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On Some Problems of Variable Population Poverty Comparisons

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Abstract

This paper demonstrates that the property of 'replication invariance', generally considered to be an innocuous requirement for the extension of fixed-population poverty comparisons to variable-population contexts, is incompatible with other plausible variable- and fixed-population axioms.

Keywords: variable populations, fixed populations, Replication invariance, population focus, income focus, impossibility result

JEL classification: I32, D63, D30

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1 Introduction

Much of the canonical work in poverty measurement considers desirable properties of poverty indices in the context of populations of fixed size. Recently, however, several scholars, motivated by philosophical work on population ethics,[1] have demonstrated that some of these properties have implausible consequences in variable population contexts (Kundu and Smith 1983; Subramanian 2002, 2005a, 2005b; Paxton 2003; Chakravarty, Kanbur and Mukherjee 2006; Kanbur and Mukherjee 2007; Hassoun 2009).

It is a standard feature of the poverty measurement literature to invoke the *replication invariance axiom*, which allows poverty comparisons in fixed-population contexts to be extended to poverty comparisons in variable population contexts. Replication invariance is the purportedly unexceptionable requirement that the extent of poverty in a situation should remain the same if the population is replicated any number of times. Many of the common measures of poverty satisfy replication invariance because they incorporate within themselves the headcount ratio (or the proportion of the population in poverty).[2] The axiom occupies a central place in all of distributional, welfare, and poverty analysis: phenomena such as Lorenz dominance, generalized Lorenz dominance, and Stochastic dominance—which are essential ingredients of the sorts of assessments under review—depend for their meaningfulness on acceptance of the replication invariance axiom. (With specific reference to the context of poverty comparisons, the test of Stochastic dominance is employed to verify that an unambiguous poverty ranking of distributions can be effected—one that does not, for instance, depend on what particular poverty line, or what particular poverty index, is used.)

This paper argues, however, that the replication invariance axiom may be problematic. Others have made similar points (see the earlier-cited work by Subramanian 2002; Paxton 2003; Chakravarty, Kanbur and Mukherjee 2006; Hassoun 2009).[3] Specifically, it turns out that, if one accepts certain other canonical axioms for poverty measurement, one would be compelled to choose between replication invariance and some sort of *population focus axiom*, for it is impossible to hold them all. The population focus axioms at issue suggest that the magnitude of poverty ought not to be sensitive (in various ways) to additions to the *non*-poor population of a society. Variations of this desideratum are reflected in Subramanian's (2002) 'strong focus axiom', in Paxton's (2003) 'poverty non-invariance axiom', and in Hassoun's (2009) 'no mere addition axiom'.

To see why one might be disposed favourably towards some population focus axiom, consider an argument in favour of the strongest version of this axiom on which changes in the non-poor population do not affect poverty in any way. One should take this

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1 For a canonical text on population ethics, see Parfit (1984). Two other important contributions are: Blackorby and Donaldson (1984) and Blackorby, Bossert and Donaldson (2005).

2 A prominent exception is the income-gap ratio, which measures the proportionate shortfall of the average income of the poor from the normative income level that separates the poor from the non-poor.

3 The axiom also acquires salience in the context of welfare and inequality comparisons across populations of different sizes (as discussed in, among other works, Blackorby and Donaldson (1984); Bossert (1990); Trannoy and Weymark (2009)). In this connection, see also Subramanian (2010).

population focus axiom seriously for the same reason, we suggest, that what is popularly known as the *focus axiom* has been traditionally taken so seriously in the poverty measurement literature. The focus axiom is really an *income focus axiom*, which demands (in a fixed-population context) that measured poverty ought not to be sensitive to increases in *non*-poor incomes. The rationale for this requirement is that poverty is a feature of the poor, and not of the general population: in making poverty comparisons, one ought, in this view, to focus attention only on the income distribution of the poor. If there is merit in this contention, its scope ought to extend also to a population-focus view of the matter. That is to say, if poverty is a characteristic of the poor, then additions to the non-poor *population*—exactly like additions to non-poor *incomes*—ought not to affect the magnitude of poverty. Both income-focus and population-focus requirements can then be seen as being allied to Broome's (1996) 'constituency principle' in population ethics, a principle which, in the present context, would require assessments of the extent of poverty to be based exclusively on information regarding the *constituency of the poor*. If this argument is persuasive, and one accepts any of the canonical (fixed-population) axioms for poverty measurement relied upon in the elementary inconsistency proofs that follow, then there is reason to question any routine acceptance of replication invariance.

2 Concepts

An income distribution is an *n*-dimensional vector $\mathbf{x} = (x_1,...,x_i,...,x_n)$, in which the typical element x_i stands for the (non-negative) income of person i in a community of n individuals, n being a positive integer. The poverty line, designated by z, is a positive level of income such that anybody with an income less than z is labelled *poor*.[4] The set of all individuals whose incomes are represented in the distribution $\mathbf{x}$ is $N(\mathbf{x})$, while, given that the poverty line is z, $Q(\mathbf{x};z)$ is the set of poor people in $\mathbf{x}$, that is, $Q(\mathbf{x};z) \equiv \{i \in N(\mathbf{x}) | x_i < z\}$. For any $\mathbf{x}$, and any $z > 0$, the vector of poor incomes is designated by $\mathbf{x}_z^P$, while the vector of non-poor incomes is designated by $\mathbf{x}_z^R$. If $\mathbf{X}_n$ is the set of all *n*-dimensional income vectors, then the set of all conceivable income vectors is given by $\mathbf{X} \equiv \bigcup_{n=1}^{\infty} \mathbf{X}_n$.

We shall let Π stand for a weak binary relation of poverty defined on $\mathbf{X}$: specifically for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, we shall write $\mathbf{x}\Pi\mathbf{y}$ to signify that there is no more poverty in $\mathbf{x}$ than in $\mathbf{y}$. The asymmetric and symmetric factors of Π are represented by $\overline{\Pi}$ and $\underline{\Pi}$ respectively:

$$\forall \mathbf{x}, \mathbf{y} \in \mathbf{X} : \mathbf{x}\overline{\Pi}\mathbf{y} \leftrightarrow [\mathbf{x}\Pi\mathbf{y} \;\&\; \neg(\mathbf{y}\Pi\mathbf{x})]; \text{ and}$$

$$\forall \mathbf{x}, \mathbf{y} \in \mathbf{X} : \mathbf{x}\underline{\Pi}\mathbf{y} \leftrightarrow [\mathbf{x}\Pi\mathbf{y} \;\&\; \mathbf{y}\Pi\mathbf{x}].$$

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4 This is what Donaldson and Weymark (1986) call the 'weak' definition of the poor, which excludes an individual on the poverty line from the count of the poor.

That is, we shall write $\mathbf{x}\overline{\Pi}\mathbf{y}$ to signify that there is less poverty in $\mathbf{x}$ than in $\mathbf{y}$, and $\mathbf{x}\underline{\Pi}\mathbf{y}$ to signify that there is exactly as much poverty in $\mathbf{x}$ as there is in $\mathbf{y}$. It will be assumed that Π is *reflexive* ($\forall \mathbf{x} \in X : \mathbf{x}\Pi\mathbf{x}$) and *transitive* ($\forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbf{X} : \mathbf{x}\Pi\mathbf{y} \,\&\, \mathbf{y}\Pi\mathbf{z} \rightarrow \mathbf{x}\Pi\mathbf{z}$). We shall not, however, insist that Π be *complete* (which is the requirement that $\forall \mathbf{x}, \mathbf{y} \in \mathbf{X} : \mathbf{x}\Pi\mathbf{y}$ or $\mathbf{y}\Pi\mathbf{x}$). Π, in other words, will be assumed to be a *quasi-order*.

Throughout, we shall assume that the binary relation Π is *anonymous*, that is, for any given poverty line and all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $\mathbf{x}$ is merely a permutation of $\mathbf{y}$, then $\mathbf{x}\underline{\Pi}\mathbf{y}$ - which is just another way of saying that poverty assessments are impervious to the personal identities of individuals. The anonymity axiom enables variable population (either cross-section or time-series) poverty comparisons to be performed in such a way as to suggest that, of a pair of distributions under comparison, one distribution can be seen to have been derived from the other through a population increment or decrement.

3 Axioms for poverty comparisons

Presented below is a set of axioms for comparisons of poverty across both fixed and variable populations. Since the axioms are generally well-known, and in any event their import is reasonably clear, we shall not spend much time in explaining their meaning and significance. It will be taken as read that in everything that follows, z is a strictly positive scalar. Also, Π will be taken to belong to the set $\mathcal{R}$ of all anonymous quasi-orders.

Income-focus

For all z, and for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $N(\mathbf{x}) = N(\mathbf{y})$ and $\mathbf{x}_z^P = \mathbf{y}_z^P$, then $\mathbf{x}\underline{\Pi}\mathbf{y}$.

This axiom is usually just called the focus axiom. It requires that any change in non-poor incomes that leaves the numbers of individuals on either side of the poverty line unchanged, ought not to have any impact on the extent of poverty.

Monotonicity

For all z, and for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $N(\mathbf{x}) = N(\mathbf{y})$ and $x_i = y_i \forall i \in N(\mathbf{x}) \setminus \{j\}$ for some j such that $j \in Q(\mathbf{y}; z) \,\&\, x_j > y_j$, then $\mathbf{x}\overline{\Pi}\mathbf{y}$.

The monotonicity axiom states that, other things equal, an increase in a poor person's income should reduce poverty.

Transfer

For all z, and for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $N(\mathbf{x}) = N(\mathbf{y})$, and $x_i = y_i \forall i \in N(\mathbf{x}) \setminus \{j, k\}$ where j and k are such that $j \in Q(\mathbf{y}; z), k \in N \setminus Q(\mathbf{x}; z)$, $x_j = y_j + \delta, x_k = y_k - \delta$, and $0 < \delta \leq (y_k - y_j)/2$, then $\mathbf{x}\overline{\Pi}\mathbf{y}$.

The transfer axiom presented here is weaker than Donaldson and Weymark's (1986) weak downward transfer axiom. As we have stated it, the transfer axiom demands that,

ceteris paribus, a mutually rank-preserving transfer of income from a non-poor person to a poor person that keeps the former non-poor, should reduce poverty.

Replication invariance

For all z, for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and any positive integer k, if $\mathbf{y} = (\mathbf{x},...,\mathbf{x})$ and $n(\mathbf{y}) = kn(\mathbf{x})$, then $\mathbf{x} \underline{\Pi} \mathbf{y}$.

Again, replication invariance requires the extent of poverty to remain unchanged by any k-fold replication of a population.

Weak poverty growth

For all z, and for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $\mathbf{x}_z^P$ is the q-vector (x^o,…, x^o) for some non-negative real number x^o and positive integer q; $\mathbf{y}_z^P$ is the (q+1)-vector (x^o,..., x^o); and $\mathbf{x}_z^R = \mathbf{y}_z^R \neq \{\}$; then $\mathbf{x} \overline{\Pi} \mathbf{y}$.

This axiom was introduced in Subramanian (2002). It is a diluted version of the Kundu and Smith (1983) poverty growth axiom. It requires that in a situation where there is at least one non-poor person and all the poor have the same income,[5] the addition of another poor person with this same income ought to increase poverty.

Non-poverty growth

For all z, and for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $\mathbf{x}_z^P = \mathbf{y}_z^P$ and $\mathbf{y}_z^R = (\mathbf{x}_z^R, x)$ (for any $x \geq z$), then $\mathbf{y} \overline{\Pi} \mathbf{x}$.

This axiom is due to Kundu and Smith (1983). It stipulates that poverty should decline with the addition of a non-poor person to the population.

Weak population-focus

For all z, and for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $\mathbf{x}_z^P = \mathbf{y}_z^P$ and $\mathbf{y}_z^R = (\mathbf{x}_z^R, x)$ (for any $x \geq z$), then $\neg[\mathbf{y} \overline{\Pi} \mathbf{x}]$.

This axiom, which is diametrically opposed in spirit to non-poverty growth, corresponds to what Hassoun (2009) calls 'the no mere addition axiom': it says that poverty is not reduced by changes in the non-poor population which leave the income distribution amongst the poor unchanged.

Population-focus

For all z, and for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if $\mathbf{x}_z^P = \mathbf{y}_z^P$ and $\mathbf{y}_z^R = (\mathbf{x}_z^R, x)$ (for any $x \geq z$), then $\mathbf{x} \underline{\Pi} \mathbf{y}$.

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5 The requirement of at least one person being non-poor is a weakening intended to give the headcount ratio some chance of surviving a strengthened version of the axiom: if everybody is poor, then the headcount ratio will be unvarying, at unity, irrespective of the dimensionality of the income vector. The additional stipulation that all the poor (including the addition to the population) have the same income, is to avoid any ambiguity that might arise from the additional poor person having an income greater than the initial average income of the poor: in this event, the headcount ratio would increase and the income-gap ratio would decline, and in principle, it may not be straightforward to suggest that poverty, on net, has increased. (See Subramanian 2002 for a discussion.)

Population-focus is a strengthened version of weak population-focus, and corresponds to what Paxton (2003) calls the 'poverty non-invariance axiom. It requires an addition to the non-poor population to leave the extent of poverty unchanged (and not just to leave it un-reduced). Motivationally, it is very much in the spirit of the standard (income-) focus axiom, which says poverty is not affected by changes in non-poor incomes which leave the income distribution amongst the poor unchanged.

4 Three impossibility results

We first present and prove three propositions on the existence of quasi-orders satisfying specified sets of variable- and fixed-population properties, and then comment on these results.

Proposition 1. There exists no $\Pi \in \mathcal{R}$ which satisfies replication invariance (RI), weak poverty growth (WPG), and weak population-focus (WPF).

Proof. Let $z = 2$. Consider the income distributions $\mathbf{a}$ = (1,3), $\mathbf{b}$ = (1,3,3) and $\mathbf{c}$ = (1,1,3,3).

By WPG,

$$\mathbf{b}\overline{\Pi}\mathbf{c}. \qquad (1.1)$$

By RI,

$$\mathbf{c}\underline{\Pi}\mathbf{a}. \qquad (1.2)$$

From (1.1) and (1.2), transitivity of Π dictates:

$$\mathbf{b}\overline{\Pi}\mathbf{a}. \qquad (1.3)$$

However, by WPF,

$$\neg[\mathbf{b}\overline{\Pi}\mathbf{a}]. \qquad (1.4)$$

(1.4) contradicts (1.3). ●

Proposition 2. There exists no $\Pi \in \mathcal{R}$ which satisfies monotonicity (M), replication invariance (RI), and weak population-focus (WPF).

Proof. Let $z = 2$. Consider the income distributions $\mathbf{a}$ = (1,1), $\mathbf{b}$ = (1,3) and $\mathbf{c}$ = (1).

By M,

$$\mathbf{b}\overline{\Pi}\mathbf{a}. \qquad (2.1)$$

By RI,

$$\mathbf{a}\underline{\Pi}\mathbf{c}. \qquad (2.2)$$

Transitivity, in conjunction with (2.1) and (2.2), yields

$$\mathbf{b}\overline{\Pi}\mathbf{c}\,. \tag{2.3}$$

However, WPF requires that

$$\neg[\mathbf{b}\overline{\Pi}\mathbf{c}]\,, \tag{2.4}$$

and we have a contradiction. ●

Proposition 3. There exists no $\Pi \in \mathcal{R}$ which satisfies transfer (T), replication invariance (RI), and population-focus (PF).

Proof. Let $z = 2$. Consider the income distributions $\mathbf{a}$ = (1), $\mathbf{b}$ = (1,3), $\mathbf{c}$ = (1,3,3), $\mathbf{d}$ = (1,1,5), and $\mathbf{e}$ = (1,1).

By PF:

$$\mathbf{a}\underline{\Pi}\mathbf{b}\,; \tag{3.1}$$

and

$$\mathbf{b}\underline{\Pi}\mathbf{c}\,. \tag{3.2}$$

Given (3.1) and (3.2), by transitivity of Π over the triple $\{\mathbf{a},\mathbf{b},\mathbf{c}\}$:

$$\mathbf{a}\underline{\Pi}\mathbf{c}\,. \tag{3.3}$$

By T:

$$\mathbf{c}\overline{\Pi}\mathbf{d}\,. \tag{3.4}$$

Given (3.3) and (3.4), by transitivity of Π over the triple $\{\mathbf{a},\mathbf{c},\mathbf{d}\}$:

$$\mathbf{a}\overline{\Pi}\mathbf{d}\,. \tag{3.5}$$

By PF:

$$\mathbf{d}\underline{\Pi}\mathbf{e}\,. \tag{3.6}$$

Given (3.5) and (3.6), by transitivity of Π over the triple $\{\mathbf{a},\mathbf{d},\mathbf{e}\}$:

$$\mathbf{a}\overline{\Pi}\mathbf{e}\,. \tag{3.7}$$

RI, however, dictates that:

$$\mathbf{a}\underline{\Pi}\mathbf{e}\,, \tag{3.8}$$

in contradiction of (3.7). ●

Remark 1. Proposition 1 is (effectively) a slightly strengthened version of one in Subramanian (2002), which (again effectively) shows that it is impossible to combine weak poverty growth, replication invariance and population-focus—note that weak population-focus is weaker than population-focus. (Also, Subramanian employs a real-valued measure of poverty, whereas here we employ only a quasi-order—on which more in Remark 3.)

Remark 2. It is interesting to note that Kundu and Smith (1983) came very close to proving our Proposition 1. The Kundu-Smith impossibility theorem asserts that there is no real-valued representation of poverty which simultaneously satisfies three properties, which they call, respectively, upward transfer, poverty growth, and non-poverty growth. In the course of commenting on their result, Kundu and Smith observe, in footnote 7 of their paper (1983: 431), that their impossibility outcome would be retained if non-poverty growth were replaced by replication invariance. This, as they point out, is because poverty growth and replication invariance together imply non-poverty growth. [To see this, consider again the example presented in the proof of Proposition 1: by poverty growth (which is a strong version of what we have called weak poverty growth and requires, simply, that, other things equal, an addition to the poor population should increase poverty), $\mathbf{b}\overline{\Pi}\mathbf{c}$, and by replication invariance, $\mathbf{c}\underline{\Pi}\mathbf{a}$, whence, by transitivity, $\mathbf{b}\overline{\Pi}\mathbf{a}$ - which is exactly what non-poverty growth demands.] From here, it is but a single short step to the impossibility result of Proposition 1: our weak population focus axiom is in direct opposition to non-poverty growth, and thus our Proposition 1.

Remark 3. In regard to Propositions 3 and 4, it is relevant to note that Sen's (1976) seminal work, reflected in the quest for income-responsive and distribution-sensitive poverty measures, was motivated precisely by the failure of the headcount ratio to satisfy fixed-population axioms like monotonicity and transfer. However, in a variable population context, the headcount ratio is the archetypal replication invariance-satisfying measure, and this note has shown that when we employ a population-focus axiom, it is impossible to hold replication invariance and monotonicity (Proposition 2) or transfer (Proposition 3). Hassoun (2009) suggests that similar results hold for specific real-valued poverty indices which incorporate the headcount ratio: she shows that a decline in a poor person's income, or a regressive transfer of incomes between two poor persons, is nevertheless compatible with a decline in poverty if these changes are accompanied by a sufficiently large decline in the headcount ratio owing to an increase in the non-poor population.

Remark 4. In respect of all three of our results, it should be reiterated that these impossibilities are, in one important way, different from the Kundu-Smith impossibility mentioned above (Kundu and Smith 1983). The Kundu-Smith theorem points to a *representational* problem, not an ordering problem.[6] As the authors state explicitly (Kundu and Smith 1983: 429): '…the weight of the argument rests directly on the structural properties of the real number system'. We ourselves do not insist on anything as demanding as real-valued representation: the fact that there does not exist even a quasi-order satisfying sets of specified properties points to a much more foundational conflict in the axiom systems we employ. No doubt our results are mathematically very

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6 Although Subramanian employs a real-valued measure of poverty, his result does not reflect an essentially 'representational' problem.

simple and straightforward, but they are significant as they point to a fundamental (as opposed to merely representational) incompatibility between the axioms used in our proofs.

5 Concluding observations

This paper has argued that if some sort of population-focus axiom is accepted, then one has to reject either replication invariance or each of the following: weak poverty growth, monotonicity, and transfer. The case in favour of a population-focus axiom has been discussed at some length in the introductory section, and will not be repeated here. Much of the quest for 'sophisticated poverty measures' in the poverty literature has been motivated precisely by the appeal of the monotonicity and transfer axioms, and few will reject, at this late date of the proceedings, these canonical properties. (It is worth emphasizing that the version of the transfer axiom we employ is a particularly weak one—weaker even than Donaldson and Weymark's (1986) 'weak downward transfer': indeed, our version of the transfer axiom is implied—as a moment's reflection will confirm—by the monotonicity and income focus axioms.) Some would probably argue that there is good reason to reject weak poverty growth. For, the axiom seems to suggest that since poverty declines when the poor are removed from a population, it is acceptable for poverty reductions to be brought about by excess mortality among the poor (through, say, undernutrition and untreated disease). (This problem is reviewed in Kanbur and Mukherjee 2007.) However, this worry is misplaced because it overlooks the distinction between a measure of how much poverty there is in a situation and a metric for guiding policy decisions. Although poverty indices are often used to guide policy, they must be used with at least a modicum of sound judgment. There may be less poverty in a situation where no one is poor, but that does not give us reason to allow the poor to die from neglect or, worse still, to kill them off![7]

This leaves one with replication invariance. It is not immediately obvious that there is some inherently indispensable *ethical* appeal attaching to the property. And yet, rejecting replication invariance is not a painless option to implement. As we have seen, commonly employed procedures for poverty comparison, relying on dominance criteria such as stochastic dominance, do not lend themselves to meaningful interpretation if one does not subscribe to replication invariance. There may be ways of modifying these methods for comparing poverty in variable populations along the lines of Trannoy and Weymark (2009), though we are not aware of attempts to extend this kind of analysis to poverty measurement. This is an avenue of research that awaits further exploration.

Briefly, then**,** it would appear that variable populations pose some difficult problems for poverty comparisons. This paper, which serves also to consolidate important emerging trends in the literature, has demonstrated that the property of replication invariance, generally considered to be an innocuous requirement for the extension of fixed-population poverty comparisons to variable-population contexts, is incompatible with other plausible variable- and fixed-population axioms. We hope it will invite further

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[7] To put the point another way, the binary relation Π we employ in this note is not a 'betterness' relation, only an 'at most as much poverty as' relation. That is, weak poverty growth only says that (under certain conditions) an increment to the poor population leads to more poverty, not to a better state of affairs. Seen in this light, it is hard to quarrel with weak poverty growth.

reflection on variable-population poverty comparisons, with specific reference to the proper role and possible limits of replication invariance in measuring poverty.

References

Blackorby, C., W. Bossert, and D. Donaldson (2005). *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*. Cambridge: Cambridge University Press.

Blackorby, C., and D. Donaldson (1984). 'Social Criteria for Evaluating Population Change'. *Journal of Public Economics*, 25 (1-2): 13-33.

Bossert, W. (1990). 'Population Replications and Ethical Poverty Measurement'. *Mathematical Social Sciences*, 19: 143-58.

Broome, J. (1996). 'The Welfare Economics of Population'. *Oxford Economic Papers*, 48 (2): 177-93.

Chakravarty, S., S. R. Kanbur, and D. Mukherjee (2006). 'Population Growth and Poverty Measurement'. *Social Choice and Welfare*, 26 (3): 471-83.

Donaldson, D., and J. A. Weymark (1986). 'Properties of Fixed Population Poverty Indices'. *International Economic Review*, 27 (3): 667-88.

Hassoun, N. (2009). 'Another Mere Addition Paradox? A Problem for Some Common Poverty Indexes in Variable Populations'. Carnegie Mellon University Working Paper. Pittsburgh: Carnegie Mellon University.

Kanbur, S. R., and D. Mukherjee (2007). 'Premature Mortality and Poverty Measurement'. *Bulletin of Economic Research*, 19 (4): 339-59.

Kundu, A., and T. E. Smith (1983). 'An Impossibility Theorem on Poverty Indices'. I*nternational Economic Review*, 24 (2): 423-34.

Parfit, D. (1984). *Reasons and Persons*. Oxford: Clarendon Press.

Paxton, J. (2003). 'A Poverty Outreach Index and its Application to Microfinance'. *Economics Bulletin*, 9 (2): 1-10.

Sen, A. K. (1976). 'Poverty: An Ordinal Approach to Measurement'. *Econometrica*, 33 (2): 219-31.

Subramanian, S. (2002). 'Counting the Poor: An Elementary Difficulty in the Measurement of Poverty'. *Economics and Philosophy*, 18 (2): 277-85.

Subramanian, S. (2005a). 'Fractions versus Whole Numbers: On Headcount Comparisons of Poverty across Variable Populations'. *Economic and Political Weekly*, 40 (43). 4625-28.

Subramanian, S. (2005b). 'Headcount Poverty Comparisons'. International Poverty Centre *One Pager* No. 18. Brasilia: United Nations Development Programme.

Subramanian, S. (2010). 'Variable Populations and Inequality-Sensitive Ethical Judgments'. In B. Basu, B. K. Chakrabarti, S. R. Chakravarti and K. Gangopadhyay

(eds), *Economics of Games, Social Choices and Quantitative Techniques*. Milano: Springer-Verlag Italia.

Trannoy, A., and J. A. Weymark (2009). 'Dominance Criteria for Critical-Level Generalized Utilitarianism'. In K. Basu and R. Kanbur (eds), *Arguments for a Better World: Essays in Honour of Amartya Sen; Volume I: Ethics, Welfare, and Measurement*. Oxford: Clarendon.