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The elasticity of substitution and labour-displacing technical change in post-apartheid South Africa

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Abstract: This paper uses normalized constant elasticity of substitution production functions to estimate the elasticity of substitution and labour-augmenting technical change in South Africa over the period 1994-2012. We find elasticities of 0.6-0.9 and positive labour-augmenting technical change, which results in an increase in capital's income share relative to labour. More broadly, we find total factor productivity (TFP) growth rates of between 1 and 2 per cent across industries, although we find no TFP growth in the mining sector. We also find that the sector with the highest TFP growth—agriculture—achieved this through shedding labour while steadily increasing output.

Keywords: elasticity of substitution, labour demand, production functions, technical growth
JEL classification: E24, O47, O41

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1 Introduction

Despite unemployment being a major economic challenge in South Africa, there is surprisingly little academic research on labour demand and the degree of substitutability between labour and other factors of production such as capital. Estimates based on firm-level data (Behar 2010) suggest that capital and labour are substitutes, whereas those at a more aggregated level generally find an elasticity of substitution below unity. However, these previous estimates of the constant elasticity of substitution (CES) production function for South African using economy-wide or industry data have produced varying results. Bonga-Bonga (2009) uses ARDL (autoregressive distributed lag) techniques to estimate a CES production function with Quantec's[1] data for the period 1970-2006 and finds an elasticity of 0.125 and a labour coefficient of 56 per cent. In contrast, Fedderke and Hill (2006), using a similar method, finds a CES parameter between 0.63 and 0.7 in the manufacturing sector for the period 1970 to 2004.

Outside of South Africa there has been a renewed interest in estimating the elasticity of substitution and its implications for factor-augmenting technical change. While the Cobb-Douglas production function, which assumes an elasticity of unity between labour and capital, has become standard in dynamic macroeconomics there is ample evidence suggesting an elasticity of substitution well below unity for the US (Klump et al. 2007), sub-Saharan Africa (SSA) (Mallick 2012) and for South Africa (Bonga-Bonga 2009; Fedderke and Hill 2006). This suggests a more flexible functional form is more appropriate for production function estimates.

In this paper we estimate a normalized CES production function for the period from 1994 to 2012 using data at the one-digit Standardized Industrial Classification (SIC) level. These estimates incorporate some of the methodological innovations made recently (see Klump et al. 2012 for a survey of these innovations). We find elasticities of substitution between 0.6-0.9 and positive labour-augmenting technical change for the industries analysed. These results imply that technical change in South Africa favours capital and results in an increase in capital's income share relative to labour's. More broadly we find that total factor productivity (TFP) growth is between 1 and 2 per cent for most industries, with the mining sector showing no TFP growth, and that high TFP growth in the agricultural sector is a result of dramatic labour reductions combined with steadily increasing output.

The paper begins by discussing the CES production function and how to obtain economically meaningful estimates of its parameters through normalization. It goes on to discuss empirically estimating the function in Section 3 and the dataset used in the paper in Section 4. Section 5 reports the estimation results and Section 6 concludes.

2 The CES function

2.1 The standard CES production function

The standard CES production function, as in (1), describes how the combination of labour, $L_{i,t}$, and capital, $K_{i,t}$, results in output $Y_{i,t}$ for each industry i in period t.

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[1] Quantec is a database for South African data.

$$Y_{i,t} = F(K_{i,t}, L_{i,t}) = \left(\delta_{i,t}(\Gamma l_{i,t} L_{i,t})^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \delta_{i,t})(\Gamma k_{i,t} K_{i,t})^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{\lambda_i \sigma_i}{\sigma_i - 1}} \quad (1)$$

In (1) $\delta_{i,t}$ represents labour's share of total income, while σ_i denotes the (constant) elasticity of substitution. λ_i indicates returns to scale, with $\lambda_i < 1$ indicating decreasing returns to scale, $\lambda_i = 1$ indicating constant returns to scale and $\lambda_i > 1$ indicating increasing returns to scale. $\Gamma l_{i,t}$, $\Gamma k_{i,t}$ represent labour- and capital-augmenting technology, respectively. Through these variables, TFP can include industry fixed effects, unrestricted time effects or reveal linear time trends. With non-linear estimation of this function, as in this paper, returns to scale cannot be identified simultaneously with the CES parameter and technology. Since these are the focus of this paper we set $\lambda_i = 1 \; \forall \; i$.

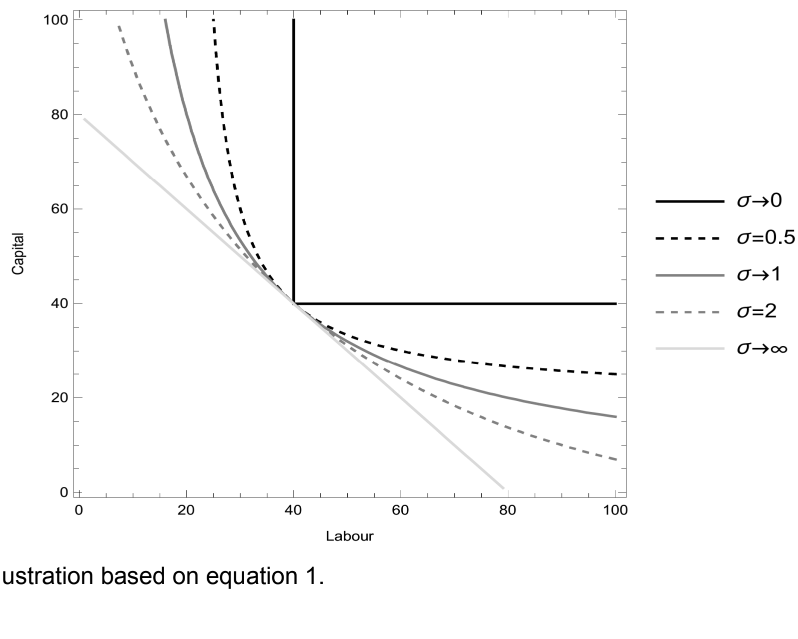
2.2 The elasticity of substitution

The elasticity of substitution is a unit-less measure of the percentage change in the proportion of two inputs associated with a percentage change in the marginal rate of technical substitution when holding all other inputs and outputs constant (Jehle and Reny 2011: 128-9).

$$\sigma \in [0, \infty) = \frac{d(K/L)/(K/L)}{d(F_L/F_K)/(F_L/F_k)} = \frac{d\log(K/L)}{d\log(F_L/F_K)} = \frac{d\log(K/L)}{d\log(w/r)}$$

The elasticity of substitution is thus a measure of the curvature of a particular isoquant going through a particular baseline point at that particular point, with a higher elasticity indicating easier substitutability between the factors of production (Jehle and Reny 2011: 129). It is then from this baseline point that the entire system of non-intersecting isoquants is defined (Klump et al. 2012: 773). A change in the elasticity of substitution informs an entire new system of non-intersecting isoquants; following such a change the old and new isoquants are not intersecting, but tangent at the baseline point as in Figure 1. At this point, the old and the new CES production function are still characterized by the same factor proportion and marginal rate of technical substitution (MRTS) (Klump et al. 2012: 773).

Figure 1: The elasticity of substitution



Source: Authors' illustration based on equation 1.

2.3 Dimensionality and normalization

The elasticity of substitution is a unit-less measure of the curvature of an isoquant at a particular baseline point and normalization is the process whereby the production function is fixed with respect to a particular baseline point, with specific values of the factors of production, the MRTS and the factor shares, so that all isoquants with different elasticities are tangent at the same point (Klump et al. 2012: 776). It can be shown that where the production function is not normalized the CES production function's share parameters have no economic meaning as they are dependent on underlying dimensions, that is they are dependent on the normalization point and the elasticity of substitution itself (Cantore and Levine 2012: 1932; Klump et al. 2012: 770). Normalization describes the representation of production relations in a consistent indexed number form, meaning that the variables in the production function become of the same dimension which allows for the de La Grandville hypothesis (1989; see also Klump and de La Grandeville 2000) of increasing growth with respect to higher substitution elasticity to hold (Klump et al. 2012: 770, 782). In this context normalization thus means that the production function is defined in a theoretically consistent manner. The normalized CES production function is given in (2).

$$\frac{Y_{i,t}}{Y_{i,0}} = F(K_{i,t}, L_{i,t}) = \left(\delta_{i,0}\left(\frac{\Gamma l_{i,t}}{\Gamma l_{i,0}}\frac{L_{i,t}}{L_{i,0}}\right)^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\delta_{i,0})\left(\frac{\Gamma k_{i,t}}{\Gamma k_{i,0}}\frac{K_{i,t}}{K_{i,0}}\right)^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i - 1}} \quad (2)$$

2.4 The elasticity of substitution and biased technical change

Under perfect competition profits are maximized where the marginal return of each factor equals its marginal cost. The ratio of the cost of capital to wages at the point of profit maximization can thus be expressed as in (3). After multiplying by the ratio of capital to labour on both sides, the relative factor income shares can then be expressed as in (4).

$$\frac{r_{i,t}}{w_{i,t}} = \frac{(1-\delta_{i,0})}{\delta_{i,0}}\left(\frac{\Gamma k_{i,t}/\Gamma k_{i,0}}{\Gamma l_{i,t}/\Gamma l_{i,0}}\right)^{\frac{\sigma_i - 1}{\sigma_i}}\left(\frac{K_{i,t}/K_{i,0}}{L_{i,t}/L_{i,0}}\right)^{\frac{-1}{\sigma_i}} \quad (3)$$

$$\Theta_{i,t} = \frac{r_{i,t}}{w_{i,t}}\frac{K_{i,t}}{L_{i,t}} = \frac{(1-\delta_{i,0})}{\delta_{i,0}}\left(\frac{\Gamma k_{i,t}/\Gamma k_{i,0}}{\Gamma l_{i,t}/\Gamma l_{i,0}}\right)^{\frac{\sigma_i - 1}{\sigma_i}}\left(\frac{K_{i,t}/K_{i,0}}{L_{i,t}/L_{i,0}}\right)^{\frac{\sigma_i - 1}{\sigma_i}} \quad (4)$$

From (3) and (4) the impact of biased technical change under different values of the substitution elasticity can be easily identified. In both (3) and (4) the impact of biased technical change can be shown to depend on the sign of $\sigma_i - 1$ (Klump et al. 2012: 781). Where $\sigma_i > 1$ factors are gross substitutes so that factor $J \in \{K, L\}$ augmenting technical change favours factor J. In this context an increase in capital-augmenting technical change, for example, will increase the return on capital relative to wages and increase capital's share relative to labour's share in total income. Furthermore, capital deepening will also lead to an increase in capital's share of total income. Where $\sigma_i < 1$, factors are gross complements, that is factor J augmenting technical change will favour the other factor. Thus, capital-augmenting technical change will tend to favour labour by increasing its factor income share and relative income. Capital deepening will also tend to increase labour's income share. It is further clear that different values of the elasticity of substitution and factor-biased technical change will yield observationally equivalent outcomes. For example, an elasticity above unity with $\Gamma k_{i,t} > \Gamma l_{i,t}\ \forall i, t$ would account for the same outcome as an elasticity below unity with $\Gamma k_{i,t} < \Gamma l_{i,t}\ \forall i, t$, in that in both cases technical change is capital biased, meaning an increase in capital's return and income share.

$$sgn\{\frac{r_{i,t}/w_{i,t}}{\Gamma k_{i,t}/\Gamma l_{i,t}}\}, sgn\{\frac{\Theta_{i,t}}{\Gamma k_{i,t}/\Gamma l_{i,t}}\}, sgn\{\frac{\Theta_{i,t}}{K_{i,t}/L_{i,t}}\} = sgn\{\sigma_i - 1\}$$

3 Estimation of the CES production function

In this paper, the CES production function will be estimated using the normalized systems approach, which is shown by León-Ledesma et al. (2010a) and Klump et al. (2007) to allow for the simultaneous identification of factor-augmenting technical change and the elasticity of substitution. The systems approach is favoured over the single equation approach as it makes explicit the assumption of profit maximization underlying the production function (León-Ledesma et al. 2010a: 1342). Econometrically, estimating the production function as a system increases the degrees of freedom, allowing for greater efficiency, as well as the application of cross-equation parameter restrictions (León-Ledesma et al. 2010a: 1342). Following the literature it is assumed that the factors of production earn their marginal revenue product so that $\delta_{i,t}$ is replaced with an estimate of the labour income share (Klump et al. 2007).[2]

3.1 The systems approach, the CES parameter, and factor-augmenting technology

The normalized system to be estimated takes the form of equations (5) to (7), where $\bar{\delta}$ is the arithmetic mean of labour share per industry, $\bar{t}$ is the arithmetic mean of the time period, and $\widetilde{X_i}$ represents the geometric mean of capital, labour, and output.[3] The data for each industry is normalized separately, thereby controlling for industry-specific fixed effects.[4] $w_{i,t}$ and $r_{i,t}$ represent the natural logarithm of wages and income respectively. ξ_i is the normalization constant that attempts control for the fact that geometric means are employed when normalizing the data. The value of ξ_i is expected to be around unity (León-Ledesema et al. 2010a: 1341; Klump et al. 2007: 101-2). The nature of factor-augmenting technical progress will be introduced with the results in the next section.

$$\mathrm{Log}(\frac{Y_{i,t}}{\widetilde{Y_i}}) = \mathrm{Log}(\xi_i) + \frac{\sigma_i}{\sigma_i - 1}\mathrm{Log}(\bar{\delta}_i(e^{\gamma_{l,i}(t-\bar{t})}\frac{L_{i,t}}{\widetilde{L_i}})^{\frac{\sigma_i-1}{\sigma_i}} + (1-\bar{\delta}_i)(e^{\gamma_{k,i}(t-\bar{t})}\frac{K_{i,t}}{\widetilde{K_i}})^{\frac{\sigma_i-1}{\sigma_i}}) \quad (5)$$ [5]

$$w_{i,t} = \mathrm{Log}(\bar{\delta}_i\frac{\widetilde{Y_i}}{\widetilde{L_{,i}}}) + \frac{1}{\sigma_i}\mathrm{Log}(\frac{Y_{i,t}/\widetilde{Y_i}}{L_{i,t}/\widetilde{L_{,i}}}) + \frac{\sigma_i - 1}{\sigma_i}(\mathrm{Log}(\xi_i) + \gamma_{l,i}(t-\bar{t})) \quad (6)$$ [6]

$$r_{i,t} = \mathrm{Log}((1-\bar{\delta}_i)\frac{\widetilde{Y_i}}{\widetilde{K_{,i}}}) + \frac{1}{\sigma_i}\mathrm{Log}(\frac{Y_{i,t}/\widetilde{Y_i}}{K_{i,t}/\widetilde{K_{,i}}}) + \frac{\sigma_i - 1}{\sigma_i}(\mathrm{Log}(\xi_i) + \gamma_{k,i}(t-\bar{t})) \quad (7)$$

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[2] One potential limitation of using this in the South African context is if wages and productivity are 'de-linked' or only weakly correlated, as may be the case in South Africa.

[3] This follows the methodology used by León-Ledesma et al. (2010b) and Klump et al. (2007).

[4] Except for dramatically increasing the amount of time taken to converge, the inclusion of industry-specific normalization constants does not change the estimated values of the CES parameter or the technological growth terms.

[5] We have attempted specifying flexible technological growth of the box-cox form used in Klump et al. (2007). Using this flexible form results in failure to converge in most cases. Where the model does converge, results are often implausible or non-robust with respect to sample period.

[6] Note the constant of this equation, while being set to ξ_1 in estimation, is defined as $\mathrm{Log}(\bar{\delta}_i\frac{\widetilde{Y_i}}{\widetilde{L_{,i}}}) + \frac{1}{\sigma_i}\mathrm{Log}(\frac{\widetilde{L_{,i}}}{\widetilde{Y_i}})$. Similarly in the equation below the constant will be $\mathrm{Log}(\bar{\delta}_i\frac{\widetilde{Y_i}}{\widetilde{L_{,i}}}) + \frac{1}{\sigma_i}\mathrm{Log}(\frac{\widetilde{L_{,i}}}{\widetilde{Y_i}})$

In the functions above $\gamma_{l,i}$ and $\gamma_{k,i}$ reflect labour-augmenting and capital-augmenting technological growth so that $\Gamma j_{i,t} = e^{\gamma_{j,i}(t-\bar{t})}$.[7] We estimate the system above under different assumptions of technical growth, specifically Harrod-neutral, Solow-neutral, Hicks-neutral and factor-augmenting technical change. In the Harrod-neutral technical growth specification we are assuming that technical change is only labour-augmenting so that, by construction, $\gamma_{k,i} = 0$ and we estimate $\gamma_{l,i}$ freely. The assumption of Solow-neutral technical growth implies capital-augmenting technical change so that we fix $\gamma_{l,i} = 0$ and estimate $\gamma_{k,i}$. Hicks-neutral technical change implies that we set both labour- and capital-augmenting technical change to be equal $\gamma_{k,i} = \gamma_{l,i} > 0$. Finally, the assumption of factor-augmenting technical change allows us to estimate $\gamma_{k,i} \neq \gamma_{l,i}$ simultaneously. TFP growth implied by the CES production function can be obtained through the Kmenta approximation (Klump et al. 2012: 789-91). In our formulation the TFP of the economy can be approximated by (8) (Klump et al. 2007: 186).

$$TFP_i = \bar{\delta}_i\gamma_{l,i} + \left(1 - \bar{\delta}_i\right)\gamma_{k,i} - \frac{1-\sigma_i}{\sigma_i}\frac{\left(\bar{\delta}_i(1-\bar{\delta}_i)\right)}{2}\left(\gamma_{l,i} - \gamma_{k,i}\right)^2 \quad (8)$$

3.2 The iterative feasible generalized non-linear least squares estimator

Most of the work on the estimation of the aggregate CES production function has employed non-linear feasible generalized least squares estimators (FGNLS) to estimate the system of equations (5) to (7). This approach is confirmed to be superior to the Kmenta approximation and estimation of a single equation of first-order conditions by León-Ledesma et al. (2010a).

This paper uses Stata's *nlsur* command which by default employs a two-step FGNLS estimator to estimate a non-linear system of equations (StataCorp. 2013: 1497, 1500). The FGNLS estimator is a non-linear expansion of Zellner's seemingly unrelated regression model, meaning that it allows the errors across regressions to be correlated and allows for the imposition of cross-equation restrictions (StataCorp. 2013: 1500). Luoma and Luoto (2011), however, show that this estimator is internally inconsistent so that the estimate of the elasticity of substitution is biased towards unity and thus advocate the use of a Bayesian Full Information method, which, by construction, avoids this inconsistency. In an attempt to control for this we employ Stata's iterative feasible generalized non-linear least squares (IFGNLS) estimator which is equivalent to maximum likelihood estimation.[8]

4 Data sources and descriptive statistics[9]

4.1 Labour

The labour data used in this study is taken from the Post-Apartheid Labour Market Series (PALMS) 1994-2012 compiled by Datafirst[10] from Statistics South Africa's (StatsSA) October Household Surveys (OHS), Labour Force Surveys (LFS), and Quarterly Labour Force Surveys (QLFS) (Kerr et al. 2014). The OHSs were administered on an annual basis between 1995 and

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[7] Following Klump et al. (2007: 185) we set the constant efficiency levels of these parameters to zero due to normalization.

[8] As expected these results are on average further away from unity than the standard FGNLS estimates.

[9] Appendix C provides an evaluation of the Quantec and StatsSA data via Cobb-Douglas Production Functions.

[10] This data may be obtained from: http://datafirst.uct.ac.za/dataportal/index.php/catalog/43

1999, before being replaced by the biannual LFSs from 2000 to 2007. In 2008 StatsSA launched the QLFS, and we use this data up to the third quarter of 2011. This provides us with thirty-six consecutive, but unevenly spaced surveys spanning the years from 1994 to 2012. These surveys include individual responses to questions regarding employment, years of schooling completed and industry of employment that can be used to estimate the number of formal sector employees working in different industries as well as their average years of completed schooling.

Many papers have investigated the problems in comparing the StatsSA household surveys (Altman 2008; Burger and Yu 2006; Casale et al. 2004; Kingdon and Knight 2005) and particularly the effect that modifications in questionnaire design and sampling methodology may have had on the comparability of the household surveys over time. The most serious comparability problems occur for informal sector or self-employed workers, so that the effect of these inconsistencies can be limited by omitting these workers from the sample and restricting our dataset to formal sector employees only. Since the capital and output data are gathered using a sampling frame of formal sector firms, omitting individuals who are known to be employed in the informal sector is also likely to improve the internal consistency of our dataset.

After omitting all the unemployed, economically inactive, and self-employed workers from the dataset, aggregate industry employment per period is calculated from the individual responses to the questions regarding industry of employment and the survey weights.[11] Since many of the measurement issues are ameliorated by aggregation, we will primarily focus on the nine different industries[12] using the SIC one-digit[13] categories. These nine industries are therefore used as the cross-sectional units of observation for our production function model. Appendix A provides a brief overview of potential areas of measurement error.

We also calculate the total number of hours worked per industry as follows, where the mean number of hours worked per week per industry is ascribed to all workers, adjusted to reflect monthly values and then multiplied by total labour in the industry:

$$H_{i,t} = \overline{hours\ worked_{\iota,t}} \times 4 \times L_{i,t}$$

As shown in Figure 3, the employment trends do not differ qualitatively from those in Figure 2. Furthermore, employing total hours worked instead of total employment per sector does not substantially alter any estimates below.[14]

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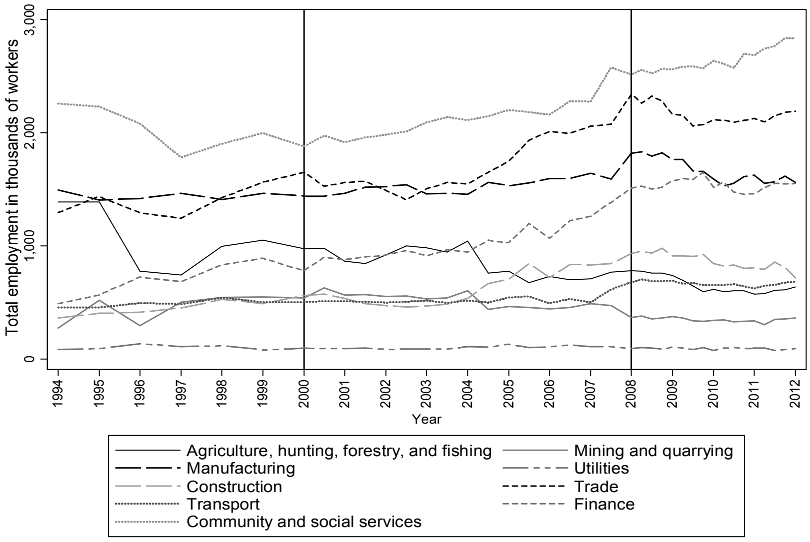
[11] Unless otherwise stated, we use the second set of cross-entropy weights calculated by Branson and Wittenberg (2014) provided by DataFirst. The results obtained using StatsSA's original survey weights were mostly indistinguishable from those reported.

[12] The nine industries are (1) agriculture, forestry, and fishing, (2) mining and quarrying, (3) manufacturing, (4) electricity, gas, and water, (5) construction, (6) wholesale and retail trade, catering and accommodation, (7) transport, storage, and communication, (8) finance, insurance, real estate, and business services, and (9) community, social, and personal services.

[13] Although these variables are also available at the two-digit and three-digit sector level (as used by Fedderke 2005), constructing the employment variable at this lower level of aggregation would mean using fewer observations for each estimate and compounding any measurement error and sampling variation in these variables. Furthermore, the output and capital stock measures are only released at the annual frequency *annually* at this lower level of disaggregation, so increasing the number of cross-sectional observations comes at the cost of reducing the time dimension of our analysis.

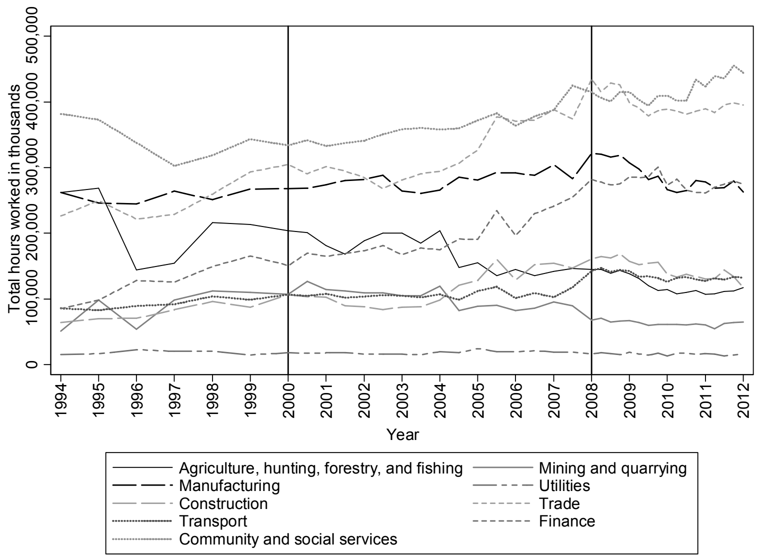
[14] These estimates are available from the authors upon request.

Figure 2: Total employment, by industry



Source: Authors' own calculations, based on the PALMS.

Figure 3: Total employment in hours worked, by industry



Source: Authors' own calculations based on PALMS.

Labour quality and education

In the estimation of production functions it is preferable to have labour not only reflect the number of people working, but also the quality of work done. In an attempt to control for differences in labour quality over time the Ho-Jorgenson (1999) methodology is employed to create a labour quality series. Ho and Jorgenson (1999: 4-6) argue that a constant quality index of labour input may capture the substitution of different labour types by scaling the components with respect to their marginal products, that is wages, and thereby allow for the separation of substitution and labour quality growth. This methodology has support in the empirical literature, with most of the literature on normalized CES production functions employing the quality index (Klump et al. 2007). While this methodology is attractive in the South African case, the lack of industry-specific earnings data for workers with different education levels is a binding constraint as it means that the indices, which are in general identified by using the changes in labour composition per industry over time, cannot be applied to the entire sample. These breaks in the earnings data imply that an index can only be generated for 14 consecutive periods. As an experiment, we created a Ho-Jorgenson index controlling for the education composition of labour only; using the index resulted in dramatically worse results statistically. Should more labour data become available, or if annual data only is used, there may be potential for labour quality indices accounting for both education and, specifically, age in an attempt to control for the perceived differences in educational quality of younger persons. The creation of this index is, however, beyond the scope of the present paper.

4.2 Capital stock

Annual fixed capital stock data by industry is obtained from Quantec, and this data corresponds to the capital stock series reported by the South African Reserve Bank (SARB). Furthermore, quarterly fixed capital formation for all of the industries is used to calculate the quarterly capital stock by simultaneously solving the equations below for capital stock $k_{t,q}$ in quarter q of year t. K_t and K_{t-1} are the gross fixed capital stock values reported at the end of the year t, while $I_{t,q}$ is the gross fixed capital formation for the quarter q in year t. The solution of these equations also allow for quarterly depreciation, d, to be identified per sector. The depreciation rates can then be used to construct quarterly capital series. Figure 4 shows gross fixed capital stock calculated for all industries in the included sample.

$$k_{t,1} = (1 - d)K_{t-1} + I_{t,1}$$

$$k_{t,2} = (1 - d)k_{t,1} + I_{t,2}$$

$$k_{t,3} = (1 - d)k_{t,2} + I_{t,3}$$

$$K_t = (1 - d)k_{t,3} + I_{t,4}$$

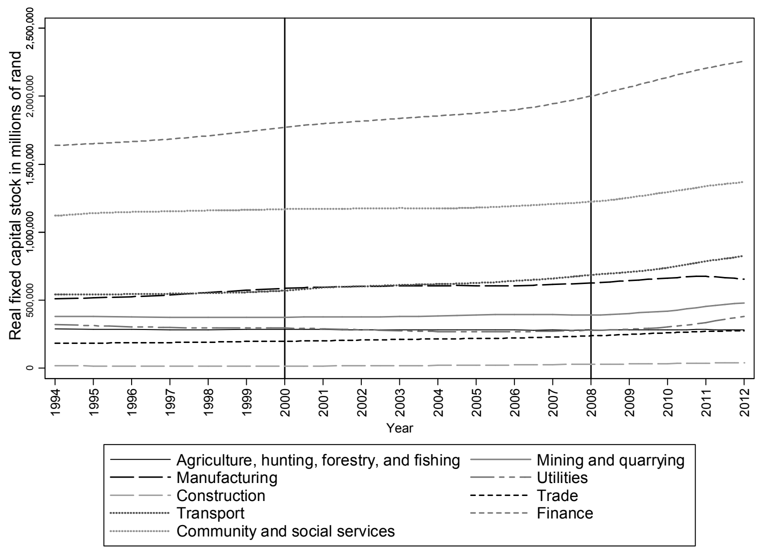
4.3 Output

The output series is obtained from StatsSA's P0441—Gross Domestic Product (GDP) series. This dataset is used as it includes total compensation of employees per industry per quarter from 1994. The deflators for each industry were also calculated from this dataset using real and nominal output. The deflators are thus defined using the below equation.[15] Real GDP per industry for the sample period is provided in Figure 5.

$$deflator_{i,t} = \frac{Nominal\ GDP_{i,t}}{Real\ GDP_{i,t}}$$

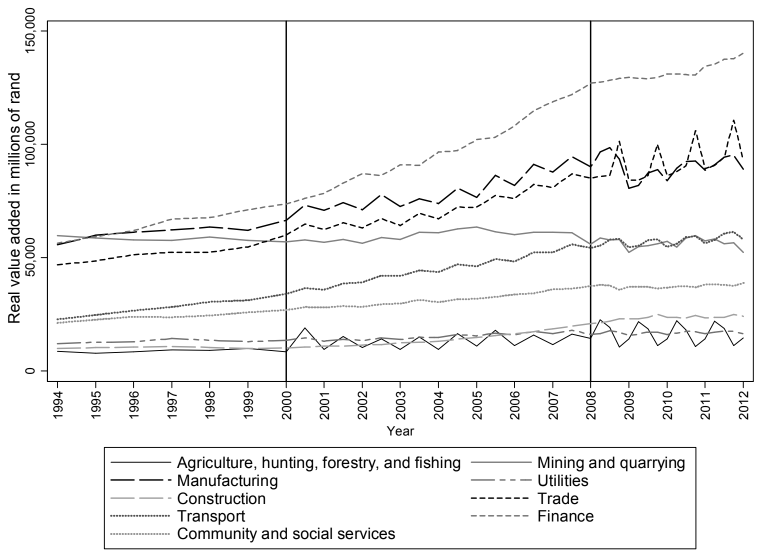
[15] These deflators were used to adjust the nominal earnings data in PALMS.

Figure 4: Fixed capital stock, by industry



Source: Authors' own calculations based on Quantec and SARB data.

Figure 5: Output, by industry



Source: Authors' own calculations based on StatsSA P0441.

4.4 Wages and labour's share of income

As already discussed, the estimates of CES production function parameters can be substantially improved if a measure of labour's income share is incorporated into the estimation. Further gains can be achieved if, instead of only estimating the effect of labour and capital on production, we simultaneously attempt to explain wages and capital costs conditional on the hiring and capital investment decisions of firms. However, these approaches will only improve the reliability and efficiency of our estimators if the additional data on the labour income share, wages and capital costs are reliable.

One of the stylized facts of international growth is that labour's share of income remains relatively constant over long periods of time, and is around two-thirds in most countries. According to the National Accounts data (SARB code KBP6295L), this share has been fluctuating between 47 per cent and 60 per cent for South Africa since 1970, which is low but not completely outside the norm by international standards. However, this economy-wide average hides substantial between-industry variation in the income share that accrues to workers. Whereas workers in the community and social services industry earn around 70 per cent of total income, their counterparts in the transportation and utilities industries only receive around 30 per cent. The labour income shares for all the industries are presented in Appendix B.

Of course, it ought to be possible to obtain these shares by combining total income data from the national accounts data with wage and employment data from the household surveys. This approach would be particularly useful if we wanted the production function to include different types of workers, since the national accounts data does not provide the income shares by worker type. Unfortunately, there are indications that workers, on average, tend to under-report their wages by a substantial margin, and this leads to implausibly low estimates of workers' share in income (Wittenberg 2014). The same issue also affects attempts to obtain more efficient parameter estimates via a systems estimator that attempts to explain simultaneously output, employment, and capital levels.

Imputing of wage and cost of capital data

We impute wage data for each period in the model using the equation below, so that total income per reported quarter is equal to the value of labour's share of total income, $\delta_{i,t}Y_{i,t}$, divided by the total number of employed persons in the industry at the given time period. In Figure B4 in Appendix B the wages imputed from the equation below are compared to those from the PALMS dataset. The figure indicates that, compared to the national accounts, wages in the PALMS data are under-reported by a factor of around 50 per cent for the agriculture and construction industries, and in the area of 30 per cent for almost all other industries except community and social services, which seem to report the most accurately. While a clear increase in reported income in PALMS relative to income inferred from the national accounts is observed from the period before 2008 and the period after 2010, these differences are likely due to changes in the LFS and QLFS survey design and not due to a fundamental shift in income. Table B1 in Appendix B shows the coefficients of the regression of the PALMS data on the National Accounts wages per industry. The results indicate a strongly significant, albeit small coefficient for all industries. This implies that while the two sources differ on the exact amount of wages and the exact trends, they do not differ in the direction of wage growth over the sample.

$$W_{i,t} = \frac{\delta_{i,t}Y_{i,t}}{L_{i,t}}$$

The cost of capital $R_{i,t}$, is then defined as in:

$$R_{i,t} = \frac{(1-\delta_{i,t})Y_{i,t}}{K_{i,t}}$$

Wages by education category in nested regressions

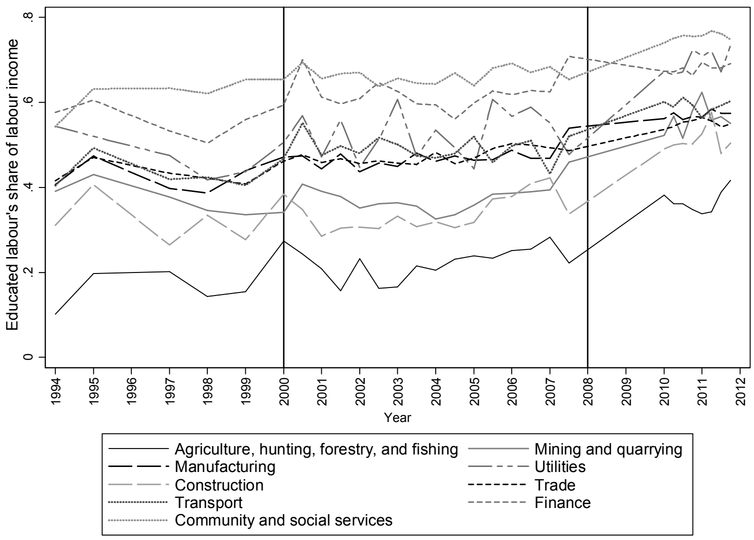
We follow a similar approach as before in the calculation of wages per education group when estimating the nested CES production function. Wages per education grouping are defined in the equation below. The PALMS data is used to inform the total weighted earnings of education group e in industry i in period t, the share of people within this category is then determined by dividing the value by the sum of total earnings for all education categories. This approach allows us to scale wages upwards to those reported in the national accounts. The approach ignores individuals with missing education values and may thus overestimate the share of educated labour.

$$W_{e,i,t} = \frac{Total\ earnings_{e,i,t}}{\sum_{e=1}^{E} Total\ earnings_{e,i,t}} W_{i,t}$$

We separate labour into two categories, more educated labour and less educated labour (Figure 6). With more educated labour being those with 12 or more years of education and less educated labour defined as those with less than 12 years of education. This choice was made due to the low numbers of workers with more than 12 years of education.[16] In Appendix B we report the share of labour income earned by those with some tertiary education (Figure B5), those with only matric (Figure B6), those with between 10 and 11 years of education (Figure B7), and those with less than 9 years of education (Figure B8). From these figures it can be seen that the share of labour income earned by highly educated workers is constant up to 2007, increasing dramatically after 2010. This increase in income share of educated workers seems to come at the expense of less educated workers for the most part, as the income shares of persons with some high school education are relatively constant over the period.

[16] Further disaggregation in terms of education was attempted, with those with primary or no schooling being separated from those with some high school education. The resulting complexity of the system of equations often results in Stata rejecting the system outright, due to the length of the command. Where restrictions are made, the models fail to converge in general.

Figure 6: More educated labour's share of labour income[17]



Source: Authors' own calculations based on PALMS and National Accounts data.

4.5 Data structure

The dataset used is a mix of annual, biannual, and quarterly data. As our production function is inherently static, there is no distinction between long- and short-run elasticities. In the normalization approach, however, we do control for time variation such that the technological growth terms are expressed in terms of annual rates. As we will see, the introduction of cyclical variation due to higher frequency data has an impact on some of the estimates.

4.6 Normalized values

As already noted, we use the standard conventions in normalizing the data by taking the geometric mean of the factors of production and the arithmetic mean of labour's share and the time period (León-Ledesma et al. 2010b; Klump et al. 2007). In Table 1 these values are reported.

Table 1: Normalization values per industry

Industry	Time	Labour share	Output*	Capital*	Labour
Agriculture	2005	0.339	13,327	283,000	780,392
Mining	2005	0.412	58,131	402,000	424,355
Manufacturing	2005	0.516	79,991	615,000	1,570,072
Utilities	2005	0.369	15,550	297,000	98,924
Construction	2005	0.478	16,488	25,200	676,406
Trade	2005	0.436	75,136	229,000	1,824,770
Transport	2005	0.342	45,807	659,000	575,007
Finance	2005	0.366	102,751	1,940,000	1,143,757
Services	2005	0.646	32,305	1,220,000	2,313,580

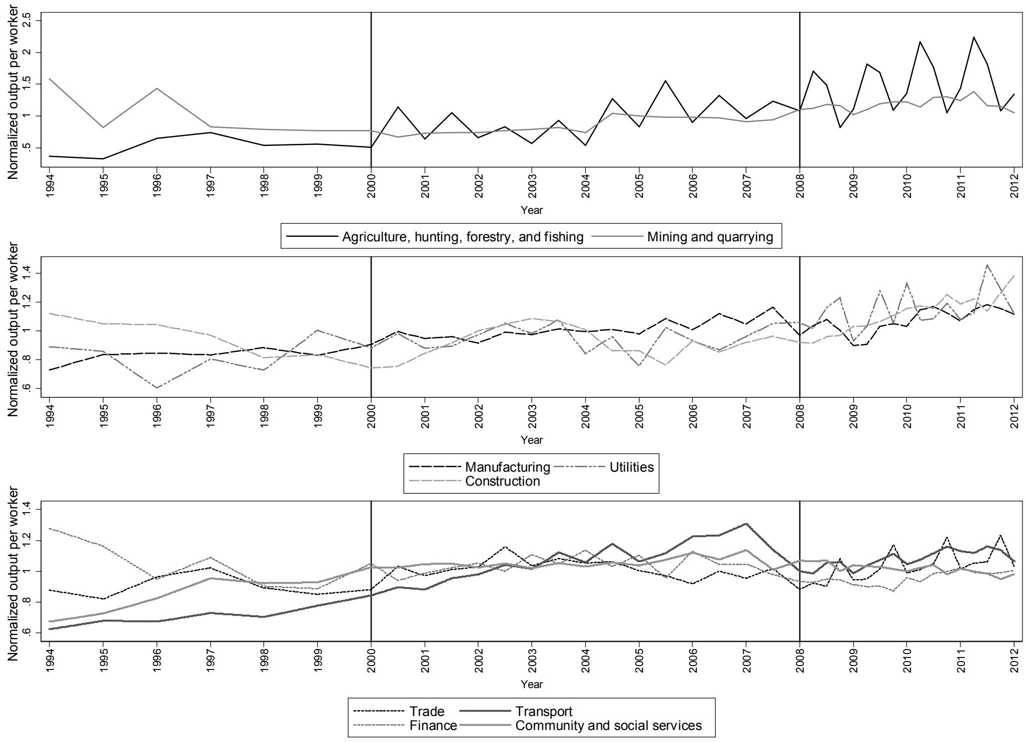
Note: * Value is expressed in millions of Rand.

Source: Authors' own calculation based on PALMS, Quantec, and StatsSA data.

[17] No wage data is available for 1996, or from 2008 to fourth quarter of 2009. Problematically, this period also seems to coincide with a dramatic increase in the share of educated labour in total labour income.

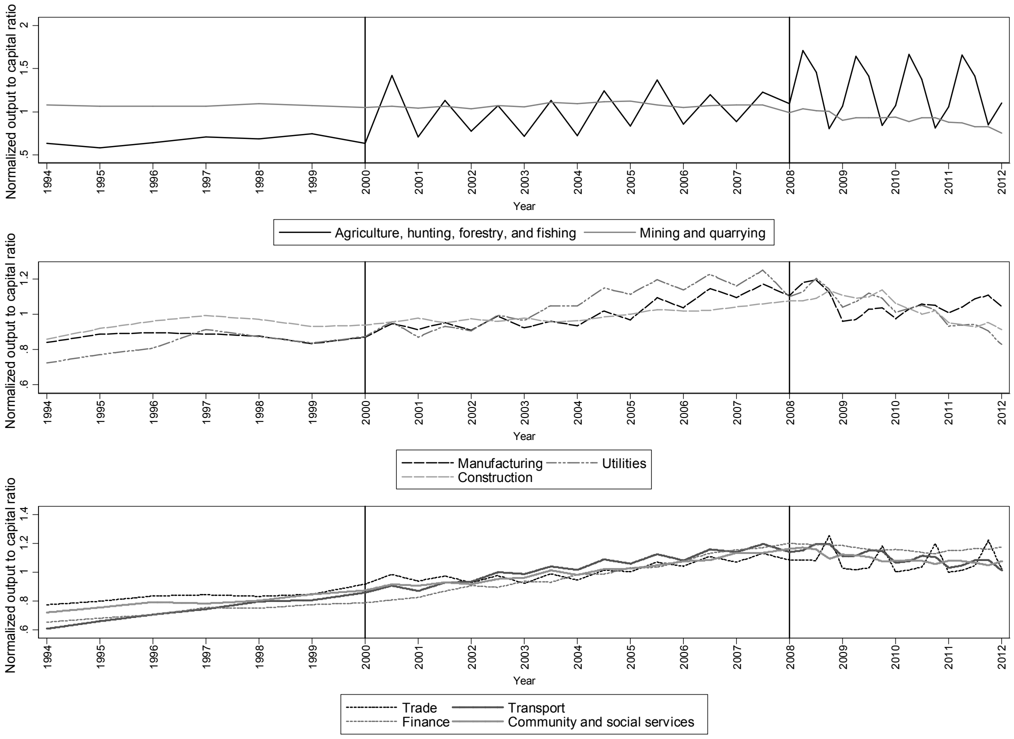
When normalizing the data certain trends are more easily comparable, specifically the substantial increase in output per worker observed in the agricultural sector, compared with the relatively constant rise in output per worker in the secondary sector and trade, transport, and community and social services industries (Figure 7). The rise in the output per worker appears to have taken hold following 2004. Against this output per capital (Figure 8) has grown at a slower pace in the agricultural sector while growing sharply in the tertiary sector. In the mining sector there are signs of capital intensification, with output per worker growing while output and output per unit of capital is decreasing.

Figure 7: Normalized output per worker



Source: Authors' own calculations based on StatsSA and PALMS data.

Figure 8: Normalized output per unit of capital



Source: Authors' own calculations based on StatsSA and Quantec data.

5 Results

All estimates reported in this section will refer to the CES value in its standard form so that it may be directly interpreted. For the same reason all industry-specific estimates are also already computed.

5.1 Previous estimates

Previous attempts to estimate the CES production function for South African have produced varying results. Bonga-Bonga (2009) uses ARDL techniques to estimate a CES production function using Quantec's data for the period 1970-2006.[18] He finds an elasticity of 0.125 and a labour coefficient of 56 per cent. In contrast, Fedderke and Hill (2006), using a similar method, finds a CES parameter between 0.63 and 0.7 in the manufacturing sector for the period 1970 to 2004. Mallick (2012) uses the normalized approach to estimate the elasticity of substitution for ninety countries in SSA, and finds an average elasticity of around 0.275 but results for individual countries show significant variation. The elasticity of substitution for Nigeria, Lesotho, Brazil, and Sri Lanka are found to be 0.83, 0.17, 0.13, and 0.42, respectively.

[18] One potential explanation for the relatively low estimate of Bonga-Bonga (2009: 335) is his linearization of the CES production function. In estimating a linear function measurement error would bias the coefficients downwards. Running Monte-Carlo Simulations suggests that measurement error will likely bias the CES estimate towards unity in the non-linear systems case. These simulations were run using Hicks-neutral technological growth.

5.2 Estimates of the elasticity of substitution and factor-biased technical change[19]

We start the analysis by estimating the system of equations (5) to (7) by specifying Harrod-neutral technical change, that is setting $\gamma_{k,i} = 0$, for all industries. We find an elasticity of substitution of 0.839 and labour-augmenting technical growth of around 4 per cent per year. Using equation (8), this implies a TFP growth rate of around 1.3 per cent and 2.5 per cent per industry. Where factor-augmenting technical change is allowed to be capital-augmenting, Hicks-neutral or factor-augmenting the elasticity of substitution is found to be 1.519, 0.992, and 1.32 respectively (Table 2). The technical change parameters read together with the estimated elasticities imply that technical change in South Africa is capital biased, increasing capital's relative income share. Based on (4), where wages are unresponsive, the reduction in labour's share of total income must come from a relative reduction in employment. In Table 3, the estimated TFP growth rate per industry is shown to be constant for each industry independent of specification, with TFP growing at between 1.35 per cent and 2.55 per cent per year.

Table 2: Results[20]

	Labour-augmenting	Capital-augmenting	Hicks-neutral	Labour- and capital-augmenting
ξ	0.983*** (0.00570)	0.981*** (0.00552)	0.982*** (0.00538)	0.980*** (0.0055)
σ	0.839*** (0.00616)	1.519*** (0.0205)	0.992*** (0.00905)	1.32*** (0.015)
γ_l	0.0396*** (0.00244)		0.0179*** (0.00108)	-0.0196*** (0.0039)
γ_k		0.0368*** (0.00191)		0.05*** (0.0034)
R2 (Y) R2 (W) R2 (R)	0.680 0.955 0.991	0.720 0.970 0.987	0.710 0.961 0.990	0.72 0.97 0.989
N	351	351	351	351

Note: *** p<0.001, **p<0.01, *p<0.05; standard errors in parentheses.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

[19] All of these estimates are reported by using total number of employed people; we did include total hours worked per sector as defined in Section 2.1. The results are not changed dramatically.

[20] Full regression output for all equations available on request. 'R2 y' refers to the R-squared of the equation for output, 'R2 w' refers to the R-squared of the wage regression and 'R2 r' refers to the R-squared of the cost of capital regression.

Table 3: Implied TFP growth by industry and type of specification

Industry	Labour-augmenting	Capital-augmenting	Hicks-neutral	Labour- and capital-augmenting
Agriculture	0.0134	0.0135	0.0134	0.0136
Mining	0.0163	0.0164	0.0163	0.0165
Manufacturing	0.0204	0.0205	0.0204	0.0206
Utilities	0.0146	0.0147	0.0146	0.0147
Construction	0.0189	0.0190	0.0189	0.0191
Trade	0.0172	0.0173	0.0173	0.0174
Transport	0.0135	0.0136	0.0135	0.0137
Finance	0.0145	0.0145	0.0145	0.0146
Services	0.0255	0.0256	0.0256	0.0257

Source: Authors' own calculations based on regression results in Table 2 and equation (8).

In the next specification (Table 4), we estimate the same system but allow for industry differences within labour-augmenting technical growth rates. We find a slightly lower elasticity for all industries. The technical change parameters change considerably with labour-augmenting technical change being the highest in the agricultural, utilities, transport, and finance sectors. These sectors are also those with the most variation in labour share of income over the sample period. This is true for all sectors except the finance sector, for which the income share remains fairly constant over the period. The mining sector is the only sector showing negative, albeit insignificant, factor-augmenting growth.

Table 4: Estimates with industry varying Harrod-neutral technological growth rates[21]

Parameters		Coeff.	SE	Implied TFP
ξ		0.991***	(0.005)	
σ		0.793***	(0.006)	
γ_l	Agriculture	0.150***	(0.008)	0.051
	Mining	-0.004	(0.007)	-0.002
	Manufacturing	0.020***	(0.005)	0.011
	Utilities	0.057***	(0.008)	0.021
	Construction	0.027***	(0.005)	0.013
	Trade	0.031***	(0.006)	0.014
	Transport	0.083***	(0.009)	0.029
	Finance	0.062***	(0.007)	0.023
	Services	0.033***	(0.004)	0.021
R2 (Y)		0.799		
R2 (W)		0.952		
R2 (R)		0.991		
N		351		

Note: *** *p*<0.001, ***p*<0.01, **p*<0.05.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

Table 5 provides estimates when the CES parameter is allowed to vary by industry. In this specification the Harrod-neutral technical growth parameter is slightly higher than that of the model presented in Table 3, and the estimated TFP parameters now range between 1.5 per cent and 3 per cent per year over the industries. The estimates of the CES parameters range between 0.73 and 0.94.

[21] Note that the estimates for technical change have already been adjusted and can be interpreted directly.

Table 5: Estimates with industry varying CES

Parameters		Coeff.	SE	Implied TFP
ξ		0.989***	(0.0057)	
σ	Agriculture	0.869***	(0.008)	0.016
	Mining	0.810***	(0.017)	0.019
	Manufacturing	0.732***	(0.029)	0.024
	Utilities	0.771***	(0.026)	0.017
	Construction	0.813***	(0.030)	0.022
	Trade	0.771***	(0.028)	0.020
	Transport	0.818***	(0.021)	0.016
	Finance	0.934***	(0.048)	0.017
	Services	0.781***	(0.030)	0.030
γ_l		0.046***	(0.003)	
R2 (Y)		0.679		
R2 (W)		0.957		
R2 (R)		0.99		

Note: *** p<0.001; ** p<0.01; * p<0.05.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

In Table 6 we allow for industry-specific technology and CES parameters and find broadly similar results to those in Tables 4 and 5. While this approach allows for the identification of industry level differences in growth and the elasticity of substitution, the estimates of technical growth for agriculture, utilities, and transport seem implausibly high.

Table 6: Estimates with industry varying technical change and CES

Industry	σ_i	$\gamma_{l,i}$	Implied TFP
Agriculture	0.809*** (0.008)	0.146*** (0.008)	0.050
Mining	0.771*** (0.021)	-0.009 (0.007)	-0.004
Manufacturing	0.784*** (0.032)	0.022*** (0.005)	0.011
Utilities	0.721*** (0.021)	0.057*** (0.008)	0.021
Construction	0.715*** (0.031)	0.021** (0.006)	0.010
Trade	0.717*** (0.027)	0.031*** (0.007)	0.014
Transport	0.754*** (0.018)	0.084*** (0.009)	0.029
Finance	0.904*** (0.039)	0.064*** (0.006)	0.023
Services	0.697*** (0.022)	0.027*** (0.004)	0.018
ξ	0.994*** (0.004)		
R2 (Y)	0.8026		
R2 (W)	0.9532		
R2 (R)	0.9907		
N	351		

Note: *** p<0.001; ** p<0.01; * p<0.05; standard errors in parentheses.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

5.3 Nested CES systems

The system discussed treats all labour as similar. We have experimented with allowing for different types of labour by separating labour between those with and without a matric (the data does not allow us to break this down further). We have tested several specifications with different controls for technology, substitution elasticity, and the definition of labour (labour hours and/or total people employed per sector). However, very few of the models converge, with those that provide solutions indicating extremely high residuals.[22] Furthermore, identification is hampered by missing wage data in the QLFS.

6 Conclusion

In this paper we estimate a CES production function for South Africa using industry level data and a variety of different assumptions. Our CES estimates are in the region of 0.7-0.85 on average and between 0.7 and 0.94 at the industry level. Overall these estimates suggest that TFP growth is in the region of 2 per cent annually over this period, but varies across industries with mining having almost no TFP growth and high TFP growth in agriculture driven by falling employment and rising output. We also find that technical change in South Africa favours capital and results in an increase in capital's income share relative to labour.

These results suggest at least two salient implications for South African economic growth and employment creation. The first is that TFP growth is not particularly high in general, and is highest in agriculture, which is shedding labour. This suggests that an important mechanism driving higher productivity in South Africa is the relative decline in labour demand. This has obvious negative implications for job creation. The second is that technical change favours capital over labour. This too suggests that South African production is becoming relatively less labour intensive over time. Both of these suggest that with current trends South Africa will not be able to create the number of jobs it needs to substantially reduce unemployment levels. To do this it is likely that significant reforms will be required to make labour more attractive to employ.

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[22] These results are available on request.

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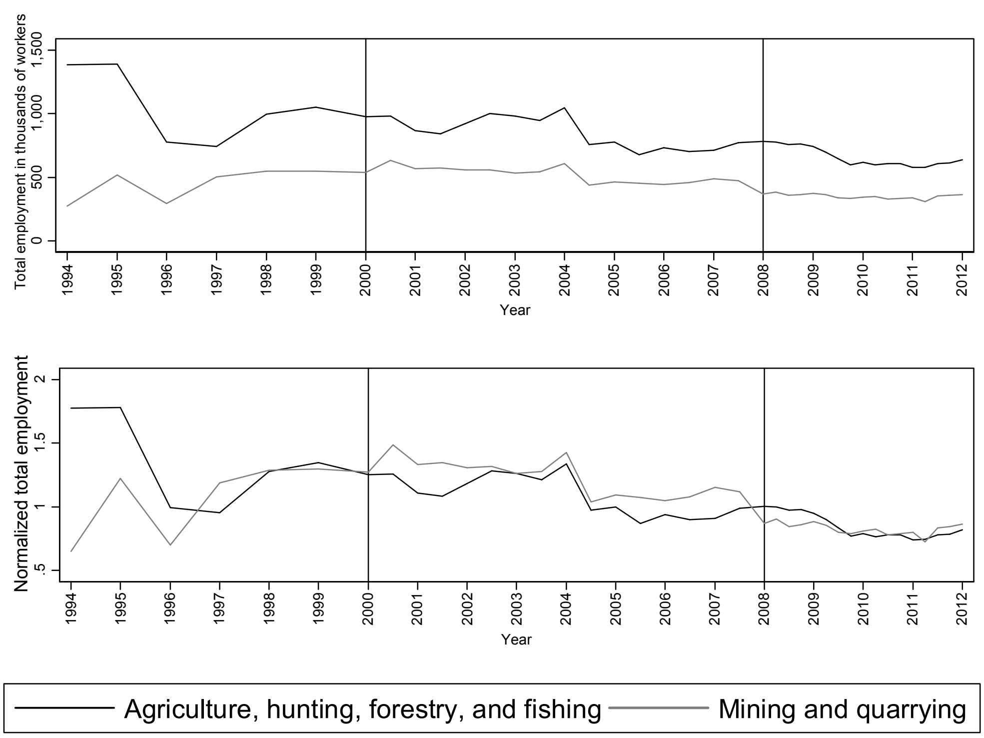
Appendix A: Measurement errors in employment

A.1.1 Labour in the primary sector

In the primary sector a sharp decline in the total number of employed people is observed in both agriculture and mining and quarrying in 2004 (Figure A1). It is unclear whether this reduction is due to survey design. This jump is more readily observed when normalizing employment in each industry by the mean of total employment from the period 2000-2011Q3. StatsSA (2004: vii) attributes the decline to severe drought in the second half of the year. While this may explain some of the shift, it does not explain the persistence of this reduction over the rest of the sample period. A further explanation may be a change in the sampling frame used by StatsSA at the end of 2003.

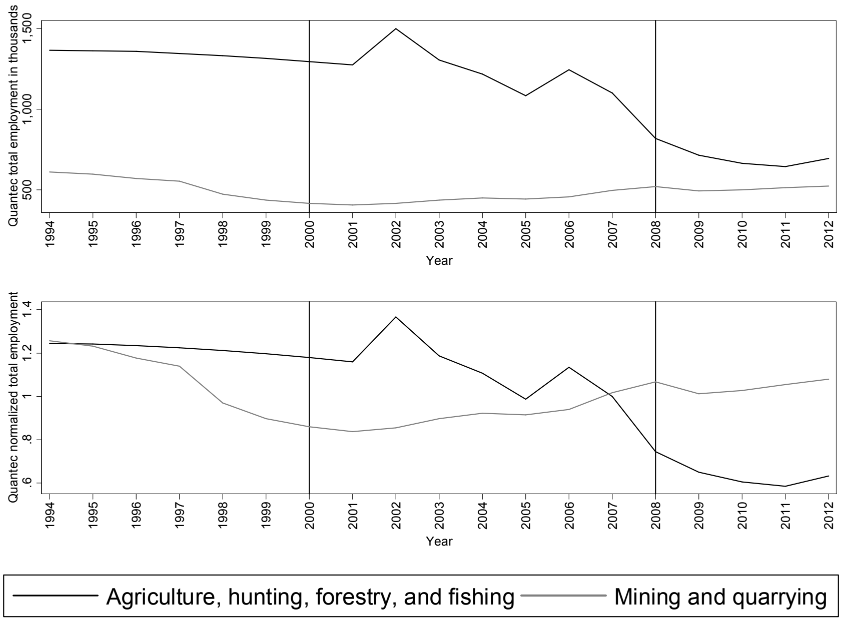
Figure A1: Total employment in the primary sector (PALMS)



Source: Authors' own calculations based on PALMS data.

The annual labour data from Quantec shows a much less pronounced trend (Figure A2), although the decline in employment in the mining sector is in contradiction with that observed in the PALMS data. The Quantec labour data also indicates a dramatic reduction in agricultural employment starting around 2007, a trend not observed to the same extent in the PALMS data.

Figure A2: Total employment in the primary sector (Quantec)



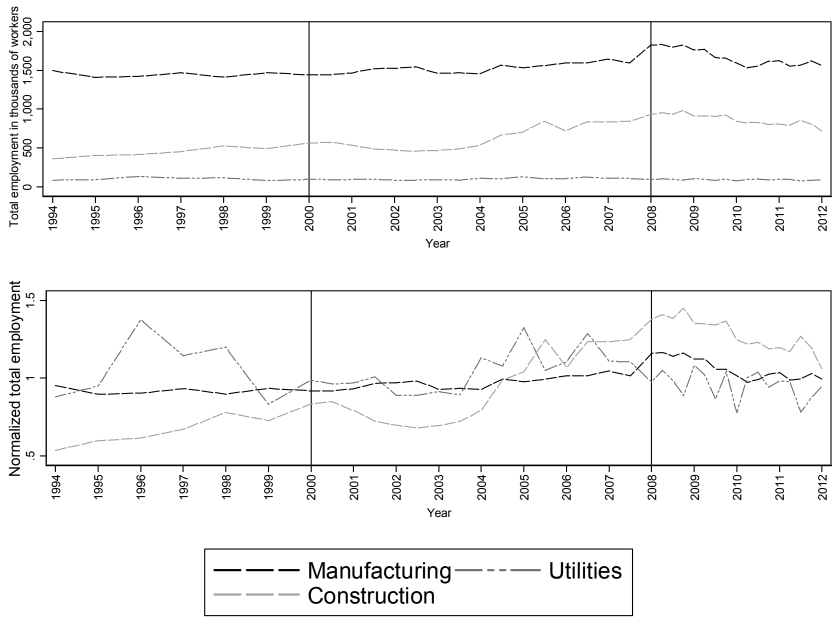
Source: Authors' own compilation based on Quantec labour data.

A.1.2 Labour in the secondary sector

Labour in manufacturing, utilities and construction is much more stable (Figure A3), although the sharp increase in manufacturing employment between the September version of the 2007 LFS and the first QLFS of 2008 may be a statistical artefact resulting from a change in the surveys. A small jump in manufacturing and construction employment is observed at 2004, the same period as the dramatic reduction in agriculture employment.

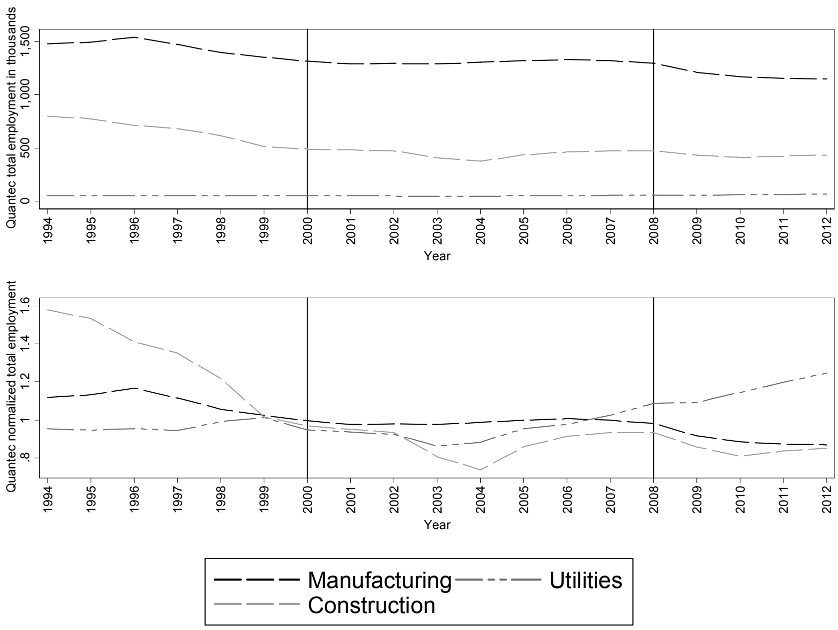
The normalized employment figures provide a more detailed account of the shifts in employment during the period, with employment in manufacturing and utilities staying relatively constant, although the latter is extremely volatile. There also appears to be a persistent jump in total persons employed in utilities after 2004, with employment in the sector being 1.2 times above the mean in the period until the first QLFS. Employment in manufacturing increased steadily, apart from a sudden 15 per cent employment increase observed in the first quarter of 2007. Employment in construction was growing at a rapid rate in the period under consideration. The Quantec data (Figure A4), on the other hand, indicates a dramatic reduction in the total number of workers in the construction sector.

Figure A3: Total employment in the secondary sector (PALMS)



Source: Authors' own calculations based on PALMS data.

Figure A4: Total employment in the secondary sector (Quantec)

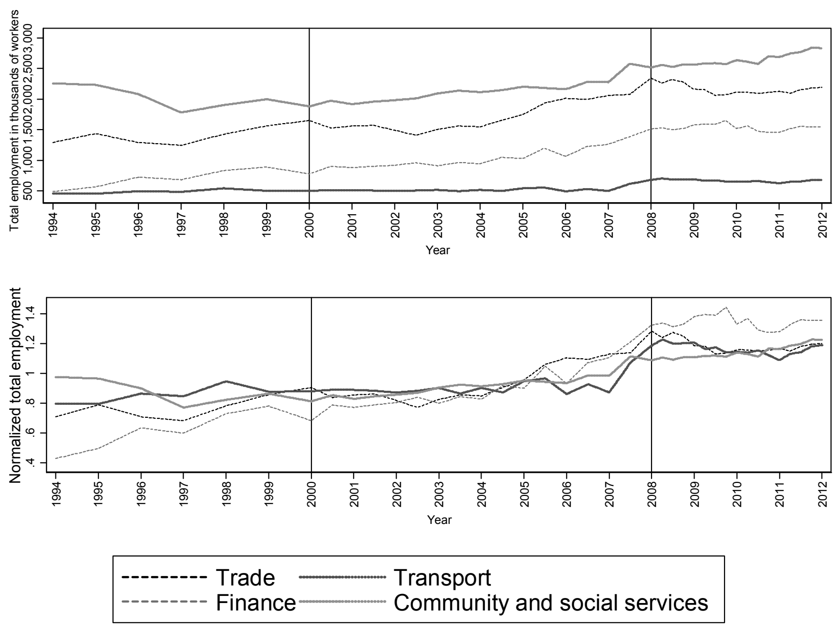


Source: Authors' own calculations based on Quantec labour data.

A.1.3 Labour in the tertiary sector

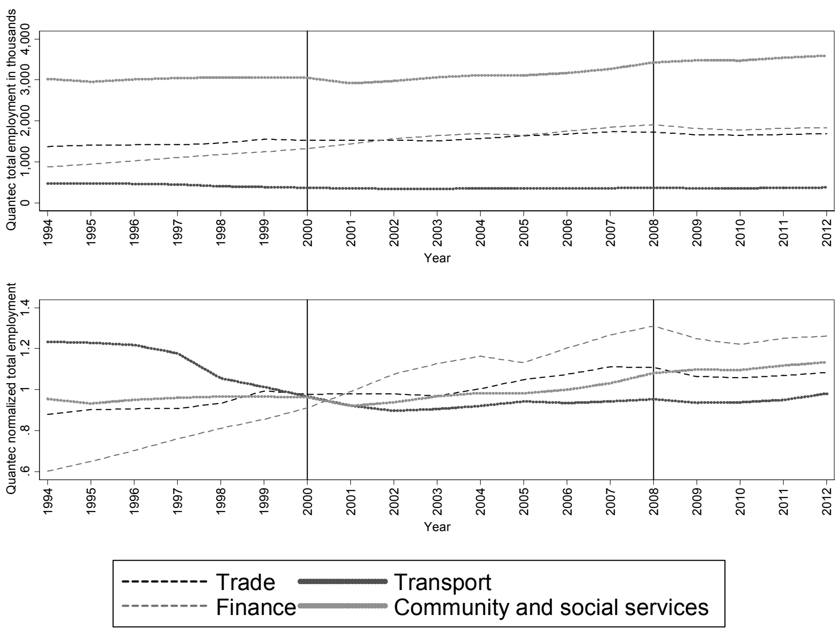
Labour in the tertiary sector, while more volatile in absolute terms, maintains a steady upward trend over the period in question (Figure A5). Aside from the noticeable jumps in labour as it approaches the first QLFS. The labour shifts in the Quantec data (Figure A6) are again inconsistent with the largest deviation observed in the transport sector.

Figure A5: Total employment in the secondary sector (PALMS)



Source: Authors' own calculations based on PALMS data.

Figure A6: Total employment in the secondary sector (Quantec)

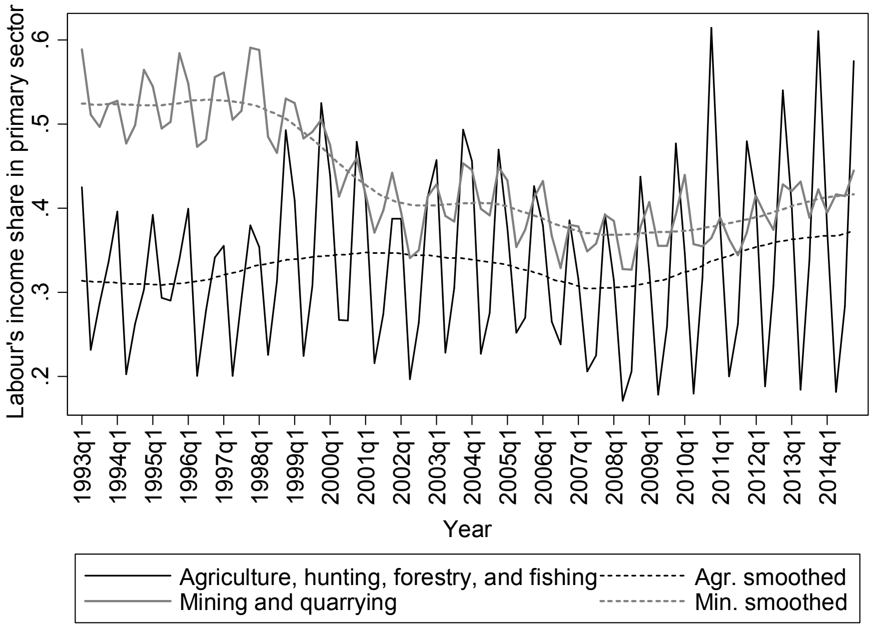


Source: Authors' own calculations based on Quantec labour data.

Appendix B: Labour's income share

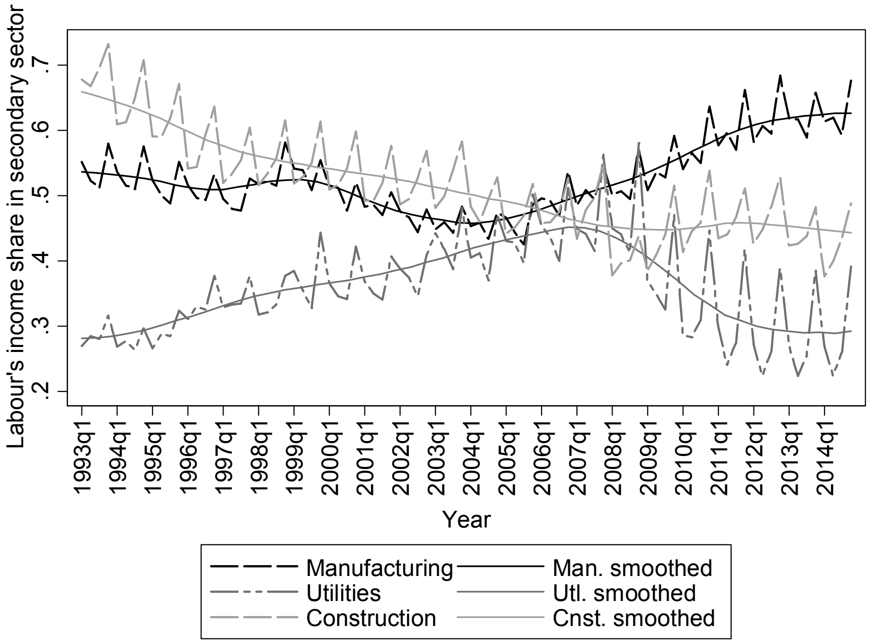
Figure B1 compares the labour income shares for the agriculture and mining industries over time. We observe that this share is extremely seasonal in agriculture, with a mean of around 0.35. On the other hand, the labour share in mining reveals much less seasonal fluctuation, but a long-run decrease from around 0.6 in the early 1990s to about 0.4 more recently.

Figure B.1: Labour income share, primary sector industries



Source: Authors, based on South African National Accounts.

Figure B2: Labour income share, secondary sector industries

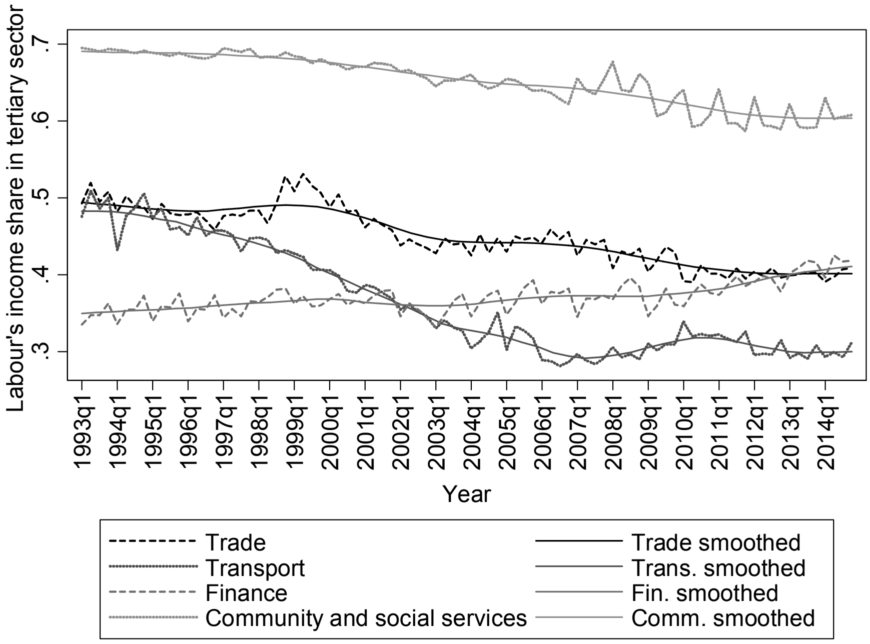


Source: Authors, based on South African National Accounts.

Figure B2 graphs the same measures for the manufacturing, utilities, and construction industries. The share of income accruing to workers in manufacturing initially decreased before it increased sharply, whereas workers in the utilities industries experienced exactly the opposite trend. The labour share of income declined steadily from 0.7 to 0.45 in construction.

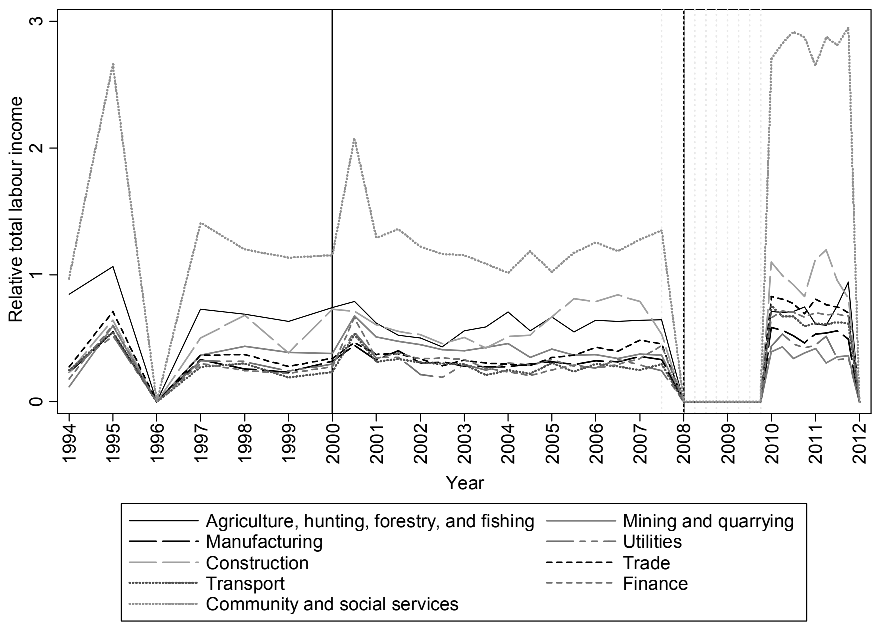
Finally, Figure B3 reports the income shares for internal trade, transportation, finance, and community and social services. Community and social services pay a comparatively high share of about 0.65 to workers, whereas the remaining industries all paid between 0.5 and 0.3.

Figure B3: Labour income share, tertiary sector industries



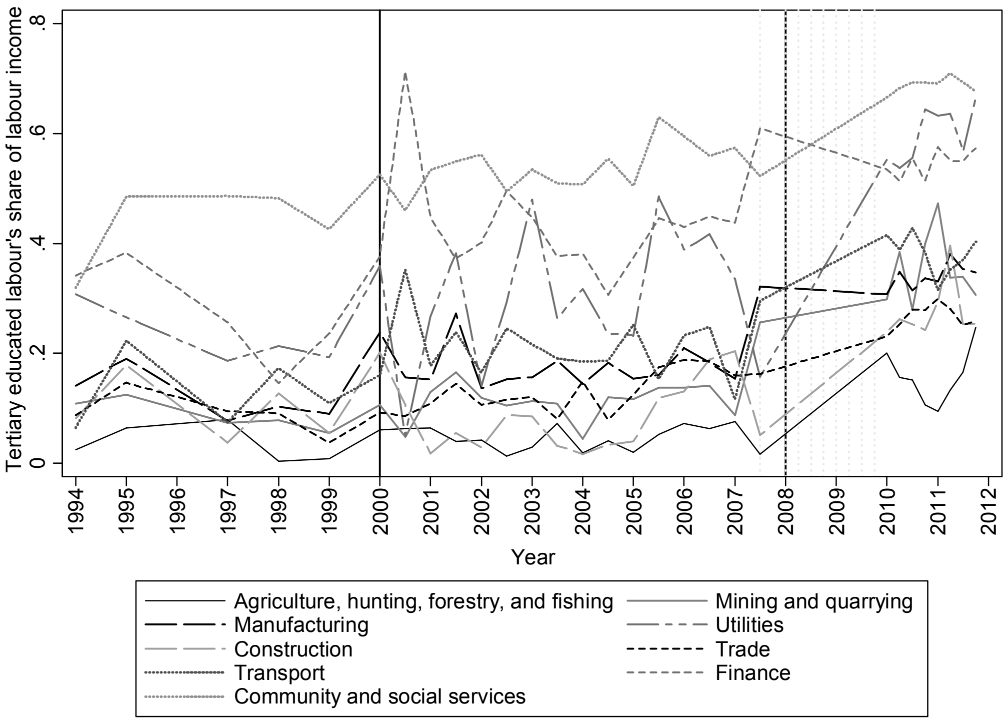
Source: Authors, based on South African National Accounts.

Figure B4: Ratio of total labour income reported in National Accounts against total labour income reported in PALMS



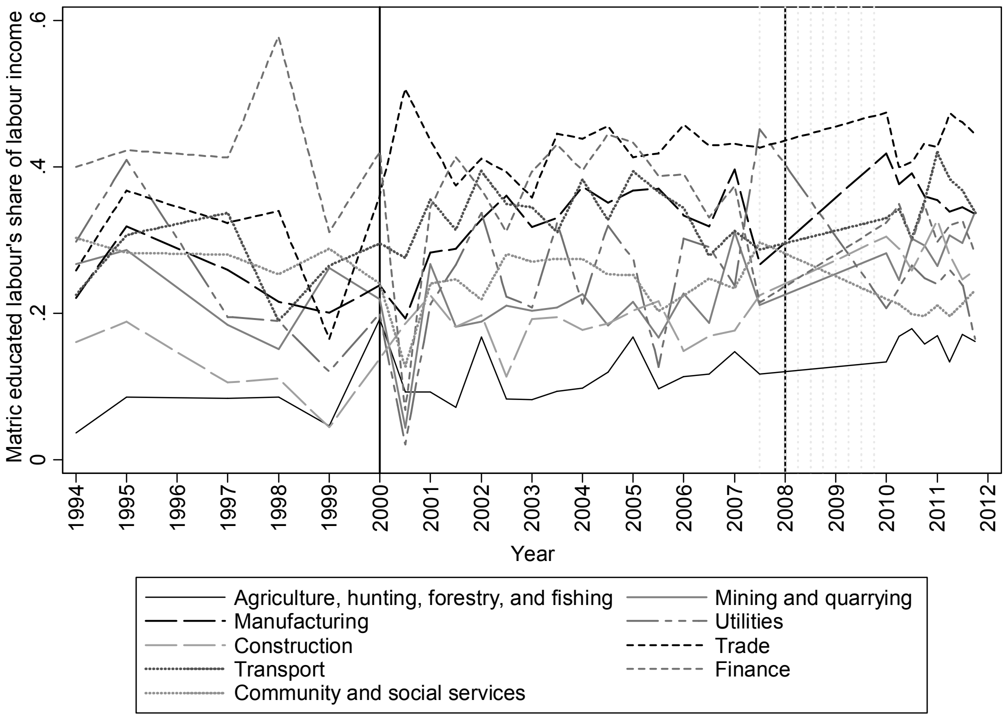
Source: Authors, based on PALMS and National Accounts data.

Figure B5: Tertiary-educated labour's share of labour income



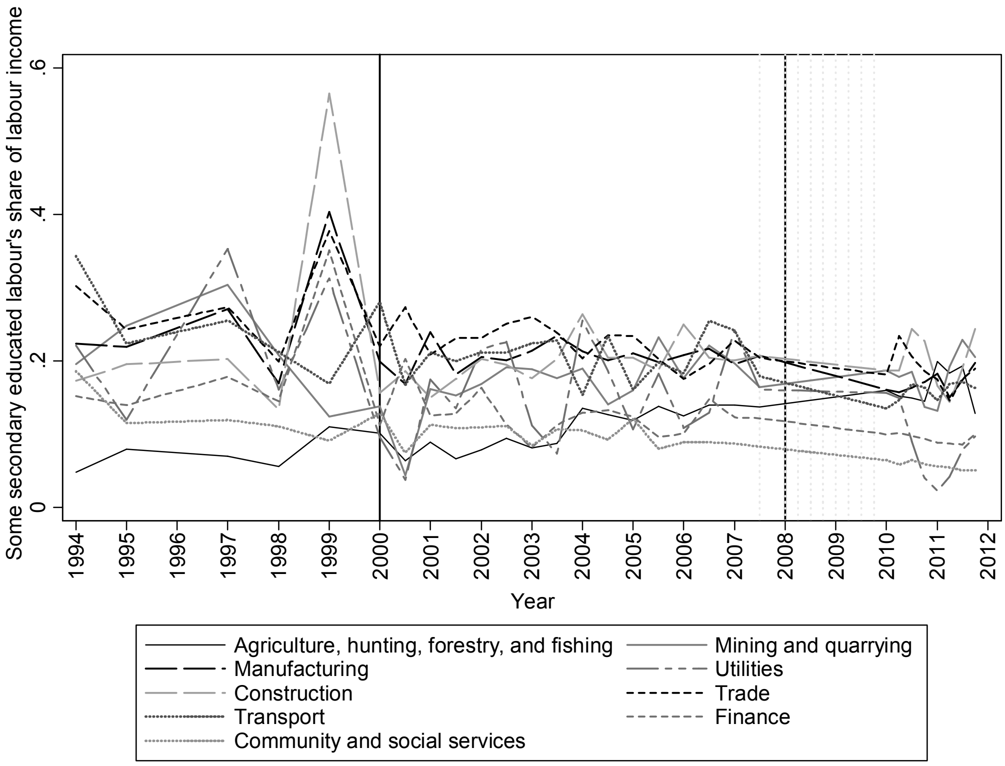
Source: Authors' own calculations based on National Accounts and PALMS data.

Figure B6: Matric-educated labour's share of labour income



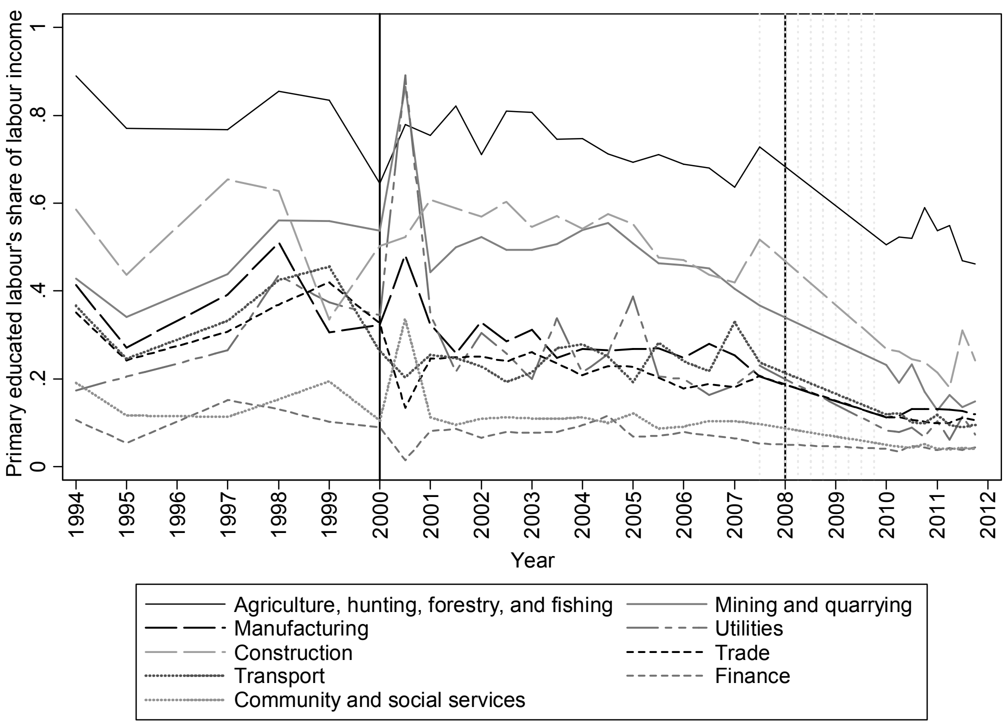
Source: Authors' own calculations based on National Accounts and PALMS data.

Figure B7: Secondary-educated labour's share of labour income



Source: Authors' own calculations based on National Accounts and PALMS data.

Figure B8: Primary-educated labour's share of labour income



Source: Authors' own calculations based on National Accounts and PALMS data.

Table B1: Regression estimates of total labour income in National Accounts and total labour income in PALMS

	Log total labour income in National Accounts								
	Agriculture	Mining	Manufacturing	Utilities	Construction	Trade	Transport	Finance	Comm. serv.
Log PALMS Wages	0.555***	0.233*	0.428***	0.388**	0.455***	0.270***	0.242***	0.379***	0.254***
	(5.16)	(2.73)	(10.81)	(3.08)	(9.18)	(6.42)	(6.51)	(7.75)	(5.31)
Constant	10.09***	18.56***	14.40***	14.17***	12.59***	17.88***	18.02***	15.45***	17.77***
	(4.30)	(9.46)	(15.57)	(5.29)	(11.42)	(18.25)	(21.68)	(13.57)	(15.33)
R2	0.506	0.223	0.818	0.267	0.764	0.613	0.620	0.698	0.520
N	28	28	28	28	28	28	28	28	28

Note: *** p<0.001; **p<0.01; *p<0.05; standard errors in parentheses.

Source: Authors' own calculations based on National Accounts and PALMS data.

Appendix C: Cobb-Douglas and data quality

We start our analysis by estimating the simpler Cobb-Douglas production function. This allows us to explore the effects of measurement error and parameter restriction in a familiar linear regression context.

C.1 Aggregate production function

Our econometric model is based on standard production theory: industries combine physical capital, K, labour, L, and Hicks-neutral technology, A, to produce output (measured as value-added), Y:

$$Y_{it} = A_{it} K_{it}^{\alpha} L_{nt}^{\delta} \quad (1)$$

where i denotes a generic industry and time is indexed by t. This equation can be expressed in logarithmic form as:

$$y_{it} = a_{it} + \alpha k_{it} + \delta l_{it} \quad (2)$$

where lower case roman letters denote logarithmically transformed variables.

In the empirical analysis that follows, we will experiment with two aspects of this specification. First, if industry production occurs under constant returns to scale, then this implies the parametric restriction that $\alpha = 1 - \delta$. Applying this restriction provides two potential benefits and one potential drawback. The drawback is that it could bias our parameter estimates if constant returns to scale is not a feature of South African production. However, if valid, this assumption will improve our estimator efficiency as well as potentially reducing the effects of measurement error bias. The second aspect that we experiment with is the nature of the TFP term, a_{it}. We can allow this variable to consist of time or industry fixed effects, or to have a constant growth rate over time.

Table C1 reports the parameter estimates from these Cobb-Douglas regressions. Column 1 represents the unrestricted regression, and reveals capital and labour coefficients of 0.39 and 0.42 respectively. The labour coefficient is substantially lower than estimates of labour's share of total income, which may reflect attenuation bias due to measurement error in our employment measure. Although these coefficients are consistent with decreasing returns to scale, this outcome is inconsistent with the firm-level observation that larger firms are typically more productive than small firms (although pricing power, which is more likely for larger firms, will also bias estimated productivity upwards for larger firms). A more plausible interpretation is therefore that these coefficient estimates suffer from attenuation bias due to measurement error in the capital or labour variables. Applying constant returns to scale (column 2) has the effect of moving the labour coefficient much closer to its income share. This result is consistent with Krueger and Lindahl (2001), who find that more reliable cross-country production function estimates are obtained when fixing either the capital or labour coefficients to reasonable values.

Columns 3 and 4 repeat the regressions in columns 1 and 2, but now allow for TFP to grow at a constant rate. The results indicate technological growth of around 2 per cent per year, while the labour and capital coefficients are not substantially affected. The estimates of columns 1-4 of the PALMS data and those of the annual Quantec data show very similar results, although the labour coefficient is usually slightly smaller in magnitude for the estimates on the Quantec data. This may be indicative of the Quantec employment measure suffering from larger measurement error. This hypothesis is tested by adding industry fixed effects to the regression, which is known to exacerbate the attenuation bias of measurement error. The results are shown in Table C2.

Table C1: Cobb-Douglas production function estimates

Dataset	PALMS				Quantec			
Regression	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
<u>Variable</u>								
Constant	7.960***	4.507***	8.218***	4.456***	6.995***	5.136***	7.281***	5.195***
	(0.770)	(0.313)	(0.760)	(0.311)	(0.753)	(0.286)	(0.726)	(0.277)
Log labour	0.424***	0.536***	0.417***	0.539***	0.421***	0.464***	0.426***	0.474***
	(0.033)	(0.025)	(0.033)	(0.025)	(0.028)	(0.023)	(0.026)	(0.022)
Log capital	0.390***		0.379***		0.487***		0.470***	
	(0.028)		(0.028)		(0.029)		(0.028)	
Technological growth			0.021***	0.017**			0.021***	0.019***
			(0.006)	(0.006)			(0.005)	(0.005)
R2	0.584	0.554	0.599	0.564	0.795	0.787	0.813	0.802
N	333.000	333.000	333.000	333.000	180.000	180.000	180.000	180.000
RMSE	0.541	0.559	0.531	0.553	0.408	0.415	0.391	0.401

Note: ^ *p*<0.125; ^^ *p*<0.1; * *p*<0.05; ** *p*<0.01; *** *p*<0.001.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

Table C2: Cobb-Douglas estimates with industry fixed effects[23]

Fixed effects and time trends		Industry fixed effects		Industry fixed effects with time trend		Industry fixed effects with industry time trends	
		(1)	(2)	(1)	(2)	(1)	(2)
PALMS	Log labour	0.248*** (0.048)	0.224*** (0.046)	0.279*** (0.038)	0.335*** (0.042)	0.101* (0.048)	0.283*** (0.044)
	Log capital	0.838*** (0.060)		0.153* (0.068)		0.119 (0.094)	
Quantec	Log labour	-0.033 (0.064)	0.061 (0.041)	0.039 (0.052)	0.249*** (0.049)	0.088^ (0.048)	0.336*** (0.068)
	Log capital	0.872*** (0.053)		0.373*** (0.068)		0.025 (0.062)	

Note: ^ p<0.125; ^^ p<0.1; * p<0.05; ** p<0.01; *** p<0.001.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

We observe that controlling for industry fixed effects substantially reduces the labour coefficients, regardless of whether we apply constant returns to scale, or control for either an exhaustive set of time effects or a linear time trend. The effect is observed to be more severe for the Quantec employment series. This is consistent with our hypothesis that both measures are somewhat noisy, but that the Quantec industry employment measure is less reliable than the PALMS measure. For this reasons the rest of this document focuses solely on the results obtained from the PALMS series as complemented by the Quantec Capital and StatsSA output data.

C.2 Industry production functions

The data quality issues already discussed mean that attempting to estimate industry-specific production function parameters is very ambitious, even after applying the constant returns to scale restriction. Such regressions only use the (much less informative) time-series variation in the data. The results are reported in Table C3.

[23] Full results are reported in Appendix D.

Table C3: Cobb-Douglas production function estimates

		Industry								
		Agriculture	Mining	Manufacturing	Utilities	Construction	Trade	Transport	Finance	Community and social services
PALMS	Constant	-11.861***	2.615***	-4.682	3.010**	1.857**	3.769^	-10.035*	8.016***	-7.866**
		(2.545)	(0.528)	(3.751)	(0.981)	(0.531)	(1.988)	(4.653)	(0.680)	(2.881)
	Labour's share	-0.787***	0.368***	-0.229	0.431***	0.246***	0.439*	-0.531	0.769***	-0.350
		(0.215)	(0.040)	(0.302)	(0.068)	(0.052)	(0.175)	(0.339)	(0.051)	(0.228)
	R2	0.241	-2.826	0.649	0.242	0.978	0.873	0.724	0.950	0.823
	N	37.000	37.000	37.000	37.000	37.000	37.000	37.000	37.000	37.000
	RMSE	0.284	0.081	0.094	0.101	0.050	0.078	0.149	0.065	0.074
		Industry								
		Agriculture	Mining	Manufacturing	Utilities	Construction	Trade	Transport	Finance	Community and social services
Quantec	Constant	-5.696***	9.110***	-6.348***	17.247***	2.011***	-5.665***	-9.257***	11.809***	-0.779
		(0.820)	(1.889)	(0.941)	(2.088)	(0.185)	(1.413)	(1.039)	(2.462)	(0.459)
	Labour's share	-0.399***	0.761***	-0.467***	1.260***	0.125***	-0.511***	-0.556***	0.975***	0.004
		(0.071)	(0.144)	(0.075)	(0.137)	(0.018)	(0.123)	(0.074)	(0.189)	(0.037)
	R2	0.544	-13.256	0.901	0.420	0.984	0.949	0.936	0.849	0.993
	N	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000	20.000
	RMSE	0.088	0.121	0.052	0.086	0.046	0.052	0.082	0.127	0.012

Note: ^ $p<0.125$; ^^ $p<0.1$; * $p<0.05$; ** $p<0.01$; *** $p<0.001$.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

As expected, the coefficient estimates are very different and often lie, implausibly, outside of the unit interval. We interpret this as evidence of the unreliable estimates obtained from attempting to estimate more parameters than the variation in the data can reliably identify.

Appendix D: Cobb-Douglas estimates

Table D1: Cobb-Douglas regressions on PALMS data

	Industry fixed effects		Industry fixed effects with time trend		Industry fixed effects with industry time trends	
	(1)	(2)	(1)	(2)	(1)	(2)
Share parameters						
Log labour	0.248***	0.224***	0.279***	0.335***	0.101*	0.283***
	(0.048)	(0.046)	(0.038)	(0.042)	(0.048)	(0.044)
Log capital	0.838***		0.153*		0.119	
	(0.060)		(0.068)		(0.094)	
Industry fixed effects						
Agriculture	-1.798	0.100	15.053***	1.345**	18.251***	0.538
	(1.291)	(0.547)	(1.571)	(0.498)	(2.556)	(0.510)
Mining	0.568***	0.602***	1.129***	0.759***	1.277***	1.003***
	(0.077)	(0.074)	(0.072)	(0.067)	(0.075)	(0.069)
Manufacturing	0.746***	0.842***	1.602***	0.908***	1.857***	1.059***
	(0.074)	(0.045)	(0.084)	(0.040)	(0.122)	(0.048)
Utilities	-0.088	-0.100	0.397***	0.200^	0.179	0.293*
	(0.129)	(0.129)	(0.107)	(0.118)	(0.117)	(0.124)
Construction	1.637***	1.527***	0.458***	1.352***	0.343^	1.710***
	(0.106)	(0.081)	(0.118)	(0.073)	(0.205)	(0.073)
Trade	1.183***	1.224***	1.391***	1.170***	1.599***	1.406***
	(0.049)	(0.042)	(0.042)	(0.038)	(0.044)	(0.037)
Transport	0.125	0.212*	1.196***	0.422***	1.168***	0.451***
	(0.108)	(0.094)	(0.114)	(0.085)	(0.135)	(0.095)
Finance	0.534***	0.655***	1.773***	0.819***	1.881***	0.890***
	(0.107)	(0.077)	(0.122)	(0.070)	(0.163)	(0.088)
Community and social services	-0.871***	-0.731***	0.362**	-0.644***	0.690***	-0.481***
	(0.100)	(0.051)	(0.118)	(0.046)	(0.174)	(0.054)
Time and industry varying Hicks-neutral technology						
Constant or agriculture			0.029***	0.015***	0.044***	0.052***
			(0.002)	(0.002)	(0.004)	(0.004)
Mining					-0.046***	-0.063***
					(0.005)	(0.005)
Manufacturing					-0.018**	-0.036***
					(0.005)	(0.005)
Utilities					-0.026***	-0.047***
					(0.005)	(0.005)
Construction					0.005	-0.055***
					(0.010)	(0.005)
Trade					-0.010	-0.042***
					(0.007)	(0.005)
Transport					0.004	-0.025***
					(0.006)	(0.005)
Finance					0.003	-0.029***
					(0.007)	(0.006)
Community and social services					-0.015*	-0.041***
					(0.006)	(0.005)
R2	0.967	0.967	0.979	0.974	0.987	0.985
N	333.000	333.000	333.000	333.000	333.000	333.000
RMSE	0.155	0.155	0.122	0.136	0.099	0.106

Note: ^ p<0.125; ^^ p<0.1; * p<0.05; ** p<0.01; *** p<0.001.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.

Table D2: Cobb-Douglas regressions on Quantec data

	Industry fixed effects		Industry fixed effects with time trend		Industry fixed effects with industry time trends	
	(1)	(2)	(1)	(2)	(1)	(2)
Share parameters						
Log labour	-0.033	0.061	0.039	0.249***	0.088^	0.336***
	(0.064)	(0.041)	(0.052)	(0.049)	(0.048)	(0.068)
Log capital	0.872***		0.373***		0.025	
	(0.053)		(0.068)		(0.062)	
Industry fixed effects						
Agriculture	3.149^	-0.081	15.593***	2.177***	24.336***	3.274***
	(1.754)	(0.500)	(1.923)	(0.599)	(1.531)	(0.836)
Mining	-0.545***	-0.320***	-1.382***	-0.461***	-1.972***	-0.636***
	(0.129)	(0.054)	(0.136)	(0.055)	(0.100)	(0.065)
Manufacturing	-0.234^	0.013	-0.618***	0.170**	-0.807***	0.318***
	(0.140)	(0.056)	(0.120)	(0.058)	(0.086)	(0.061)
Utilities	0.265***	0.386***	0.046	0.446***	-0.129**	0.409***
	(0.078)	(0.047)	(0.067)	(0.044)	(0.043)	(0.035)
Construction	-1.369***	-0.901***	-1.665***	-0.352*	-1.854***	-0.057
	(0.274)	(0.127)	(0.224)	(0.150)	(0.179)	(0.201)
Trade	1.006***	1.423***	-0.669*	1.090***	-1.968***	0.875***
	(0.233)	(0.085)	(0.257)	(0.096)	(0.215)	(0.145)
Transport	0.802***	0.971***	0.092	0.818***	-0.455***	0.706***
	(0.104)	(0.056)	(0.112)	(0.057)	(0.089)	(0.064)
Finance	-0.698***	-0.477***	-0.697***	-0.136	-0.808***	-0.093
	(0.143)	(0.086)	(0.116)	(0.098)	(0.093)	(0.121)
Community and personal services	-0.084	-0.009	-0.042	0.131*	-0.111*	0.118^
	(0.066)	(0.054)	(0.054)	(0.055)	(0.045)	(0.065)
Time and industry varying Hicks-neutral technology						
Constant or agriculture			0.022***	0.013***	0.021***	0.004
			(0.002)	(0.002)	(0.002)	(0.003)
Mining					0.002	0.032***
					(0.004)	(0.005)
Manufacturing					-0.023***	-0.022***
					(0.003)	(0.004)
Utilities					0.006*	0.019***
					(0.003)	(0.004)
Construction					-0.006*	-0.011**
					(0.003)	(0.004)
Trade					0.037***	0.018*
					(0.005)	(0.007)
Transport					0.015***	0.012**
					(0.003)	(0.004)
Finance					0.031***	0.033***
					(0.003)	(0.005)
Community and social services					0.028***	0.022***
					(0.003)	(0.005)
R2	0.977	0.976	0.985	0.980	0.997	0.993
N	180.000	180.000	180.000	180.000	180.000	180.000
RMSE	0.140	0.141	0.113	0.129	0.052	0.079

Note: ^ $p<0.125$; ^^ $p<0.1$; * $p<0.05$; ** $p<0.01$; *** $p<0.001$.

Source: Authors' own calculations based on StatsSA, PALMS, and Quantec data.