth. Inequality in wealth converges

[9] Starting from zero assets, it is possible that low initial realization of income trigger no accumulation. We ignore these complications here and focus on the long-run distribution of N_t^w only.

to an asymptotic value N^w. This is illustrated in Figure 1.[10]

This implies that the conditional distribution of N^w_{t+1} gets more and more narrowly defined around N^w_t as time passes. There is 'lock-in': changes in inequality becomes smaller and smaller over time. Even though both agents might have had the same economic opportunities ex ante, over time they diverge so that, with time, their economic prospects become highly contrasted. At the limit, relative prosperity becomes permanent. Social mobility – that is, the chance of changing one's rank – disappears over time.

Which value N^w_t converges to is path dependent: if agent 1 is lucky early on, he gets a headstart, and vice versa. Ex ante, there are many possible asymptotic values N^w can take – essentially any value between zero and infinity. Are all these values equally likely? In gneral the answer is no. To characterize the ex ante (unconditional) distribution of N^w we need to impose more structure on the model. The distribution of N^w is also complicated by the fact that it is a ratio. It is easier to characterize the distribution of the share of total wealth $S^w_t \equiv \frac{W^1_t}{W^1_t+W^2_t}$. By construction, $S^w_t \in [0, 1]$. Since N^w_t converges to a single value, so does S^w_t.

With identical non-traded assets $Z^1 = Z^2$, independent income shocks, identical initial wealth $W^1 = W^2$, and identical utility functions $U^1(.) = U^2(.)$, both agents are ex ante equivalent and their economic opportunities are the same. If agents have equal initial endowments and identical preferences, it should be possible to show that $E[S^w] = 0.5$. The realized value of S^w, however, can take any value between 0 and 1. In the symmetric case, agents have equal opportunities ex ante but once inequality sets in it gets reinforced over time.

Having characterized the distribution of wealth inequality, we now turn to other inequality

[10] Figure 1 is constructed using a convex savings function (lower savings rate at low values of cash-in-hand). Both agents have the same savings function. Labor incomes are independent and uniformly distributed. Three paths are generated corresponding to three sets of labor income realizations. With a constant savings rate, the distribution of convergence shares S^w is more concentrated around 0.5. This is because lucky labor income early on provides less of a headstart in wealth accumulation.

measures. By definition, we have $N_t^x = \frac{\omega_t^2+(1+\gamma)W_t^2}{\omega_t^1+(1+\gamma)W_t^1}$ and $N_t^y = \frac{\omega_t^2+\gamma W_t^2}{\omega_t^1+\gamma W_t^1}$. As W_t^1 and W_t^2 tend to infinity over time, both ratios tend to N_t^w. We thus have $\lim_{t\to\infty} ||N_t^x - N_t^w|| = \lim_{t\to\infty} ||N_t^y - N_t^w|| = 0$: inequality in cash-in-hand and in income tends to inequality in wealth. Of course, since wealth increases without bounds, inequality is not synonymous with poverty: at the limit, both agents are infinitely wealthy.

Given that there is no risk sharing, each agent consumes exclusively from its own cash-in-hand $X_t^i \equiv y_t^i + W_t^i$. Let $c^i(X_t^i)$ denote the consumption function of individual i. We have $N_t^c = \frac{c^2(X_t^2)}{c^1(X_t^1)}$. Suppose that both consumption functions are asymptotically linear, e.g., that $\lim_{X\to\infty} c^i(X) = k^i X$. In this case, $\lim_{t\to\infty} ||N_t^c - \frac{k^2}{k^1} N_t^x|| = 0$: inequality in consumption tends to a multiple of wealth inequality.

More generally, there exist a function of wealth inequality $f^c(N_t^x)$ to which consumption inequality converges: $\lim_{t\to\infty} ||N_t^c - f(N_t^x)|| = 0$.[11] This implies that, at the limit, the ratio of consumption is a function of the ratio of wealth. The same reasoning applies to welfare inequality, i..e., there exist a function $f^u(N_t^x)$ such that $\lim_{t\to\infty} ||N_t^c - f(N_t^x)|| = 0$. Inequality in wealth leads to inequality in consumption and in welfare. Other conclusions apply as well: path dependence; lock-in; ex ante unpredictability of long-term inequality. When both agents asymptotically save at the same rate, all inequality measures converge to a single number.

These results can be summarized as follows:

Proposition 5 *With unbounded accumulation ($\gamma > 0$), we have:*

(1) $\lim_{t\to\infty} ||N_t^x - N_t^w|| = \lim_{t\to\infty} ||N_t^y - N_t^w|| = 0$

(2) There exist a function $f^c(N_t^x)$ such that $\lim_{t\to\infty} ||N_t^c - f(N_t^x)|| = 0$.

(3) There exist a function $f^u(N_t^x)$ such that $\lim_{t\to\infty} ||N_t^u - f(N_t^x)|| = 0$.

[11] To see why, note that in the long run, the path of inequality depends less and less on income, i.e., converges to a deterministic path. Consider the deterministic path of cash-in-hand inequality. Along this deterministic path, to each inequality ratio corresponds a ratio of consumption.

(4) $\lim_{t\to\infty} Var[N^w_{t+1}|N^w_t] = 0$, $\lim_{t\to\infty} Var[N^x_{t+1}|N^x_t] = 0$, $\lim_{t\to\infty} Var[N^y_{t+1}|N^y_t] = 0$, $\lim_{t\to\infty} Var[N^c_{t+1}|N^c_t] = 0$, $\lim_{t\to\infty} Var[N^u_{t+1}|N^u_t] = 0$.

(5) Ex ante, N^w is a random variable $\in (0, \infty)$. By extension, the same applies to other inequality measures.

(6) If agents asymptotically save at the same rate, all inequality measures converge to a single number $N^w \in (0, \infty)$.

(7) If agents asymptotically save at a different rate, all inequality measures converge either to 0 (if agent 2 saves less than 1) or to ∞ (if agent 2 saves more than 1).

3.2 Bounded accumulation with unproductive assets

The situation is very different if $\gamma < 0$. In this case, wealth accumulation is costly. Agents engage in it only to insure themselves against income shocks – the precautionary savings motive. This case is more relevant for poor or pre-industrial societies where opportunities to invest are few and returns to assets are low.

Accumulation in such models has been analyzed elsewhere (e.g. Deaton 1991, Zeldes 1989, Deaton 1990, Deaton 1992). Assets are known to follow a renewal process. As long as accumulated wealth is positive, it follows a random walk. For sufficiently large negative shocks, agents deplete all their wealth, at which point the process is 'renewed', that is, it forgets the past and starts anew. How much accumulation takes place depends on the marginal propensity to save (MPS) out of cash-in-hand. Kimball (1990) has shown how the MPS is related to the third derivative of the utility function via what he calls 'prudence', defined as $\phi \equiv -\frac{U'''}{U''}$. Other things being equal, more prudent agents – higher ϕ – save more.

We seek to characterize the distribution of cash-in-hand, income, and consumption. In this model as in the previous one, all inequality measures are closely related. This is because, in the absence of risk sharing, an agent's consumption depends exclusively on individual cash-in-hand X_t^i. Inequality in cash-in-hand thus translate into inequality in consumption and welfare. However, because agents use wealth to smooth consumption, inequality in consumption is typically much less than inequality in wealth (e.g. Paxson 1992, Townsend 1994).[12]

We begin by noting that, since income is bounded, wealth is also bounded. This is because $\gamma < 0$. To see why, let $\bar{\omega}^i$ denote the maximum level of labor income. The maximum rate at which wealth can accumulate is when agents save all and income is always at its maximum: $W_{t+1}^i = \bar{\omega}^i + (1+\gamma)W_t^i$. Since $\gamma < 0$, we see that wealth cannot exceed $-\frac{\bar{\omega}^i}{\gamma}$.

Inequality in cash-in-hand N_t^x in general follows an AR1 stochastic process between 0 and ∞. This is because income realizations induce random changes in the wealth of both agents. If an agent gets a temporarily high cash-in-hand level, he saves and his wealth goes up. The opposite is true if realized cash-in-hand is low. Cash-in-hand inequality is thus a transient phenomenon: since each agents' wealth is bounded from above and from below, all agents eventually run out of funds in finite time. This implies that no agent can indefinitely stay ahead of the other. By this reasoning, the distribution of N_t^x is stationary. By extension, all inequality measures have stationary distributions. Unlike in the case with infinite accumulation, there is no lock-in here.

Can we be more precise and characterize the distribution of some of the inequality measures? Consider consumption inequality. If agents were perfectly able to smooth consumption thanks to precautionary saving, consumption would be constant as well as consumption inequality. This outcome, however, is generally not achievable. The next best outcome is if preferences are quadratic and households face no credit constraint. In this case, certainty equivalence applies and agents respond to income shocks by consuming the annuity value of their total wealth (e.g.

[12] Because wealth can fall to 0 for both agents, the distribution of N_t^w is not strictly speaking defined. For this reason, we ignore it here and focus instead on inequality in cash-in-hand. The latter is always defined since income can never be 0.

Hall 1978, Zeldes 1989).

In our case, the annuity value of a (finite) income shock is 0 since agents' rate of time preference is 0.[13] The annuity value of wealth is also 0 since, with $\gamma < 0$, wealth is expected to be eliminated in finite time. We therefore get the standard permanent income result: when the rate of time preference is 0, agents consume their average income $E[\omega^i(Z_t^i, \theta_t)]$ which is also the annuity value of wealth. In the more general case, Zeldes (1989) has shown that, when wealth is sufficiently large, consumption tends to the certainty equivalent level of consumption for a large enough level of wealth. This is true for any (reasonable) utility function and holds even if agents cannot incur a negative net worth. Applying these ideas to our economy, we see that when the wealth of both agents is large, N_t^c tends to $\frac{E[\omega^2(Z_t^2,\theta_t)]}{E[\omega^1(Z_t^1,\theta_t)]}$: consumption inequality is a function of inequality in expected labor income.

In general, N_t^c is not constant. When cash-in-hand is large for both agents, certainty equivalence approximately holds and fluctuations in consumption – and thus in consumption inequality – are quite small. As Zeldes (1989) and Carroll (1992) have shown, however, certainty equivalence is violated when wealth is small and agents cannot borrow (as we assume here). In this case, a shortfall in cash-in-hand results in a drop in consumption – and a temporary increase in consumption inequality. Since these shortfalls can affect both agents, we would expect consumption inequality to fluctuates around $\frac{E[\omega^2(Z_t^2,\theta_t)]}{E[\omega^1(Z_t^1,\theta_t)]}$ and to be correlated over time (through the dependence of current consumption on accumulated wealth). The precise distribution of consumption inequality, however, is difficult to characterize without imposing more structure on the model. Figure 2 illustrates how inequality in labor income, cash-in-hand, and consumption evolve over time.[14] We see that, thanks to precautionary saving, inequality in consumption is

[13] Ignoring the case where labor income follows a random walk. Here the distribution of labor income is assumed stationary.

[14] As in Figure 1, the savings function is convex (lower savings rate at low levels of cash-in-hand). This is meant to reproduce the shape Zeldes' consumption function. Both agents have the same savings function. Labor income

always less than inequality in cash-in-hand or in labor income.

Regarding correlation in inequality over time, we first note that, in the absence of correlation in labor income, correlation in cash-in-hand inequality depends on how much agents accumulate. If they accumulate a lot, the correlation is high; if they accumulate little, the correlation is low. Consequently any factor that favors accumulation also favors the correlation of inequality over time. This means, for instance, that inequality is more persistent when the propensity to save is higher. By the same token, the correlation between N_t^x and N_{t+1}^x depends on γ. If γ is very negative, wealth dissipates rapidly and $Cor[N_t^x, N_{t+1}^x]$ is low: a wealth advantage does not last long. If γ is close to 0, the reverse is true. At the same time, if both agents accumulate more, they are better able to withstand shocks, and consumption inequality fluctuate less on average.

Second, we note that the more cash-in-hand is correlated over time, the more consumption is correlated as well. This is because consumption is a monotonically increasing function of cash-in-hand: $c_t^i = c^i(X_t^i)$ with $c'(X) > 0$ for all X. As a result, a high correlation between $\frac{X_t^2}{X_t^1}$ and $\frac{X_{t+1}^2}{X_{t+1}^1}$ results in a high correlation between $\frac{c^2(X_t^2)}{c^1(X_t^1)}$ and $\frac{c^2(X_{t+1}^2)}{c^1(X_{t+1}^1)}$. Since welfare inequality depends on consumption, the same observation applies to N_t^u.

Combining the two above observations, we note that factors that raise accumulation reduce the variance of consumption and welfare inequality but raise persistence in inequality. This is illustrated in Figure 3: when γ is lower – and accumulation less – consumption inequality is more variable but it is less correlated over time.[15]

Results can be summarized as follows:

Proposition 6 *With bounded accumulation ($\gamma < 0$) and no discounting, we have:*

(1) All inequality measures are stationary Markov processes.

(2) All inequality measures move together.

is independently distributed.

[15] Settings for Figure 3 are identical to those of Figure 2, except that the value of γ is varied.

(3) N_t^c *fluctuates around* $\frac{E[\omega^2(Z_t^2,\theta_t)]}{E[\omega^1(Z_t^1,\theta_t)]}$.

(4) $Var[N_t^x]$, $Var[N_t^y]$, $Var[N_t^c]$ *and* $Var[N_t^u]$ *are non-decreasing functions of* γ.

(5) $Cor[N_t^x, N_{t+1}^x]$, $Cor[N_t^y, N_{t+1}^y]$, $Cor[N_t^c, N_{t+1}^c]$ *and* $Cor[N_t^u, N_{t+1}^u]$ *are increasing functions of* γ.

Part (1) means that inequality is a transient phenomenon. Part (2) means that inequality in non-traded assets Z^i has a permanent effect on inequality. Parts (3) and (4) mean that easier accumulation reduces fluctuations in inequality but raises persistence in inequality. There is a trade-off between social mobility and vulnerability.

3.3 Accumulation with assets in fixed supply

In the previous subsection, agents accumulate assets independently from each other. Grain storage is an example of wealth that fits nicely in this model. Other types of wealth, such as land, do not fit as nicely. In this section, we consider accumulable assets that are in fixed supply, i.e., such that $W_t^1 + W_t^2 = \bar{W}$. Land is one possible example, provided there exists a sales market for land. Another possible example is manpower in societies that allow voluntary indenture contracts (Srinivasan 1989). In general, we assume that $\gamma \geq 0$, that is, that the asset is productive. The case where $\gamma = 0$ corresponds to the island economy discussed in Sargent (1987), chapter 3.

We begin by focusing on asset inequality. Other inequality measures ultimately depend on how the fixed asset is shared. When the asset is in fixed supply, we can no longer ignore relative prices: the price p_t at which the asset is turned into consumption varies to clear the asset market. This singularly complicates the model because fluctuations in asset prices introduce uncertainty in real asset returns and generate a speculative motive for holding wealth. In the discussion that follows, we mostly ignore these complications.

Saving is now a function of cash-in-hand and asset price: $W_{t+1}^i = s^i(\omega_t^i, W_t^i, p_t)$. To simplify the presentation, we assume that the savings function is separable so that:

$$W_{t+1}^i = \frac{s^i(\omega_t^i, W_t^i)}{g(p_t)} \tag{12}$$

with $\frac{\partial s^i}{\partial \omega^i} \geq 0$ and $\frac{\partial s^i}{\partial W^i} \geq 0$. With this assumption, the market clearing condition $W_t^1 + W_t^2 = \bar{W}$ yields the following expression for asset inequality (expressed here as a share):

$$\begin{aligned} S_{t+1}^w &\equiv \frac{W_{t+1}^2}{W_{t+1}^1 + W_{t+1}^2} \\ &= \frac{s^2(\omega_t^2, W_t^2)}{s^1(\omega_t^1, W_t^1) + s^2(\omega_t^2, W_t^2)} \\ &= \frac{s^2(\omega_t^2, S_t^w \bar{W})}{s^1(\omega_t^1, (1-S_t^w)\bar{W}) + s^2(\omega_t^2, S_t^w \bar{W})} \end{aligned} \tag{13}$$

For comparison with earlier sections, $N_t^w = S_t^w/(1-S_t^w)$. The advantage of the above formulation is that the asset price has been factored out. What the above expression shows is that future asset inequality depends on current inequality: when S_t^w is large, saving by agent 2 tends to be large relative to agent 1. This is because agents save from cash-in-hand and large wealth raises cash-in-hand. This process may lead to self-reinforcing inequality – what we call polarization. The question is under what conditions polarization arises.[16]

This can be answered by examining the law of motion of S_t^w given above. The analysis is complicated by the fact that the ω_t^i's are random variables so that the law of motion of S_t^w is stochastic. We focus on the relationship between S_t^w and $E[S_{t+1}^w]$. We note that

$$E[S_{t+1}^w] \simeq \frac{s^2(E[\omega_t^2], S_t^w \bar{W})}{s^1(E[\omega_t^1], (1-S_t^w)\bar{W}) + s^2(E[\omega_t^2], S_t^w \bar{W})} \tag{14}$$

[16] In this model as in Banerjee and Newman (1991), the rich have a less risky income stream than the poor. But in contrast to their work, we do not consider risk-taking as an entrepreneurial activity.

Consequently the relationship between S_t^w and $E[S_{t+1}^w]$ can studied by studying the above difference equation. Figure 4 illustrates what it looks like depending on the curvature of the savings function.[17] The two agents are assumed to have the same savings function. Two curves are shown. The one with a nearly linear (low curvature) savings function intersects the 45 degree line at 0.5 from above. This means that if the fixed asset is shared equally at time t, it tends to be shared equally at time $t+1$ as well. Deviation from equality tend to correct themselves over time: at low S_t^w leads to a higher $E[S_{t+1}^w]$, and vice versa. Income shocks push S_{t+1}^w up or down a bit, but inequality always gravitates around $S_t^w = 0.5$. There is a single steady state distribution of assets. Polarization does not arise. If agents have different savings rate, polarization arises: the thrifty agent eventually gets most of the asset. This finding is reminiscent of what we found with unbounded accumulation: the shave of total wealth owned by the thrifty agent rises over time, although in this case it does not converge to 1 unless the savings rate of the other agent falls to 0 for low enough wealth. However, there is still a single steady state distribution.

The story is different in the case of the high curvature savings function. Here, the curve intersects the 45 degree line three times, corresponding to three zeros of the difference equation.[18] The middle intersection is unstable because it is cut from below. This means that, starting from equal distribution of assets, small differences in income realizations induce differenciation: one agent is able to accumulate more than the other. There are therefore two steady states distributions of assets: one in which agent 1 owns most of the asset, and one in which agent 2 in the wealthy agent. There is polarization even though agents are symmetrical, i.e., have the same preferences and the same expected labor income.

[17] To compute this Figure, we use a savings function of the form $S(\omega, W) = e^{\alpha(E[\omega]+(1+\gamma)W)} - 1$. Parameter α determines the curvature of the savings function: the higher it is, the more convex savings is. Values of α are respectively 0.02 and 0.2 for the low curvature and high curvature savings functions reported in the Figure.

[18] This curve resembles Azariadis (1996)'s analysis of country-level poverty traps (see his Figure 2c). Our interpretation is quite different, however. In Azariadis, poverty traps exist for each country independently from any interaction between them. Here, the trap arises from the interaction between the two agents.

Multiple steady state distributions are illustrated in Figure 5.[19] Two alternative paths of S_t^w are shown. The only difference between the two is the sequence of labor income shocks. Which long-term distribution of inequality is chosen depends on initial realizations of labor income. When agent 2 is lucky early on, he is able to accumulate more of the fixed asset and to gain a permanent advantage. The reverse is true if agent 1 enjoys high income draws at the beginning. There is path dependence. These results are similar to those reported by Carter and Zimmerman (2000) for a more complicated simulated economy. Multiple equilibria arise even if labor incomes are strongly correlated across agents, although this correlation may delay the polarization process starting from an egalitarian distribution.

The corollary of path dependence is that the starting point – 'history' – matters. If, for exogenous reasons, an agent has most of the wealth to start with, multiplicity of equilibria disappears. The only steady state distribution is the one that favors this agent.

The above framework enables us to investigate the factors that favor multiple equilibria and thus polarization among otherwise identical agents. We have already seen that polarization is more likely when the savings rate increases sharply with cash-in-hand. In fact, polarization cannot arise if the savings rate is constant and identical for both agents. In this case we have:

$$\begin{aligned} E[S_{t+1}^w] &\simeq \frac{sE[\omega_t] + sp_t(1+\gamma)S_t^w\bar{W}}{sE[\omega_t] + sp_t(1+\gamma)(1-S_t^w)\bar{W} + sE[\omega_t] + sp_t(1+\gamma)S_t^w\bar{W}} \\ &= \frac{sE[\omega_t] + sp_t(1+\gamma)S_t^w\bar{W}}{2sE[\omega_t] + sp_t(1+\gamma)\bar{W}} \end{aligned} \tag{15}$$

which is linear in S_t^w. The intuition behind this results is that for multiple steady state distribution to arise, accumulation must get reinforced over time.

It can also be shown that all factors that increase wealth income relative to labor income

[19] The savings function is the same as for Figure 4. Here $\alpha = 0.04$.

favor multiple equilibria. This is true of an increase in γ or $\bar{W}$ or of a decrease in $E[\omega^i]$. The reason is that when the income from wealth is small, differences in wealth may not be sufficient to cause a large differentiation in savings rate. As a result, polarization does not arise.

Turning to the conditional (steady state) distribution of inequality, results from the previous section apply here aswell: the more remunerative wealth is, the more correlated inequality is over time. The rationale is the same.

Having clarified how wealth evolves over time, the extension to consumption and welfare is straightforward. In the absence of risk sharing, consumption is simply a function of individual cash-in-hand. Inequality in wealth results in inequality in cash-in-hand and thus in consumption and welfare. Consequently, all that has been said about polarization and multiple equilibria in asset inequality applies to consumption and welfare as well.

Our findings can be summarized as follows:

Proposition 7 *Suppose $W_t^1 + W_t^2 = \bar{W}$ and $\gamma \geq 0$. We have:*

(1) All inequality measures move together.

(2) Multiple steady state distributions of inequality can arise if the savings rate increases with wealth. This is true even if agents have identical savings function and distribution of labor income. This is true even if labor incomes are strongly correlated.

(3) When multiple steady state distribution exist, initial conditions may have a permanent effect on the distribution of inequality.

(4) Multiple steady state distributions are more likely if:

(a) the savings rate increases more rapidly with wealth;

(b) the return to wealth γ is high;

(c) total fixed wealth $\bar{W}$ is large;

(d) expected labor income $E[\omega^i]$ is small.

Although it is perilous to interpret the above results literally since they were obtained in a highly stylized model, they may provide an explanation for the apparent difficulty to durably affect land inequality. It has indeed been observed that land inequality often worsens after land reform. Using a complex simulation model, Zimmerman (1993) explains this outcome in terms of credit constraints and perverse fluctuations in land prices. The above model generalizes these insights.

In this section we have interpreted the accumulated factor as land. Other interpretations are feasible as well. If indenture contracts are allowed, then agents can sell their future labor force in exchange for immediate consumption. This has the effect of raising $\bar{W}$ and thus the likelihood of multiple equilibria and thus of polarization among otherwise identical agents. For this reason, certain societies might decide to 'close' or 'outlaw' markets for productive assets in order to prevent inequality. For such measures to be effective, however, alternative channels must exist that enable agents to smooth consumption. If such channels are inexistent, labor bonding and distress sales of land may be the next best alternative (e.g. Srinivasan 1989, Fafchamps 1999). Labor bonding contracts are revisited in the next section because indentured laborers would normally be insured against risk (i.e., they get a constant albeit low consumption) after they begin working as slaves.

4 Accumulation and Risk Sharing

We now consider wealth accumulation and risk sharing together. We begin by assuming that risk sharing contracts are perfectly and costlessly enforceable. The case with unbounded accumulation is ignored since, in the long run, both agents have infinite wealth and self-insure perfectly. We therefore consider what happens if agents can save in the form of an unproductive asset such as grain stocks. This is the Arrow-Debreu world studied for instance by Udry (1994) and ?. With complete markets, the Pareto efficient outcome can be sustained. This outcome ensures that

$$\frac{U_1'(c_t^1)}{U_2'(c_t^2)} = \frac{U_1'(c_{t'}^1)}{U_2'(c_{t'}^2)} = k \text{ for all } t \text{ and } t' \tag{16}$$

where k is a constant that depends on the ratio of welfare weights of each agent. An immediate corollary of the above is that, with assets and perfect risk sharing, welfare inequality is constant over time: $N_t^u = k$. Wealth and income distribution are inconsequential for welfare distribution.

We now turn to consumption inequality. Along the equilibrium path, individual consumption fluctuates only as a function of aggregate cash-in-hand $X_t \equiv X_t^1 + X_t^2$: if X_t is high, both agents consume a lot; if X_t is low, both agents consume little. Purely idiosyncratic fluctuations in income do not affect consumption. If the utility function takes a constant absolute risk aversion form, $N_t^c = N_t^u = k$ (e.g. Mace 1991, Cochrane 1991, Altonji et al. 1992). If the utility function has constant relative risk aversion, it is the ratio of the log of consumption that is constant. More generally, N_t^c is an increasing function of N_t^u.[20]

Next we note that if the economy is subject to collective income shocks, the precautionary savings motive applies: it is in the interest of all agents collectively to accumulate grain to protect themselves against future collective shortfalls. The rules governing the accumulation of precautionary saving are the same as those discussed earlier (e.g. Zeldes 1989, Deaton 1991, Kimball 1990), except that when they pool risk agents do not need assets to insure against idiosyncratic risk. The level of aggregate saving should, in general, be lower with risk sharing. We denote the (efficient) aggregate savings function $s(X_t)$. The higher collective risk is, the more aggregate saving we expect (provided agents' preference are prudent; Kimball (1990)).

Can we say something about wealth inequality over time? To this we now turn. There is a

[20] Conditional on the total cash-in-hand in the economy.

difficulty because the distinction between credit, wealth, and insurance is indeterminate in equilibrium. All that matters is the level of cash-in-hand in the economy X_t, the aggregate savings function $s(X_t)$, and the consumption splitting rule k. Many different contract combinations can deliver the first-best solution: Arrow-Debreu securities/fully contingent contracts; a combination of credit and insurance contracts (Lucas 1978); and insured credit contracts (Udry 1994). This point has been made before and need not be revisited here. For the remainder of this section, we focus on the case with a combination of credit and insurance.

Even if restrict our attention to a particular set of contracts, the distribution of wealth among agents is still indeterminate. This is because any distribution of wealth is compatible with any constant ratio of marginal utilities provided it is combined with the correct insurance contract. Put differently, it does not matter who 'owns' the wealth. This is because, whatever the distribution of wealth, insurance contracts can be found ensure that the existing cash-in-hand is distributed among agents so as to satisfy Pareto efficiency.

To reduce indeterminacy, we add a new incentive constraint, namely, that insurance contracts are renegotiation-proof. By this, we mean that at the beginning of each period, agents must regard insurance contracts as individually rational ex ante. (Commitment failure, that is, ex post voluntary participation is ignored here; it is discussed later.) An example of an insurance contract that violate this ex ante participation constraint is a contract that stipulate that agent 1 transfers consumption to agent 2 in all states of the world. For agent 1 to accept to continue an insurance contract, it must provide agent 1 with some expected future benefits.

Formally, let $\tau(X_t^1, X_t^2)$ be the transfer from agent 1 to agent 2 that is stipulated in the contract if individual cash-in-hand are X_t^1, X_t^2. The continuation payoff of a participant is defined by the following Belman equation:

$$V_i(X_t^i) = \max_{W_{t+1}^i} U_i(X_t^i + \tau^i(X_t^1, X_t^2) - W_{t+1}^i) + EV_i(\omega^i(Z_i, \theta_{t+1}) + (1+\gamma)W_{t+1}^i) \tag{17}$$

where transfers $\tau^i = -\tau$ if $i = 1$ and $\tau^i = \tau$ if $i = 2$.[21] The autarchy continuation payoff is:

$$\widetilde{V}_i(X_t^i) = \max_{\widetilde{W}_{t+1}^i} U_i(X_t^i - \widetilde{W}_{t+1}^i) + E\widetilde{V}_i(\omega^i(Z_i, \theta_{t+1}) + (1+\gamma)\widetilde{W}_{t+1}^i) \tag{18}$$

Ex ante voluntary participation requires that:

$$E\widetilde{V}_i(X_t^i) \leq EV_i(X_t^i) \tag{19}$$

Arrangements that satisfy the ex ante participation constraints belong to the core (Hildenbrand 1974).

This requirement somewhat restricts the range of insurance contracts, but not entirely. This is because, whenever all agents are risk averse, they are willing to pay for insurance. If all agents are risk averse, they are all willing to pay, which means that there are many insurance contracts that are satisfy the ex ante participation constraint – i.e., belong to the core. One category of contracts that always satisfy the participation constraint – even if one or more agents are risk neutral – is the set of contracts that are actuarially fair. For the purpose of this paper, an actuarially fair contract is when the expected payment from one agent to the others is zero, that is, when $E[\tau(X_t^1, X_t^2)] = 0$. For the remainder of this section, we restrict our attention to actuarially fair contracts.

This is still not sufficient to tie down wealth distribution. This is because, in the presence

[21] Here we assume that savings decisions can be decentralized. This is because insured decision makers face the same returns to wealth as the group. Consequently, their savings decisions are consistent with the social planner's optimum.

of assets, credit is redundant.[22] All that matters is agents' net wealth. This is immediately apparent from the above voluntary participation constraints: all that matters is net wealth/cash-in-hand X_t^i. Consequently, from now on we ignore credit and use the term 'assets' to refer to net wealth.

With all these additional assumptions, we now seek a characterization of insurance and wealth distribution as the level of transfers and current and future assets that satisfies voluntary participation constraints and ensures that $\frac{U_1'(X_t^1-\tau(\theta_t)-W_{t+1}^1)}{U_2'(X_t^2+\tau(\theta_t)-W_{t+1}^2)} = k$ for all θ_t and $E[\tau(X_t^1, X_t^2)] = 0$ in all periods. A full characterization of such plan is beyond this paper (e.g. Lucas 1978, Prescott and Mehra 1980). But a few preliminary insights are worth mentioning.

First, voluntary participation to risk sharing requires that wealth be aligned with desired consumption. Failure to align the two will either violate voluntary participation constraints or the actuarial fairness requirement. To see why, suppose that this is not case: agent 1 is expected to enjoy more consumption but have less wealth. Since agent 1 has a lower income (wealth generates income), this means that on average he will need to receive more from agent 2 in order to raise his consumption.

There are two cases to consider. First suppose that future savings requirements remain unchanged. Giving more to agent 1 will violate the actuarial fairness requirement. Second, suppose that future saving is adjusted to satisfy the actuarial fairness requirement: agent 1 is asked to save less (to leave more for consumption); agent 2 is asked to save more. This solves the problem for the current period but results in a violation of the voluntary participation constraint in subsequent periods – a richer agent 2 is more likely to opt out of an insurance arrangement – or runs into a feasibility constraint when all the wealth of agent 1 is exhausted.

An immediate corollary is that wealth must be aligned with intended distribution of con-

[22] Agents who borrow to finance productive investments have a positive net worth. Most credit observed in practice goes to agents with positive net worth, i.e., household to purchase a home or firms to purchase equipment.

sumption for a risk sharing arrangement to be renegotiation-proof. This but an application of the second welfare theorem: initial wealth determines consumption inequality. Another way of saying this is that N_t^w is an increasing function of N_t^c. Since income is a function of wealth, we also have that N_t^y is an increasing function of N_t^c.

It is difficult to be more explicit given that the level of aggregate wealth varies with collective shocks. But one remarkable consequence of efficient risk sharing is that welfare inequality N_t^u does not change over time. Inequality measures such as N_t^c, N_t^y, N_t^x, and N_t^w may change with X_t but rankings do not change.

The incompatibility of social mobility with efficiency in risk sharing means that societies face a trade-off between perfect insurance and equality of chances. Put differently, they must choose between permanent poverty for some and permanent prosperity for others but protection against idiosyncratic shocks; and social mobility, that is, the opportunity for ranks to change over time. This conflict should not come as a surprise: after all, the rich are not insured if they may lose their status. For them social mobility is not Pareto efficient. Consequently an efficient risk sharing system is always geared towards the preservation of the social status quo.

Our findings are summarized as follows:

Proposition 8 *With asset accumulation and perfect risk sharing:*

(1) N^u is constant over time. Other measures of inequality may change over time.

(2) Credit is redundant; only net wealth matters.

(3) N_t^c, N_t^y, N_t^x, and N_t^w are all non-decreasing functions of N^u.

(4) N_t^c, N_t^y, N_t^x, and N_t^w must remain 'close' to N^u for risk sharing to be renegotiation-proof.

5 Imperfect Commitment

In the previous section we assumed that risk sharing/insurance contracts can be perfectly and costlessly enforced. As argued by Posner (1980) and Platteau (1991), this need not be a reasonable assumption for poor communities. In the absence of external enforcement, risk sharing must be self-enforcing. This requires adding ex post voluntary participation constraints of the form:

$$\widetilde{V}_i(X_t^i) \leq V_i(X_t^i) \tag{20}$$

Other sources of market imperfections (e.g., information asymmetries discussed in Fafchamps (1992) and Ligon (1998)) are ignored here. For a detailed analysis of risk sharing with imperfect commitment, see Kimball (1988), Coate and Ravallion (1993), and Fafchamps and Quisumbing (1999). Ligon, Thomas and Worrall (2001) and Ligon, Thomas and Worrall (2000) present models with wealth accumulation. Our purpose here is not to present imperfect commitment models in detail but rather to discuss their implications for inequality over time.

We begin by noting that if ex post participation constraints are never binding, we are back to the previous section: risk sharing is efficient and N_t^u is constant over time. Next, if participation constraints prevent any risk sharing (i..e., all $\tau^i(X_t^1, X_t^2) = 0$ for all X_t^1, X_t^2), then we are back to section 3. A world with imperfect commitment is thus in between the two.

Ligon et al. (2000) discuss how N_t^u evolves over time when commitment constraints are binding. Following Kocherlakota (1996), they show that N_t^u stays constant as long as constraints are not binding. When a constraint becomes binding for one agent, N_t^u changes so as to incite the agent to voluntarily remain in the risk sharing arrangement. After this jump, N_t^u remains unchanged until another commitment constraint becomes binding. If there exist ratios of marginal utility such that constraints are never binding, then the economy converges to one of these ratios, after which time N_t^u remains constant. If not, then the economy stochastically cycles across a finite set of marginal utility ratios – and hence a set of N_t^u. To the extent that risk sharing is optimized subject to commitment constraints, Kocherlakota (1996) and Ligon et al. (2001) further show that the set of N_t^u among which the economy randomly cycles is as close as possible given the constraints. Thus, even though inequality changes over time, it changes just by as much as is necessary to satisfy voluntary participation.

These results show that, the more efficient risk sharing is, the more persistent poverty is. Any departure from efficient risk sharing allows for social mobility, with the natural restrictions discussed in section 3. Of course, if risk sharing is initially egalitarian, imperfect commitment may take the economy away from an egalitarian distribution of welfare. In this sense, imperfect commitment can generate social differentiation. But the opposite is also possible: imperfect commitment and the rise of opportunism may erode asymmetric risk sharing arrangements that traditionally guarantee more welfare to specific individuals, such as nobles, high castes, village chiefs, and the like.

One issue of interest is the emergence of patronage as a natural outcome of risk sharing arrangements with imperfect commitment. To see how this is possible, first note that, in the presence of imperfect commitment, rankings may change over time. This means that certain agents accumulate more assets than others, even if the distribution was initially egalitarian. Suppose that absolute risk aversion falls with cash-in-hand. Then it can be shown that agents that have accumulated sufficient wealth will refuse actuarially fair contracts (Fafchamps and Quisumbing 1999). For their ex post participation constraint to be satisfied, they must receive more than they give on average. We call this situation 'patronage', i.e., a situation in which a rich agent offers insurance and protection to poor agents in exchange for constant payments and services (e.g. Platteau 1995, Platteau 1995).

If we allow for patronage, ex post participation constraints should be easier to satisfy in sharply polarized societies provided that asset accumulation is sufficient to reduce absolute risk aversion to negligible levels among the rich while very poor households become extremely concerned by risk. To see why, consider our 2-agent economy and assume that agent 1 has all the economy's wealth W and sufficient income to be risk neutral. Agent 2, in contrast, faces a lot of income risk and is desperate to purchase insurance. It is clearly in the interest of agent 1 to become a patron and to serve as 'insurance company' for agent 2. If agent 2 is sufficiently patient, it is in her interest to pay agent 1 in exchange for the (credible) promise of protection against negative income shocks.

To demonstrate this point, consider the following example. Utility is linear except in the vicinity of 0 (e.g., below 1/4), where it falls to $-\infty$. This is meant to represent the fear of starvation. Incomes are (2,0) and (0,2) with probability 1/2. Total wealth is 100, return to wealth is 10%. Here we assume that the rich agent has a discount factor of 0.5. First suppose that both agents have equal wealth. Since they can both insure themselves survival, they are unwilling to pay for future insurance. Consequently, risk sharing does not take place. Next, suppose that agent 1 has all wealth and agent 2 has nothing. Now agent 2 is concerned about survival. His promise to pay for insurance is thus credible. The ex post participation constraint of agent 1 is:

$$\tau_1 \leq \left[-\frac{1}{2}\tau_1 + \frac{1}{2}\tau_2 \right] \frac{0.5}{1-0.5} \tag{21}$$

where τ_1 is the transfer from agent 1 to agent 2 in the first state of the world (when agent 2 has an income of 0), and τ_2 is the transfer of agent 2 to agent 1 in state 2 (when agent 2 has an income of 2). The maximum transfer agent 1 agrees to pay must satisfy $\tau_1 = \tau_2/3$. Given this constraint, the maximum agent 2 is willing to pay is an amount that ensures her constant consumption:

$$2 - \tau_2 = 0 + \tau 1 \tag{22}$$

The solution is $\tau_1 = 1/2$. This example illustrates that the fear of starvation by the poor may trigger patronage albeit imperfect commitment prevented risk sharing among agents with equal wealth.

From the point of view of inequality dynamics, patronage is interesting because it enables the rich to accumulate more and the poor to accumulate less. This is because, on average, the rich receive net transfers from the poor. The precise form of inequality fostered by patronage depends on the returns to wealth and whether aggregate wealth is bounded or not. If returns to wealth are positive and unbounded, patronage is only a transient phenomenon: even poor agents save so that, in the long run, they no longer need to purchase insurance from the patron.

If returns to wealth is negative, patronage reinforces the tendency towards inequality. But polarization is not an absorbing state because aggregate wealth gets depleted in finite time with probability one. In this configuration, patronage arises and survives for a while until aggregate wealth is spent to deal with a large (or series of) collective shock(s), at which point it disappears because impoverished patrons cannot promise sufficient protection and extract net payments from others. If returns to wealth are positive but aggregate wealth is bounded, patronage makes polarization more likely.

A similar possibility arises if returns to wealth are positive but total wealth is fixed. In this case, patronage indirectly raises the return on wealth for the rich, hence making multiple equilibria more likely. This is but an application of Proposition 4.

These findings can be summarized as follows:

Proposition 9 *With imperfect commitment:*

(1) If ex post voluntary participation constraints are never binding, risk sharing is efficient and there is no social mobility, i.e., N^u *is constant over time.*

(2) If ex post voluntary participation constraints are always binding, models discussed in section 3 apply.

(3) If risk aversion is high for agents without assets but low for agents with assets, risk sharing is more likely in polarized societies, where it takes the form of patronage.

(4) Other things being equal, patronage makes inequality in wealth and consumption more likely and long lasting, while protecting the poor from starvation.

6 Conclusion

In this paper we have examine how risk, accumulation, and insurance affect inequality. The main findings are that there are trade-offs between insurance and social mobility, and between asset markets and social polarization. We also find that, in the presence of imperfect commitment, mutual insurance might be easier to sustain in the form of patronage in unequal societies. These findings are hardly new but the contribution of this paper is show how they are related to each other within a coherent framework.

Perhaps our most novel result is the observation that, when wealth is made of assets in fixed supply, such as land or manpower, trade in these assets naturally leads to an unequal distribution of welfare and income. For this result to arise, the savings rate must increase with cash-in-hand. Factors that favor accumulation, such as a high return on wealth or large aggregate stock of wealth, also favor inequality.

From an equity point of view, there might therefore be a rationale for shutting down certain asset markets, i.e., those for which supply is finite. This is because allowing accumulation is likely to result in polarization. This conclusion applies primarily to land, manpower, mineral resources, and the environment. With the possible exception of mineral resources, these are also the markets in which restrictions to market transactions are most widespread. In contrast, the accumulation of unbounded resources such as equipment, skills, and knowledge does not raise similar fears although, in the long run, they also lead to persistent inequality. These assets are also those that are essential for growth.

Developed societies are adamantly opposed to slavery – even if it is voluntary. Yet, the U.S. has a student loan program that require graduates to work in the army for a set number of years. Unlike other labor contracts for which employees can never be forced to work, military personnel who do not report for work are regarded as having deserted. Student loans by the military are thus a modern form of indenture contracts – with the caveat that they are used to accumulate education. These loans are acceptable while other indenture contracts are not. This suggests that society intuitively understands the distinction between bounded and unbounded assets and the implication for inequality.

This paper also suggests that shutting down certain asset markets can only be effective in reducing inequality if it is combined with access to actuarially fair insurance. If a starving person cannot sell his future workforce in exchange for survival today, patronage is likely to arise as substitute (e.g. Srinivasan 1989, Platteau 1995). Given that patronage favors polarization, the end result is not very different. This implies that programs to eradicate inequality must imperatively combine asset redistribution with access to fair insurance. This is particularly true in pre-industrial societies where aggregate accumulation is small or inexistent. Redistribution of assets is not, by itself, sufficient to durably eliminate inequality.

Our analysis also offers some caution to those who see the provision of insurance mechanisms as a solution to poverty. The theory is quite clear: perfect insurance freezes inequality. An insurance system based on voluntary participation – such as a market-based approach for instance – cannot eliminate inequality. It might raise the welfare of the poor by providing protection against shocks, but it will not in general eliminate inequality that is already there.[23] A tax-based system might be necessary to build redistribution into the insurance system. Combatting inequality thus requires policy tools other than insurance. In some cases, organizing insurance might even worsen the long-term prospects of the poor by reducing social mobilty.

This paper leaves many questions unanswered. One issue of interest is the interface between accumulation, inequality, and growth. Most engines of growth require the accumulation of something – equipment, skills, knowledge. It is therefore widely recognized that economic development is intimately related to aggregate accumulation. The distribution of wealth across the population is also likely to affect the pace of growth (Aghion, Caroli and Garcia-Penalosa 1999).

This issue has received a lot of attention in the literature as far as the accumulation of human capital is concerned (e.g. Becker and Tomes 1979, Galor and Zeira 1993, Maoz and Moav 1999, Mookherjee and Ray 2000). The literature has also examined the relationship between risk taking, credit markets, and growth (e.g. Banerjee and Newman 1991, Banerjee and Newman 1993, Piketty 1997, Aghion and Bolton 1997). Less attention has been devoted to capital accumulation and returns to scale (e.g. Freeman 1996, Stiglitz 1969). For instance, if production is subject to increasing returns to scale, the concentration of capital in a few hands would ensure faster growth than a more egalitarian distribution. The same reasoning applies to human capital: if the highest returns are in technology transfer, a few highly trained individuals who can borrow technology from elsewhere might better favor growth than a large number of workers with a little bit of training. Factors that influence the evolution of wealth inequality over time, such as returns to assets, income correlation, and patronage, may thus affect growth

[23]Unless the rich's willingness to pay was quite high to start with and they were asked to pay much more for insurance than the poor. With assets, this is unlikely to be the case because then the rich can self-insure.

as well (in addition to being influenced by it). These issues are left for future research.

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Figure 1. Inequality over time
for 3 realizations of labor income

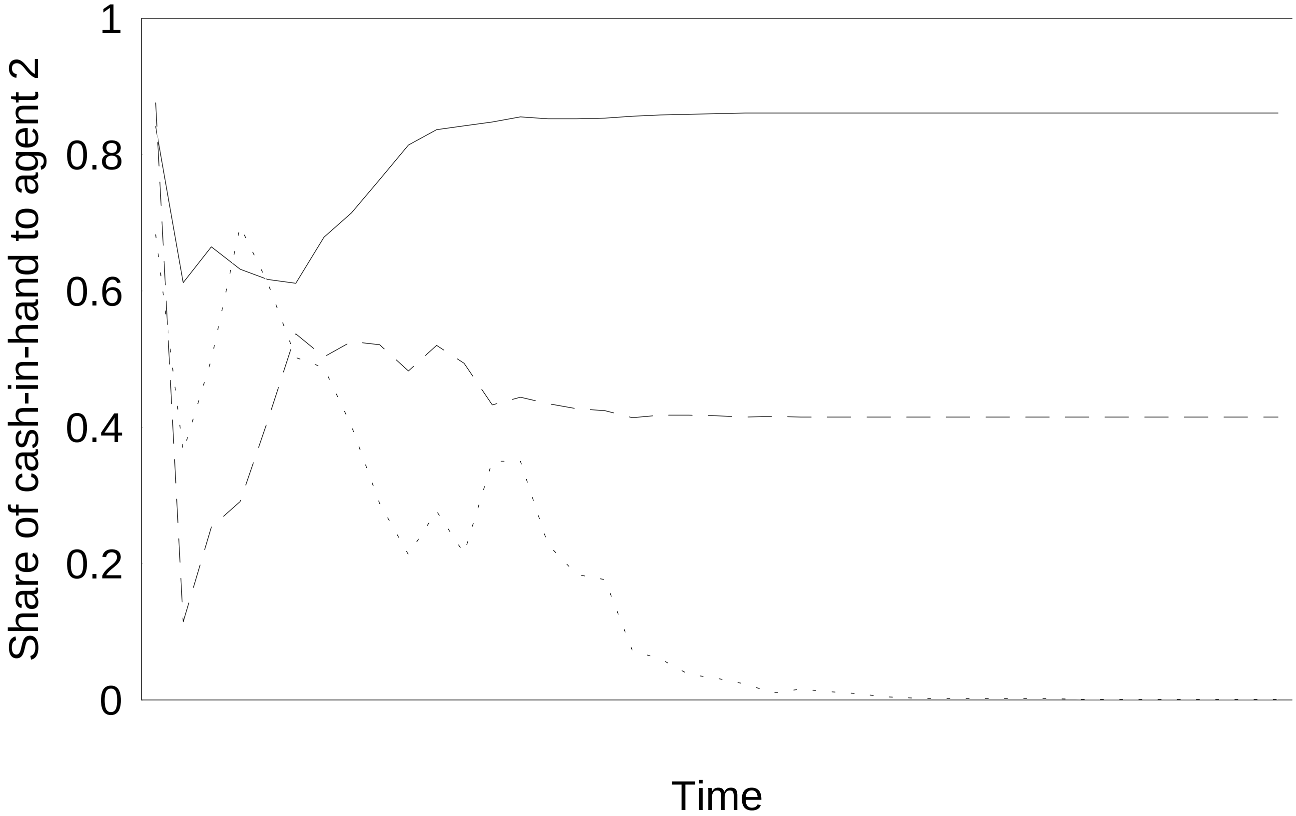

Figure 2. Inequality over time
with precautionary saving only

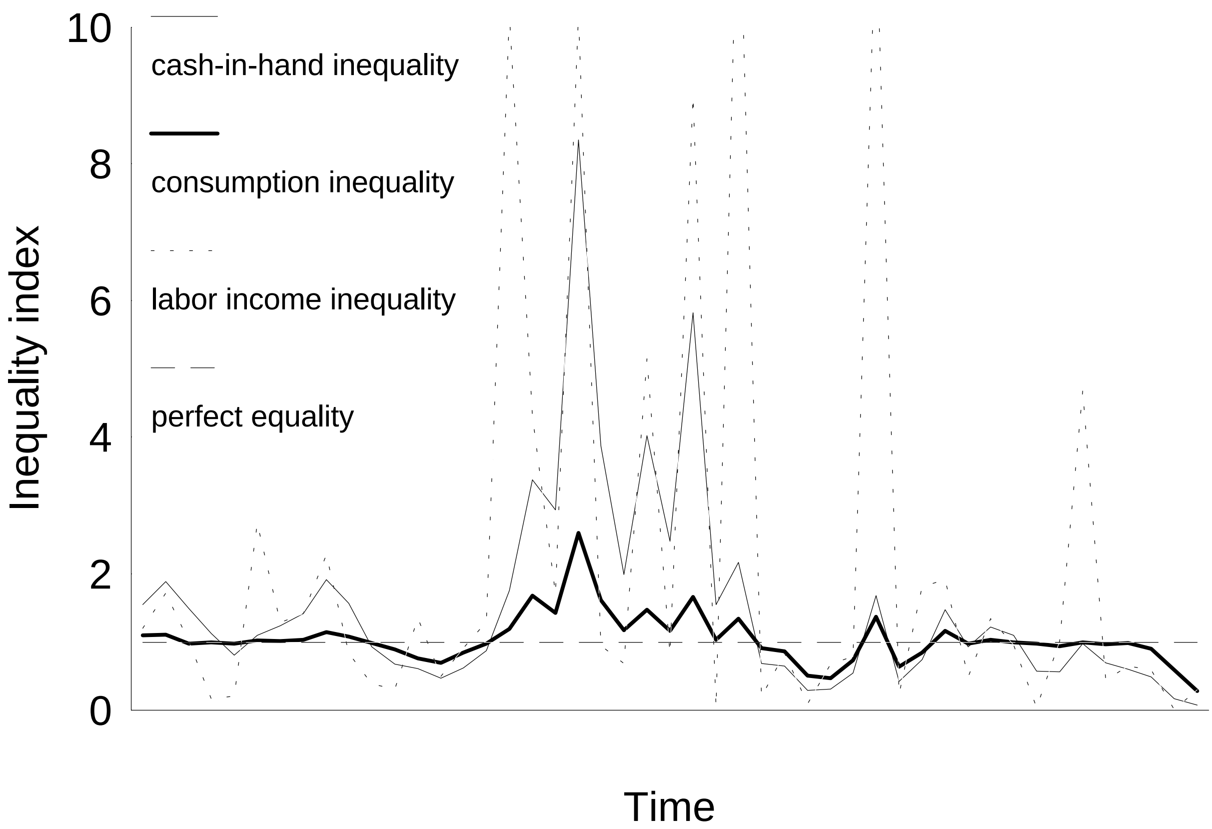

Figure 3. Consumption inequality and return on savings

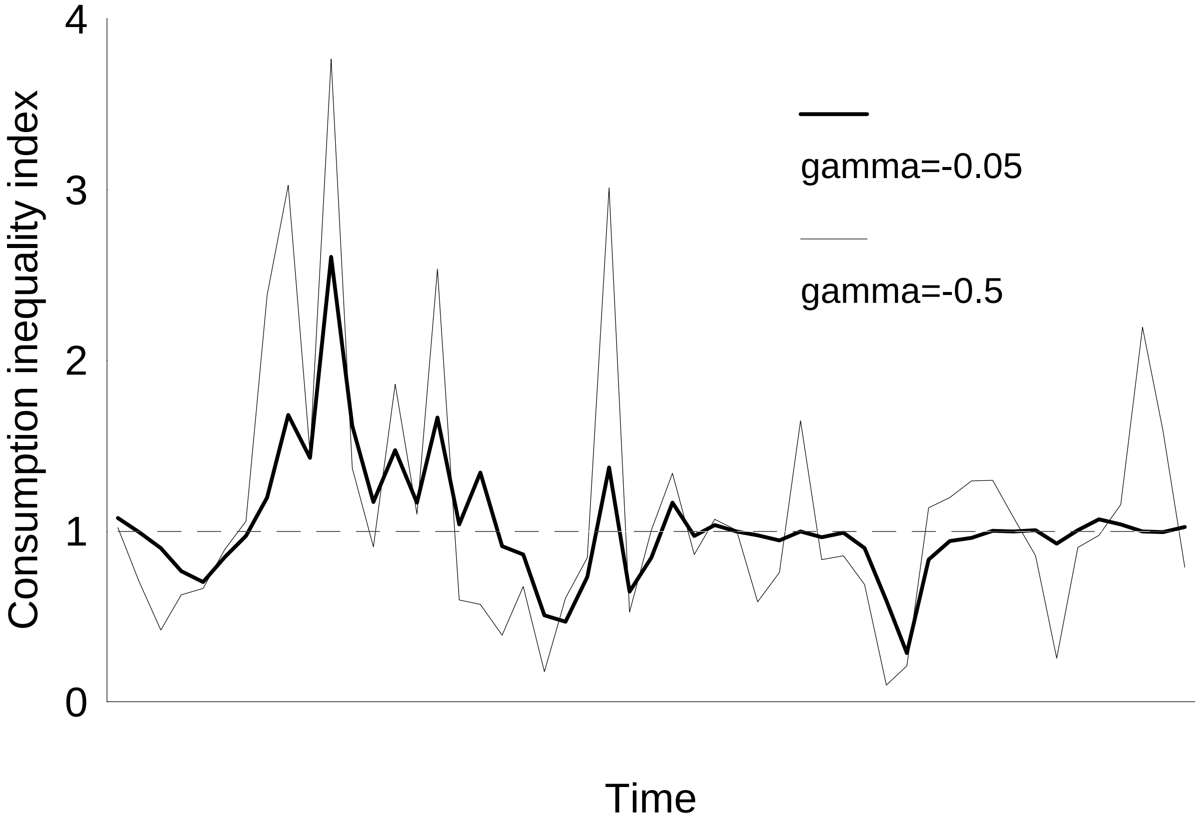

Figure 4. Law of motion of inequality with fixed total asset

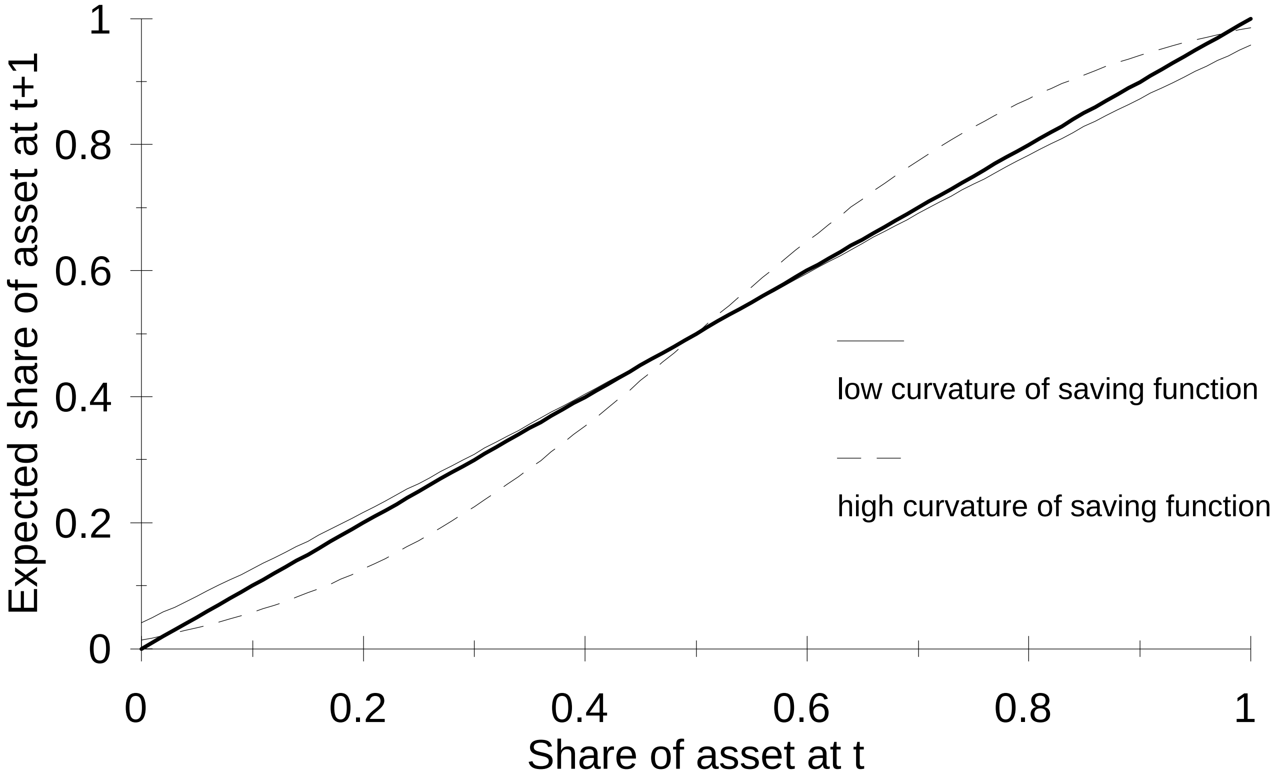

Figure 5. Polarization