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Carbon pricing under binding political constraints

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Abstract: The economic prescription for climate change is clear: price carbon dioxide (CO2) and other greenhouse gas emissions to internalize climate damages. In practice, a variety of political economy constraints prevent the introduction of a carbon price equal to the full social cost of emissions. This paper develops insights about the design of climate policy in the face of binding political constraints, formulated here as limits on the CO2 price itself, on increases in energy prices, and on energy consumer and producer surplus loss. We employ a stylized model of the energy sector to develop intuition about the welfare-maximizing combination of CO2 price, subsidy for clean energy production, and lump-sum transfers to energy consumers or producers under each constraint. We find that the strategic use of subsidies or transfers can compensate for or relieve political constraints and significantly improve the efficiency and environmental efficacy of carbon pricing policies.

Keywords: political economy, carbon pricing, environmental economics, public economics, climate change, instrument choice, carbon tax, emissions trading
JEL classification: H23, Q48, Q54, Q58

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1. INTRODUCTION

For decades, the economically-efficient prescription for the severe consequences of global climate change has been clear: establish a price on emissions of carbon dioxide (CO_2) and other greenhouse gases that internalizes the far-reaching external costs of climate change in market transactions (see e.g. Metcalf and Weisbach, 2009; Nordhaus, 1992; Stavins, 1997; Stern, 2007; and many others). Yet in sharp contrast to this prescription, a diverse patchwork of climate policy measures has proliferated, and where CO_2 pricing policies do exist, the prices established typically fall far short of the levels estimated to fully internalize the marginal cost of climate damages.

The failure of governments to establish a pricing (or equivalent market-based) approach to climate change mitigation—or to adequately price carbon when they succeed in doing so—can be largely attributed to a variety of persistent political economy challenges. In particular, climate change mitigation is a global collective action challenge (Olson, 1984), demanding coordinated action among many disparate stakeholders (e.g. nations, emitting industries, individual consumers). Meanwhile, the benefits of climate mitigation are uncertain, unevenly distributed, and accrue primarily to future generations (IPCC, 2014; Nordhaus, 1992), while the costs of climate mitigation are born immediately, with acute distributional impacts for particular constituencies (Burtraw *et al.*, 2002, Bovenberg *et al.*, 2005, Boyce, 2015, Jenkins, 2014, Rausch and Karplus, 2014). Climate mitigation thus has all the hallmarks of an intergenerational principal agent problem (Eisenhardt, 1989), with private costs of mitigation out of proportion to the private benefits for many actors. Furthermore, climate policy must be established through political processes, which invoke classic challenges in public choice (Arrow, 1970; Black, 1987; Buchanan and Tullock, 1999; Downs, 1957; Olson, 1984) and are vulnerable to capture by vested interests (Stigler, 1971). Voters typically "vote with their pocketbooks" and frequently express limited tolerance for measures that have salient impacts on their private welfare (such as tax or energy price increases) (Kotchen *et al.*, 2013; Leiserowitz *et al.*, 2013; Villar and Krosnick, 2010). Industrial sectors with high concentrations of assets that would lose considerable value under carbon pricing policies (e.g., fossil energy extraction, fossil electricity production, fuel refining, concrete production, and energy-intensive manufacturing) have also mounted vociferous and often effective opposition to climate policies (Jenkins, 2014; Murphy, 2002). As a result of these public choice dynamics, policymakers have tended to support policies that minimize direct and salient impacts on businesses and households, minimize burdens on regulated and strategically important sectors, and/or redistribute welfare and rents in a manner that secures a politically-durable coalition. As a result, policy makers have in practice preferred command-and-control regulations that are narrowly targeted (and thus allow for regulatory capture while reducing scope for opposition) and subsidies (which allow for transfers of rents while spreading policy costs broadly and indirectly across the tax base), rather than uniformly pricing CO_2 (Gawel *et al.*, 2014; Karplus, 2011).

These persistent political economy constraints motivate a search for climate policies that are politically feasible, environmentally effective, and economically efficient (Jenkins, 2014). As in many other domains of economic regulation, second best (Lipsey and Lancaster, 1956) (and third and fourth best) climate policy mechanisms abound. By paying close attention to the distributional impacts of different climate policy instruments and their interaction with potentially-binding political constraints, economists, political scientists, and policy makers can help design climate policy responses that are both palatable enough to be implemented today and economically superior to alternative second-best instruments.

In light of these challenges, this paper aims to develop general insights about the design of climate policy in the face of binding political constraints. We employ a stylized partial-equilibrium model of the energy sector to explore the welfare implications of combining a CO_2 price with the strategic application of revenues to compensate for and/or relieve several potential political constraints on carbon pricing policies. Specifically, we implement constraints of varying severity on: 1) the maximum feasible CO_2 price itself; 2) the maximum tolerable increase in final energy prices; 3) a maximum tolerable decline in energy consumer surplus; and 4) a maximum decline in fossil energy producer surplus. Under each political constraint, we identify the CO_2 price, subsidy for clean energy production, and lump-sum transfers to energy consumers or fossil energy producers that maximizes total welfare, subject to constraints, and explore parametric sensitivities.

To our knowledge this paper is the first to employ a net-benefits framework to explore the impact of multiple political economy constraints on the design of climate mitigation policy. This work builds on previous literature that considers the distributional impacts of policy as an essential component of instrument choice. Burtraw *et al.* (2002) explore the impact of different allowance allocation schemes under emissions trading programs in the electricity sector on the asset value of existing generators, while Bovenberg *et al.* (2005) employ a stylized general equilibrium model to explore the efficiency costs of environmental policies that are designed to fully offset distributional impacts on pollution-related industries. Boyce and Riddle (2007) and Boyce and Riddle (2010) explore the distributional impact of carbon pricing on U.S. households and identify revenue recycling strategies that produce net private benefits for all but the highest income deciles. Finally, Hirth and Ueckerdt (2013) employ a partial equilibrium model of the Northwestern European electricity sector to calculate the distributional effects of renewable energy support and carbon pricing policies, arguing that a mix of these policies may be preferable if policy makers wish to avoid large transfers of wealth, even if carbon pricing is preferable from an efficiency perspective.

In addition, while this paper focuses on welfare-maximizing policies in a static context, this approach to designing climate policy in the face of political constraints also interacts with literature exploring the dynamic effects of near-term policy action on both the future cost of mitigation (e.g. Bertram *et al.*, 2015; Fischer and Newell, 2008; Nemet, 2010; Newell, 2010; Trancik *et al.*, 2015) and the political durability of climate policies over time (Gawel *et al.*, 2014; Isley *et al.*, 2015). In a dynamic context, feasible but sub-optimal policies implemented today can reduce the future cost of mitigation and/or alter the relative influence of stakeholders in ways that increase support for the efficient policies over time. Future work could fruitfully explore the dynamic implications of the strategies developed herein.

This paper begins by contrasting the range of carbon pricing policies implemented across the world with estimates of the full social cost of carbon and introduces the common political economy constraints encountered in real-world climate policy-making (Part 2). We then introduce our model formulation and representations of four stylized political constraints (Part 3). We derive the analytical solution for welfare-maximizing policy under constraints on carbon and energy prices, and then present numerical results demonstrating the improvement in welfare and carbon abatement due to the application of revenues under each constraint case (Part 4). Finally, we discuss the implications of these findings for climate policy and ongoing research (Part 5).

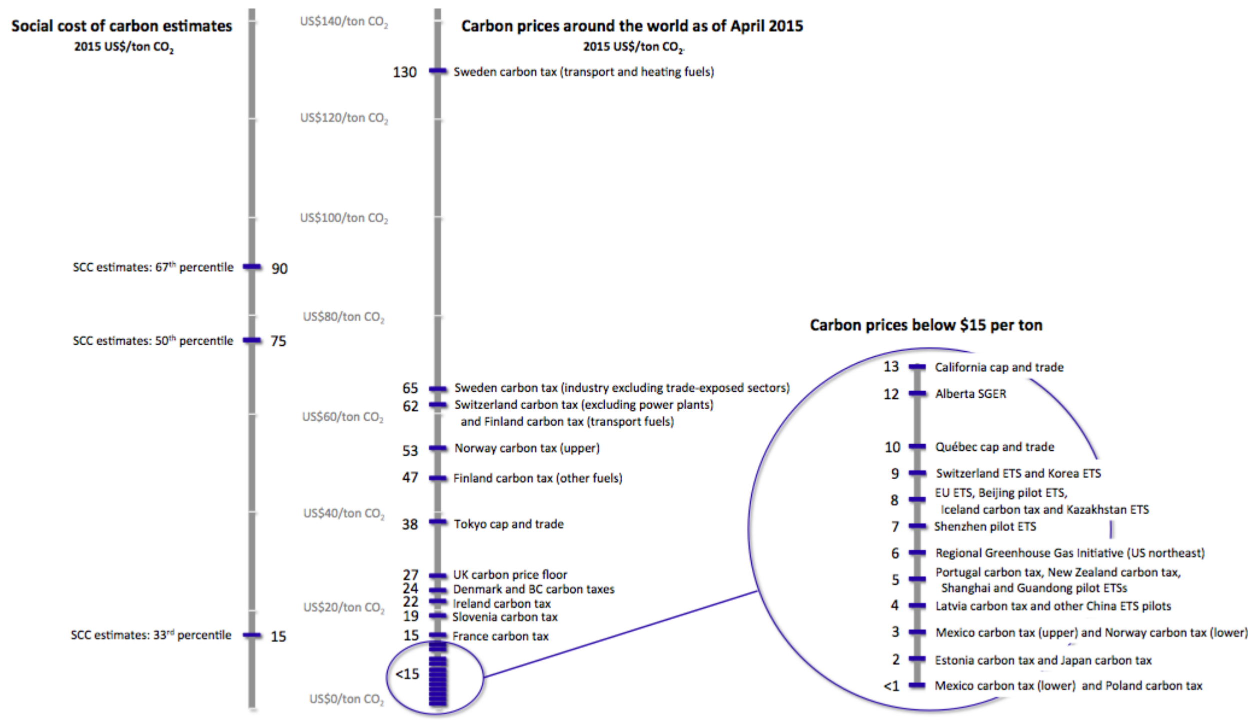
2. CARBON PRICING IN THEORY AND PRACTICE

Economists generally conceptualize climate change as a conventional environmental externality caused by emissions of GHGs, which are globally-acting stock pollutants (Nordhaus, 1992; Stavins, 1997; Stern, 2007). As such, the traditional economic prescription involves establishing a Pigouvian fee (Pigou, 1932) on GHG emissions that corrects for the unpriced externality, either via an emissions tax (Metcalf and Weisbach, 2009) or a market-based emissions cap and permit trading mechanism (Coase, 1960, Stavins, 2008). While there are conceptual and practical differences between CO_2 taxes and emissions trading programs (Aldy *et al.*, 2010; Weitzman, 1974), here we will refer to both instruments collectively as "carbon pricing policies." If these instruments successfully establish a carbon price that internalizes the full climate change-related external costs associated with emissions of CO_2 and other GHGs (Greenstone *et al.*, 2011; Tol, 2011), the private costs of GHG emitting activities will reflect their marginal social costs, theoretically restoring a level of emissions that is Pareto optimal. In other words, policy should ideally equate the marginal cost of GHG emissions control with the marginal damage caused by the climate externality.

Marginal damage estimates for climate change are expressed in terms of the social cost of CO_2 (or CO_2-equivalent) emissions, or the "social cost of carbon" (SCC). There is great uncertainty surrounding the true estimate of the SCC, both because damages from climate change under a given level of warming are highly uncertain and because calculating such figures involves normative judgments such as the appropriate inter-generational discount rate. As shown in Figure 1 below, a review of the literature (Tol, 2011) suggests a price on the order of \$75 per ton CO_2 (with a central range of \$14 to \$90 per ton CO_2, in 2015 USD) is necessary in order to internalize the full social costs of climate change. The U.S. Environmental Protection Agency has also generated estimates of the SCC for future years under different discount rate assumptions, which it and other federal agencies apply to estimate the climate benefits of regulations. Average estimates assuming a 3% discount rate increase over the period 2015 to 2050 from \$41 to \$80 per metric tons CO_2 (in 2015 USD) (EPA, 2015). Critiques of the methodologies applied and the certainty with which estimates are used in the policy debate abound. Nevertheless, there is widespread recognition that to avoid severe climate change, society should be pricing CO_2 at a level well above that observed in nations that price carbon today.

Indeed, while a variety of jurisdictions have implemented some form of carbon pricing instrument, real-world examples of CO_2 prices that fall squarely within the range of SCC estimates are few and far between, as illustrated in Figure 1. Sweden (\$130 per ton), Switzerland (\$62 per ton), Finland (\$47-62 per ton, depending on the fuel) and Norway (\$53 per ton) are all at the very high end of the spectrum. Each of these nations is relatively wealthy and has abundant supplies of low-carbon electricity. Yet even these nations frequently adjust carbon pricing policies

Figure 1. CO_2 prices in markets around the world, compared to the social cost of carbon



Sources: Social cost of carbon estimates from Tol, 2011; CO_2 prices from Kossoy *et al.*, 2015. Values adjusted to 2015 USD by authors using U.S. Bureau of Labor Statistics inflation index

in light of political constraints. Sweden, for example, appears to have the highest carbon price in the world. Yet the carbon tax was implemented as part of a series of reforms in 1991 that simultaneously reduced existing energy taxes by 50 percent. The total effect was to lower overall tax rates on fossil energy consumption (Johansson, 2000). Furthermore, Sweden exempts trade-exposed, energy-intensive industries such as pulp-and-paper, mining, and industrial horticulture from paying the carbon tax at all, while other industrial emitters pay only half the full tax rate (ibid.). Power plants and district heating plants are exempt from the tax as well and instead fall under the European Union's Emissions Trading System (EU-ETS), which imposes a CO_2 price of just $8 per ton (Driesen, 2013). Switzerland similarly allows industrial emitters to opt out of the carbon tax if they participate in the country's own ETS program, in which CO_2 permits trade for just $9 per ton (ibid.). Meanwhile, most countries and regions that have implemented CO_2 prices to date have established prices below $15 per ton (Kossoy *et al.*, 2015), including the most significant carbon pricing policies established by the world's largest emitters: the EU-ETS, Chinese ETS pilots, Japan's carbon tax, and two regional programs in the United States, the Regional Greenhouse Gas Initiative (RGGI) in the U.S. north east and California's cap-and-trade program.

The presence of one or more political economy constraints can explain why the majority of carbon pricing policies around the world today fall well below the central range of estimates of the full social cost of carbon. Any effort to transform the energy system will create economic and political winners and losers, and introducing a CO_2 price is no exception. Climate policy design and instrument choice must therefore contend not only with efficiency concerns, but also with distributional impacts and the resulting implications for political feasibility and durability. Attention to how clever policy design can manage the distributional impacts and costs associated with a clean energy transition while maximizing the efficiency and efficacy of policy measures has thus proven an important (and elusive) challenge.

Political constraints on carbon pricing policies can take many forms. We group these constraints into four broad categories which are later formalized in the model presented in Section 3 below. First, political factors may directly constrain the level of the CO_2 price itself, though several mechanisms. In any emissions trading system, for example, there is a strong incentive to base reductions on relatively conservative growth and technology projections, increasing the relative certainty that the cap will not be hard to achieve. Over-allocation of emissions permits is common in practice and can be difficult to remedy. In the first phase of the EU-ETS, producers recognized over-allocation about halfway through the compliance period and the price of CO_2 fell precipitously and stayed near zero for the remainder of the period. After a brief rise at the start of the second ETS compliance phase, permit prices crashed in 2008 and again in 2011 and have remained very low throughout the second and third phases of the ETS, as an excess of permits has again accumulated. Efforts to tighten the ETS cap by removing excess permits from the market have been highly contentious and have, to date, failed to restore higher carbon pricing levels.

Hybrid instruments—such as a cap with a price ceiling and floor—effectively convert an emissions trading system to a CO_2 tax above or below threshold prices, enhancing price certainty but explicitly constraining carbon prices. In CO_2 tax systems, the level of the tax itself becomes the carbon price, which in democracies is a negotiated outcome of the political process. CO_2 tax systems generally exhibit higher CO_2 prices than emissions trading schemes, because the price is set directly, avoiding the misalignment between cap size and price expectation that can occur in emissions trading systems. By setting the price directly, policy creates predictability but also may be more likely to draw objections, depending on perceptions of fairness and distributional burden. These concerns may directly constrain how high the carbon price is allowed to rise over time. British Columbia and the United Kingdom, for example, each established relatively robust CO_2 prices with plans to steadily increase these prices over time, only to freeze carbon price levels in place in 2013 and 2014, respectively.

While political processes may appear to limit the CO_2 price directly, these outcomes are likely the final expression of more immediate concerns about the distributional impact of carbon pricing policies. Given the political salience of energy prices to both households and industry, a second category of political constraints could limit the increase in end-use energy prices, which is the main way in which consumers and producers feel the impact of a CO_2 price in their daily lives or operations. Concern about increases in energy bills shaped the design of the Waxman-Markey Bill, which aimed to establish a national emissions cap and trading program in the United States, but failed to secure passage in the U.S. senate in 2010 (Jenkins, 2014). Under the legislation, a variety of cost containment measures would have been established, and energy-intensive heavy industries and natural gas and electricity distribution companies were slated to receive large free allocations of permits to ease impacts on energy consumers. As mentioned previously, energy-intensive industries in Sweden, Switzerland, and elsewhere are subject to similar exemptions to reduce the impact of CO_2 pricing policies.

Third, political constraints arise from concerns about the private welfare losses experienced by energy consumers, including both households as well as commercial and industrial users of energy. The vast majority of evidence suggests that public support for climate policy measures is broad but shallow, with limited tolerance for internalizing the costs of mitigation (Kotchen *et al.*, 2013; Leiserowitz *et al.*, 2013; Villar and Krosnick, 2010). Most studies suggest that consumers are willing to pay far less than the social cost of CO_2. Johnson and Nemet (2010) review 27 studies estimating consumer willingness-to-pay for mitigation, and find a central range between $80-200 per household per year. At average household emissions rates across U.S. states, this willingness-to-pay would tolerate a CO_2 price of $2-20/ton of CO_2 (Jenkins, 2014). While a constraint on energy prices reflects concern with the initial impact on households and commercial or industrial energy consumers, constraints on private welfare may be at least partially relieved by transfers which offset the initial impacts (i.e. via lump-sum rebates, reductions in other taxes, or free allocation of emissions permits). British Columbia, for example, offsets the impact of its carbon tax on households and businesses by using carbon pricing revenues to reduce personal and business income taxes. Australia's now-defunct carbon tax similarly offset impacts on energy consumers by increasing tax exemptions and providing direct payments to low and medium-income households and by offering direct compensation payments to heavy industries such as iron and steel, although these measures ultimately failed to prevent the legislation's repeal in 2014.

Opposition to CO_2 pricing from fossil energy producers presents a fourth and final category of constraints limiting reductions in fossil producer surplus. Industries suffer losses in producer surplus in proportion to their reliance on fossil-intensive production technologies and inputs. The fossil fuel industry, which feels especially acute impacts, has acted to deflect attention from climate change (Gillis and Krauss, 2015) and oppose policy measures (Jenkins, 2014). Citing impacts on production and competitiveness, fossil energy industry lobbyists in Australia first secured $1 billion in direct compensation payments from revenues generated by the short-lived CO_2 tax before successfully arguing for the policy's full repeal. The coal and oil industries in the U.S. actively opposed the Waxman-Markey Bill. And nearly every CO_2 pricing system around the world has encountered some form of opposition from the industries most affected. Given that clean energy transitions reduce the size and economic importance of fossil energy production and consumption over time, this category of constraints may be the most intractable. Compensatory measures can be used to relax or mute these constraints, including free allocation of emissions permits, lump-sum transfers, or other measures intended to compensate producers for devaluation of capital or reduce the asset specificity (Murphy, 2002) of fossil producers (such as funding carbon capture and storage development or retraining workers in the sector).

Instrument choice in the climate policy arena is thus significantly complicated by the presence of multiple political constraints on the realm of feasible policy measures. How are policy makers to achieve efficient outcomes and accelerate a clean energy transition while navigating the salient concerns of political constituencies? In the sections that follow, we analyze these four categories of constraints to identify the welfare-maximizing climate policy strategy under binding political constraints.

3. MODEL AND SCENARIO IMPLEMENTATION

In this section, we present a stylized model of the energy sector to simulate CO_2 pricing and policy strategies under the four sets of political economy constraints introduced above. We formulate and solve this model numerically, comparing the change in net benefits associated with relieving the political economy constraint through alternative strategies for revenue disposition (see Section 4). In addition, we analytically derive solutions to the carbon price and energy price increase constraint cases (see Appendices A and B for full derivations), presenting insights and comparative statics for these cases.

The model is based on a single aggregate energy demand function and two energy supply sub-sectors: a CO_2-emitting fossil energy sector and a zero-emissions clean energy sector (e.g., renewable and nuclear energy). To render the model analytically tractable, we assume constant linear slopes for both supply and demand curves. We further assume the two energy supply sub-sectors are perfectly competitive and are perfect substitutes for one another (although the model formulation is flexible enough to allow different elasticities of substitution). We parameterize the model to roughly approximate the current U.S. energy sector, with 100 Quadrillion British thermal units (Quads) of energy supplied, 80 percent of which is initially supplied by the fossil energy sub-sector and 20 percent by the clean energy sub-sector. The initial energy price is $10 billion per Quad, yielding an aggregate annual energy expenditure of $1 trillion. The fossil energy sector emits 5,276 million metric tons of CO_2, equivalent to 2013 U.S. energy-related emissions. The social cost of carbon is set to $75 per ton of CO_2 (as per the median estimate from Figure 1).

Policy decisions include the level of CO_2 price established, a subsidy per unit of energy supplied by the clean energy sub-sector, and lump-sum transfers to fossil energy producers or energy consumers to compensate for the private welfare impacts of policy decisions. The model is solved to maximize aggregate social welfare over a single time period (i.e., one year) and is subject to market clearing constraints, an optional revenue neutrality constraint, and one of four stylized representations of the real-world political economy constraints observed above: a direct constraint on the CO_2 price level; a constraint on the increase in the final energy price; a constraint on the decrease in energy consumer surplus (net of lump-sum transfers); or a constraint on the decrease in fossil producer surplus (net of lump-sum transfers). The remainder of this section describes the mathematical formulation of the core model (Section 3.1) and the political constraint scenarios explored (Section 3.2).

3.1 Model formulation

Energy demand and consumer surplus - The aggregation of household, commercial, and industrial demand for energy is represented as a single aggregate inverse demand function representing the marginal benefit of consumption:

$$MB(q) = d^{-1}(p) = \alpha_d + \beta_d q \tag{1}$$

where $q = q_f + q_c$ or the sum of both fossil (q_f) and clean (q_c) energy consumed and p is the market clearing price of energy. The marginal benefit of consumption is declining in the quantity consumed ($\beta_d < 0$) and β_d is parameterized in each scenario to equal a plausible initial elasticity of demand (ranging from -0.4 to -1.2). The intercept, α_d, is then set to yield 100 Quads of total consumption in the no-policy case at an initial price of \$10 billion per Quad. In effect we linearize the demand and supply curves, which results in a particular elasticity value around the equilibrium. Sensitivity to the choice of this point elasticity estimate is explored in the results.

Consumer surplus is then expressed as the cumulative benefit of consumption less expenditures on energy and net of the welfare value of any lump-sum transfers (r_d):

$$CS(q, r_d) = \int_0^q MB(q)\, dq - pq + \phi_d r_d = \alpha_d q + \frac{1}{2}\beta_d q^2 - pq + \phi_d r_d \tag{2}$$

The parameter ϕ_d captures the "efficiency" at which sums are transferred to consumers. If this value is set to 1.0, each unit of revenues transferred to consumers translates directly to one unit of increase in consumer surplus. Alternatively, if $\phi_d < 1.0$, consumers do not value transfers equivalently to the benefits of consumption, requiring greater lump-sum transfers to offset initial private surplus losses. This parameter can therefore be used to capture loss aversion (Kahneman and Tversky, 1984) on the part of consumers if desired.

Fossil energy supply and fossil producer surplus - Fossil energy supplies are represented via a linear marginal cost curve with final cost sensitive to the imposition of a CO_2 price (τ):

$$MC_f(q_f, \tau) = \alpha_f + \tau\rho_f + \beta_f q_f \tag{3}$$

where ρ_f is the CO_2 emissions rate of fossil energy supply. Marginal costs are increasing with the quantity produced ($\beta_f > 0$) and, as with consumer demand, β_f is parameterized in each scenario based on a linearization of a specified initial point estimate of the elasticity of supply (from 0.4 to 1.2) with α_f then set to yield 80 Quads of total fossil energy production in the no-policy case at an initial price of \$10 billion per Quad.

Fossil producer surplus is expressed as the sum of revenues less cumulative production costs and tax payments, and net of any lump-sum transfers (r_f):

$$\begin{aligned} PS_f(q_f, \tau, r_f) &= pq_f - \int_0^{q_f} MC(q_f, \tau)\, dq_f + \phi_f r_f \\ &= pq_f - \alpha_f q_f - \frac{1}{2}\beta_f q_f^2 - \tau\rho_f q_f + \phi_f r_f \end{aligned} \tag{4}$$

As with lump-sum transfers to consumers, ϕ_f represents the "efficiency" at which lump-sum transfers to producers offset producer surplus losses due to climate policy decisions.

Clean energy supply and clean producer surplus - Clean energy supply is likewise represented as a linear marginal cost curve with final costs adjusted by a per-unit production subsidy (σ) applied to all clean energy production:

$$MC_f(q_f, \sigma) = \alpha_c - \sigma + \beta_c q_c \tag{5}$$

Marginal costs are increasing with the quantity produced ($\beta_c > 0$) and β_c is parameterized in each scenario based on a specified initial elasticity of supply (from 0.4 to 1.2) with α_c then set to yield 20 Quads of total fossil energy production in the no-policy case at an initial price of \$10 billion per Quad.

Clean energy producer surplus is the sum of revenues and subsidy payments less cumulative production costs:

$$PS_c(q_c, \sigma) = pq_c - \int_0^{q_c} MC(q_c, \sigma)\, dq_c = pq_c - \alpha_c q_c - \frac{1}{2}\beta_c q_c^2 + \sigma q_c \tag{6}$$

Note that this formulation applies subsidies to both inframarginal and marginal clean energy production. A more targeted policy measure could reduce the required revenues by applying to marginal production only, reducing the required revenues (and the total transfer to clean energy producers).

Aggregate supply function - The aggregate supply curve corresponding to the marginal cost of supplying an additional unit of energy is the horizontal sum of fossil and clean energy marginal cost functions:

$$MC_t(q, \tau, \sigma) = \left(\frac{\alpha_c - \sigma}{\beta_c} + \frac{\alpha_f + \tau\rho_f}{\beta_f}\right)\left(\frac{\beta_f \beta_c}{\beta_f + \beta_c}\right) + \left(\frac{\beta_f \beta_c}{\beta_f + \beta_c}\right) q \tag{7}$$

Government revenues and climate damages - Net government revenues produced by the CO_2 tax after transfers to consumers and fossil producers or used to fund clean energy subsidies contribute to overall welfare as follows:

$$R(r_f, r_d, \sigma, \tau) = \phi_g(\tau\rho_f q_f - \sigma q_c - r_f - r_d) \tag{8}$$

In this case, $\phi_g > 1.0$ indicates that government revenues offset other distortionary taxes elsewhere and therefore deliver a "double dividend" (Bovenberg *et al.*, 2005; Goulder, 1998), increasing their net impact on social welfare. Alternatively, if net revenues are assumed to be utilized inefficiently, this value can be set such that $\phi_g < 1.0$.

Climate-related damages associated with CO_2 emissions are a simple function of the quantity of fossil energy supplied:

$$E(q_f) = \eta\rho_f q_f \tag{9}$$

where η is the full social cost of carbon.

Objective function and constraints - The objective function (10) maximizes total social welfare given as the sum of consumer and producer surplus and the welfare value of government revenues less climate-related damages from CO_2 emissions. The model is subject to equilibrium market clearing constraints (11-12). A revenue neutrality constraint (13) can also be enforced, which requires transfers and subsidies to be funded solely by revenues from the CO_2 tax and not taken from other government sources, but this constraint is ignored in the present analysis.

$$Max\, W(\cdot) = CS(q, r_d) + PS_f(q_f, \tau, r_f) + PS_c(q_c, \sigma) + R(r_f, r_d, \sigma, \tau) - E(q_f) \tag{10}$$

$$s.t.$$

$$p = MB(q) = MC_t(q) = MC_f(q_f, t) = MC_c(q_c, s) \tag{11}$$

$$q = q_f + q_c \tag{12}$$

$$\sigma q_c + r_f + r_d \leq \tau \rho_f q_f \quad \textit{(optional)} \tag{13}$$

3.2 Political economy constraint scenarios and analytical solutions

Direct CO_2 price constraint - The first political economy constraint considered is a direct constraint on the level of the CO_2 price of the form:

$$\tau \leq \bar{\tau} \tag{14}$$

where $\bar{\tau}$ is the maximum politically feasible carbon price level and where $\bar{\tau} < \eta$ (the full social cost of carbon).

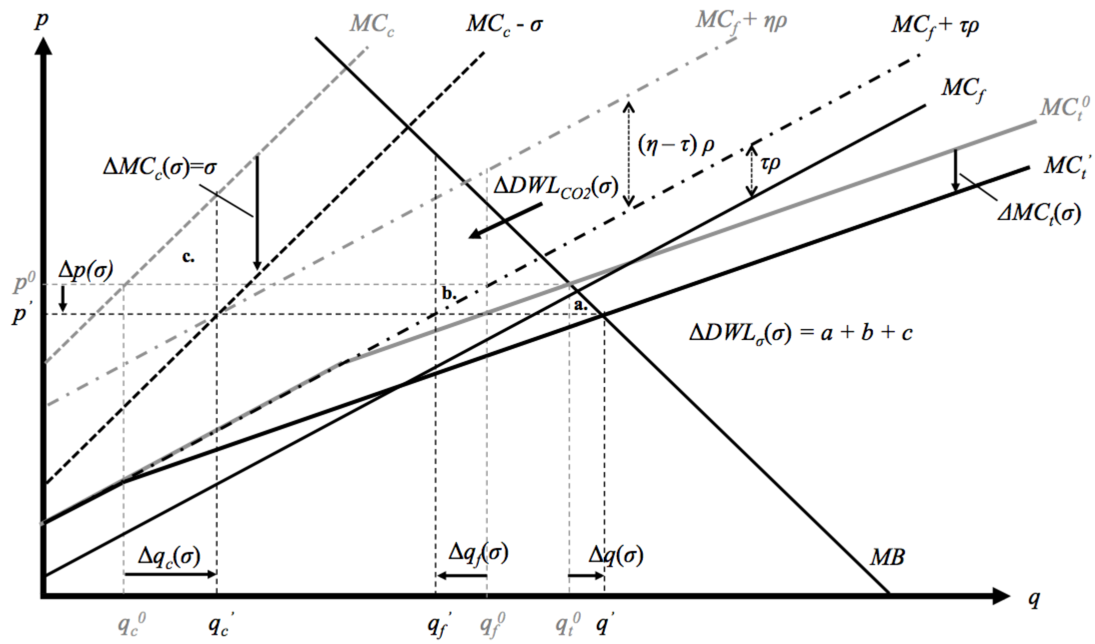
In this case, social welfare (10) is maximized when the CO_2 price approaches the SCC as closely as possible (i.e. $\tau^* = \bar{\tau}$). However, due to the political economy constraint, the carbon price remains below the full SCC (i.e. $\tau^* < \eta$). Therefore, the remaining, non-internalized climate-related damages equal:

$$DWL_{CO_2}(q_f, \tau) = (\eta - \tau)\rho_f q_f \tag{15}$$

Crucially, in the case where the full SCC is not internalized by the carbon pricing instrument, additional reductions in the deadweight loss associated with the remaining external costs of CO_2 emissions from fossil fuel consumption (15) can be achieved by subsidizing clean energy, reducing consumption of fossil energy. As illustrated in Figure 2, the reduction in deadweight loss achieved by a given clean energy subsidy level is:

$$\Delta DWL_{CO_2}(\sigma) = (\eta - \tau)\rho \Delta q_f(\sigma) \tag{16}$$

Figure 2. Impact of clean energy subsidy when CO_2 price is constrained below the social cost of carbon



However, the imposition of a subsidy creates several distortions in the market, including a distortion in total consumption (depicted in Figure 2 as the area of the triangle a.), a distortion in fossil energy production (Figure 2, b.) and a distortion in clean energy production (Figure 2, c.) In aggregate, these distortions introduce a deadweight loss equal to (17):

$$DWL_{\sigma}(\sigma) = \frac{1}{2}|\Delta p(\sigma)|\Delta q_t(\sigma) + \frac{1}{2}|\Delta p(\sigma)||\Delta q_f(\sigma)| + \frac{1}{2}\beta_c \Delta q_c(\sigma)^2 \tag{17}$$

The optimal subsidy σ^* is the value satisfying the first-order condition:

$$\frac{\partial}{\partial \sigma} DWL_{\sigma}(\sigma^*) + \frac{\partial}{\partial \sigma} \Delta DWL_{CO_2}(\sigma^*) = 0 \tag{18}$$

which occurs when the marginal deadweight loss due to distortions introduced by the subsidy equals the marginal decrease in unpriced external damage from CO_2 emissions due to the reduction in fossil fuel consumption driven by the subsidy. We can then derive an analytical solution for the optimal subsidy level (for the full derivation, see Appendix A):

$$\sigma^* = (\eta - \tau)\rho \frac{\beta_t \beta_d}{\beta_c \beta_f (\beta_d - \beta_t)} \Big[\frac{-\beta_t^2 \beta_d}{\beta_c^2 (\beta_d - \beta_t)^2}$$

$$+ \frac{\beta_t^2 \beta_d^2}{\beta_c^2 \beta_F (\beta_d - \beta_t)^2} + \frac{1}{\beta_c} - \frac{2\beta_t \beta_d}{\beta_c^2 (\beta_d - \beta_t)} + \frac{\beta_t^2 \beta_d^2}{\beta_c^3 (\beta_d - \beta_t)^2} \Big]^{-1} \tag{19}$$

Where: $\beta_t = \left(\frac{\beta_f \beta_c}{\beta_f + \beta_c} \right)$ or the slope of the aggregate supply function in (7)

It is apparent from (23) that the optimal subsidy level declines as the CO_2 price level τ approaches the full SCC η. In other words, the more the damages associated with CO_2 emissions that can be feasibily internalized into market transactions, the less of a role there is for a clean energy subsidy, and vice versa. This implies that binding political economy constraints that keep the price on carbon well below the social cost of carbon leave a significant opportunity for a clean energy subsidy to improve overall welfare. In such cases, the market distortions imposed by a well-designed subsidy are more than offset by even larger reductions in the deadweight loss due to the remaining external costs of fossil energy consumption.

Table 1 illustrates the directional impact of changes in the slopes of the supply and demand functions on the optimal subsidy level. The optimal subsidy level decreases as β_f increases and increases as β_d becomes more negative. In other words, the effectiveness of a clean energy subsidy increases with the elasticity of the fossil energy supply (and vice versa), as fossil energy producers are more responsive to the change in equilibrium energy prices induced by the clean energy subsidy, driving larger declines in the deadweight loss as CO_2 emissions fall. The subsidy is also more effective as demand becomes more inelastic (and vice versa), as the distortion in total consumption caused by the subsidy declines when demand is relatively unresponsive to price changes. Interestingly, the optimal subsidy level is insensitive to changes in the slope of the clean energy supply curve. As clean energy producers become more or less elastic, the change in β_c has offsetting impacts on both the deadweight loss associated with CO_2 emissions and the deadweight loss due to distortions caused by the subsidy.

Table 1. Impact of changes in slopes of demand and supply functions on optimal subsidy level

Parameter		Change in $\lvert\beta\rvert$	Elasticity	Change in σ^*
Slope of fossil supply	$\lvert\beta_f\rvert$	+	Less	-
Slope of fossil supply	$\lvert\beta_f\rvert$	-	More	+
Slope of clean supply	$\lvert\beta_c\rvert$	+	Less	No change
Slope of clean supply	$\lvert\beta_c\rvert$	-	More	No change
Slope of inverse demand	$\lvert\beta_d\rvert$	+	Less	+
Slope of inverse demand	$\lvert\beta_d\rvert$	-	More	-

One additional consideration in determining the optimal subsidy level is the impact on government revenues. If $\phi_g = 1.0$, then the subsidy amounts to a pure transfer from government revenues (and thus taxpayers) to clean energy producers. However, if $\phi_g > 1.0$, then a decrease in government revenues requires an increase in distortionary taxes elsewhere in the economy (or represents a foregone opportunity to reduce such taxes), which implies additional deadweight loss associated with the clean energy subsidy (see Goulder, 1998). Conversely, if government revenues used for other purposes are used inefficiently and thus contribute to deadweight loss ($\phi_g < 1.0$), using revenues for a clean energy subsidy may be more effective than a pure transfer. Thus if $\phi_g \neq 1.0$, the optimal subsidy is given by:

$$\sigma^* = \frac{(\eta - \tau)\rho}{\phi_g} \frac{\beta_t \beta_d}{\beta_c \beta_f (\beta_d - \beta_t)} \Big[\frac{-\beta_t^2 \beta_d}{\beta_c^2 (\beta_d - \beta_t)^2} + \frac{\beta_t^2 \beta_d^2}{\beta_c^2 \beta_F (\beta_d - \beta_t)^2} + \frac{1}{\beta_c} - \frac{2\beta_t \beta_d}{\beta_c^2 (\beta_d - \beta_t)} + \frac{\beta_t^2 \beta_d^2}{\beta_c^3 (\beta_d - \beta_t)^2} \Big]^{-1} \tag{20}$$

That is, the optimal subsidy is inversely proportional to the effectiveness at which net government revenues are translated to social welfare, as represented by the parameter ϕ_g.

Energy price constraint - The second political economy constraint we consider is a constraint on the change in the equilibrium energy price after policy decisions. This constraint takes the form:

$$p(\tau, \sigma) \leq p^0 (1 + \overline{\Delta p}) \tag{21}$$

where $p(\tau, \sigma)$ is the equilibrium energy price as a function of the CO_2 price and clean energy subsidy policy decisions, p^0 is the equilibrium energy price absent policy intervention (i.e. $p(\tau = 0, \sigma = 0)$) and $\overline{\Delta p}$ is the maximum percent change in energy price permitted by political economy considerations.

Under such a constraint, a CO_2 pricing instrument alone would be suboptimal. The CO_2 price would be allowed to rise only until it exhausts the political tolerance for energy price increases, internalizing a limited portion of the climate-related externality. In this case, however, welfare could be further improved by combining the carbon price with a clean energy subsidy, which by reducing final energy prices *ceteris paribus*, allows for a larger CO_2 price to be established than would otherwise be possible. At the same time, as in the direct CO_2 price constraint case above, the subsidy itself leads to substitution of clean energy for fossil energy, further reducing deadweight loss associated with any remaining unpriced climate externality. The welfare-maximizing CO_2 price τ^* and clean energy subsidy level σ^* under this constraint is thus the combination that internalizes a greater share of the climate externality and induces further reductions in unpriced damages while balancing these benefits against deadweight loss due to market distortions induced by the clean energy subsidy.

In cases where the political constraint on energy price increases (21) is binding, $p^* = p^0(1 + \overline{\Delta p})$. Using this fact, we solve the first-order conditioning maximizing social welfare (10) and derive analytical expressions for τ^* and σ^* (see Appendix B):

$$\tau^* = \frac{(p^* - \alpha_f)}{\rho_f} - \frac{\beta_f}{(\beta_f + \beta_c)} \left(\frac{\beta_c}{\beta_d} \frac{(p^* - \alpha_d)}{\rho_f} + \frac{\phi_g}{(1 - 2\phi_g)} \frac{(\alpha_f - \alpha_c)}{\rho_f} + \frac{1}{(1 - 2\phi_g)} \eta \right) \tag{22}$$

$$\sigma^* = \left(1 + \frac{\beta_c}{(\beta_f + \beta_c)} \frac{\phi_g}{(1 - 2\phi_g)} \right) (\alpha_c - \alpha_f) + \left(1 - \frac{\beta_c}{(\beta_f + \beta_c)} \right) \frac{\beta_c}{\beta_d} (p^* - \alpha_d)$$
$$- (p^* - \alpha_f) - \frac{\beta_c}{(\beta_f + \beta_c)} \frac{1}{(1 - 2\phi_g)} \eta \rho_f \tag{23}$$

Table 2 presents the comparative statics for τ^* and σ^* with respect to the maximum energy price tolerated (p^*), the social cost of carbon (η), and the welfare value of government revenues (ϕ_g). As the constraint on the maximum energy price relaxes and p^* increases, the optimal CO_2 tax rises while the optimal subsidy falls, bringing the policy strategy closer to the first-best solution. As in the carbon price constraint case, clean energy subsidies thus play a greater role the more constraining the political limits on increases in energy prices. The optimal tax and subsidy both increase, *ceteris peribus*, as the social cost of carbon increases (at least for all plausible values of ϕ_g, i.e. > 0.5). Finally, as ϕ_g increases, the optimal tax and subsidy both decrease. If government taxation induces macroeconomic distortions (i.e. $\phi_g > 1.0$), then the reductions in climate-related damages achieved by greater clean energy subsidy must trade-off against the opportunity to reduce distortions elsewhere in the economy by using carbon pricing revenues to offset other taxes. Yet as the optimal subsidy falls, so too does the level of CO_2 tax that can be established given a fixed constraint on energy price increases.

In the case where $\phi_g = 1.0$ (i.e., where government revenues are frictionless), the optimal tax

Table 2. Comparative statics for optimal CO_2 price and clean energy subsidy levels under political constraint on maximum increase in energy price

Parameter	$\partial\tau^*/\partial x$	Sign	$\partial\sigma^*/\partial x$	Sign
p^*	$\frac{1}{\rho_f}\left(1 - \frac{\beta_f \beta_c}{(\beta_f+\beta_c)}\right)$	+	$\left(1 - \frac{\beta_c}{(\beta_f+\beta_c)}\right)\frac{\beta_c}{\beta_d} - 1$	-
η	$-\frac{1}{(1-2\phi_g)}$	+	$-\rho_f \frac{\beta_c}{(\beta_f+\beta_c)} \frac{1}{(1-2\phi_g)}$	+
ϕ_g	$\frac{2\eta}{(1-2\phi_g)} - \frac{(\alpha_f-\alpha_c)}{\rho_f}\left(\frac{1}{(1-2\phi_g)} - \frac{2\phi_g}{(1-2\phi_g)^2}\right)$	-	$\frac{\beta_c}{(\beta_f+\beta_c)}\left[(\alpha_c - \alpha_f)\left(\frac{1}{(1-2\phi_g)} - \frac{2}{(1-2\phi_g)^2}\right) + \frac{2\eta\rho_f}{(1-2\phi_g)^2}\right]$	-

and subsidy expressions simplify to:

$$\tau^* = \frac{(p^* - \alpha_f)}{\rho_f} - \frac{\beta_f}{(\beta_f + \beta_c)} \left(\frac{\beta_c}{\beta_d} \frac{(p^* - \alpha_d)}{\rho_f} - \frac{(\alpha_f - \alpha_c)}{\rho_f} - \eta \right) \tag{24}$$

$$\sigma^* = \left(1 - \frac{\beta_c}{(\beta_f + \beta_c)}\right)(\alpha_c - \alpha_f) + \left(1 - \frac{\beta_c}{(\beta_f + \beta_c)}\right) \frac{\beta_c}{\beta_d}(p^* - \alpha_d) - (p^* - \alpha_f) + \frac{\beta_c}{(\beta_f + \beta_c)}\eta\rho_f \tag{25}$$

Table 3 presents the sensitivity of optimal CO_2 price and clean energy subsidy levels to the slopes or elasticities of energy supply and demand. As fossil energy supplies become less elastic (β_f increases), a given CO_2 tax has less of an impact on final energy prices. A higher tax will thus be tolerated, all else equal, without exhausting the political constraint on energy price increases. The optimal clean energy subsidy thus correspondingly falls as fossil supplies become less elastic (and vice versa). As in the carbon price constraint case above, as demand becomes more inelastic, the distortions caused by the clean energy subsidy decline and the effectiveness of a carbon tax is reduced. Thus the optimal tax level falls and clean energy subsidy level increases as demand becomes less responsive to price changes (and vice versa). Finally, as clean energy supply becomes more price elastic, the distortions introduced by a clean energy subsidy increase, reducing the optimal subsidy level and increasing the optimal carbon tax (all else equal).

Table 3. Impact of changes in slopes of demand and supply functions on optimal CO_2 tax and subsidy levels

Parameter		Change in $\lvert\beta\rvert$	Elasticity	Change in τ^*	Change in σ^*
Slope of fossil supply	$\lvert\beta_f\rvert$	+	Less	+	-
Slope of fossil supply	$\lvert\beta_f\rvert$	-	More	-	+
Slope of clean supply	$\lvert\beta_c\rvert$	+	Less	-	+
Slope of clean supply	$\lvert\beta_c\rvert$	-	More	+	-
Slope of inverse demand	$\lvert\beta_d\rvert$	+	Less	-	+
Slope of inverse demand	$\lvert\beta_d\rvert$	-	More	+	-

Consumer surplus constraint - Limits on the decrease in energy consumer surplus due to climate policy decisions form an additional political economy constraint, captured in our model as follows:

$$CS(\tau, \sigma, r_d) \geq CS^0(1 - \overline{\Delta CS}) \tag{26}$$

where $CS(\tau, \sigma, r_d)$ is final consumer surplus as a function of the carbon price and clean energy subsidy decisions and net of any lump-sum transfers, CS^0 is the consumer surplus absent policy intervention, and $\overline{\Delta CS}$ is the maximum percent change in producer surplus allowed by political economy considerations.

Assuming efficient transfers, the first-best solution is within reach under a constraint of this form. The welfare-maximizing strategy under this constraint is to establish a CO_2 price equal to the full SCC ($\tau^* = \eta$) while offsetting the impact on consumer surplus via lump-sum transfers (r_c). While a clean energy subsidy can also reduce the final impact on consumer surplus, by reducing the final energy price paid by consumers, this strategy is less efficient than a lump-sum transfer, as it introduces several distortions into the market (as described in (17)).

In the case that either $\phi_c < 1.0$ or $\phi_g > 1.0$, this strategy incurs additional efficiency losses, which must be balanced against the reduction in climate-related deadweight loss achieved by relaxing the indirect constraint on carbon prices. Interestingly, if $\phi_c < 1.0$, representing loss aversion on the part of energy consumers, the most efficient strategy to mitigate the impact on consumer surplus will include a non-zero clean energy subsidy, as the subsidy also mitigates consumer surplus loss by reducing final energy prices. Indeed, the welfare-maximizing strategy when $\phi_c < 1.0$ would equalize the marginal deadweight loss associated with distortions due to the clean energy subsidy with the deadweight loss associated with the inefficiency of compensatory transfers to consumers. Cases where $\phi_c < 1.0$ could therefore also be considered a hybrid of the energy price and consumer surplus constraints presented above and could be considered in future work.

Fossil producer surplus constraint - The final political economy constraint we consider is a constraint on the decline in fossil energy producer surplus induced by climate policy decisions:

$$PS_f(\tau, \sigma, r_f) \geq PS_f^0(1 - \overline{\Delta PS_f}) \tag{27}$$

where $PS_f(\tau, \sigma, r_f)$ is final fossil producer surplus as a function of carbon tax and clean energy subsidy decisions and net of any lump-sum transfers, PS_f^0 is the producer surplus absent policy intervention, and $\overline{\Delta PS_f}$ is the maximum percent change in producer surplus allowed by political economy considerations.

As with the consumer surplus constraint, assuming transfers are frictionless, the welfare-maximizing strategy is to impose a CO_2 price equal to the full SCC ($\tau^* = \eta$) while compensating fossil energy producers as required to satisfy the political economy constraint via lump-sum transfers (r_f). A clean energy subsidy would only further reduce fossil producer surplus, and as the subsidy drives market distortions, it would not improve overall welfare in this case. As such, $\sigma^* = 0$ under this constraint.

Again, if either $\phi_f < 1.0$ or $\phi_g > 1.0$, transfers to producers incur additional welfare losses. In this case, the optimal transfer would equalize the marginal reduction in climate-related deadweight loss achieved by offsetting producer surplus impacts and relaxing the indirect constraint on the carbon price on the one hand, and the marginal deadweight loss associated with the inefficiency of compensatory payments and the impact of distortionary taxes elsewhere in the economy on the other.

4. NUMERICAL RESULTS

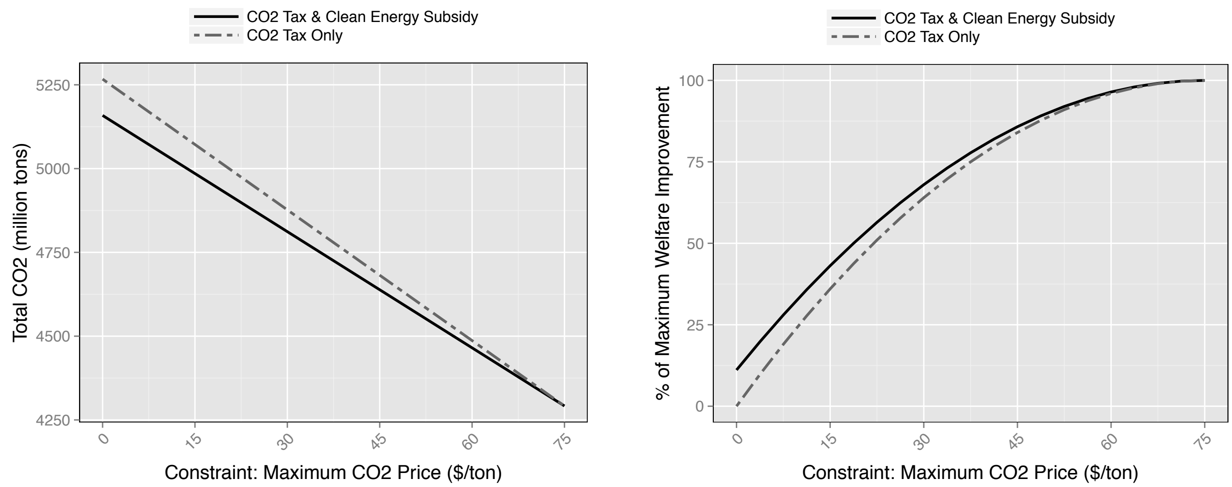
In this section, we present results for a numerical simulation using the model presented in Section 3.1 above. To demonstrate the mechanisms by which strategic allocation of carbon pricing revenues can achieve superior performance, we compare two cases for each of the four political constraint scenarios described above (Section 3.2): a case in which a CO_2 price is introduced and all revenues collected are retained by the state, and a case in which some portion of the revenues from the CO_2 charge are used to achieve either additional CO_2 reductions by subsidizing clean energy or to offset the burden on producers or consumers through government transfers. In each case comparison, we focus on CO_2 emissions and total welfare impacts. We then consider the sensitivity of outcomes, including the deployment of clean and fossil energy, the disposition of CO_2 tax revenues, and the resulting energy price, to the price elasticities of supply and demand. In all cases, we assume the full SCC is \$75 per ton of CO_2 and that ϕ_g, ϕ_f and ϕ_c equal 1.0 (i.e., all transfers are frictionless).

4.1 Constraint on the CO_2 tax increase

In a world where the politically-feasible CO_2 price remains below the social cost of carbon, using revenues to subsidize clean energy results in additional CO_2 emissions reduction and welfare gain, relative to a no-subsidy case, as shown in Figures 4a and 4b. As captured by the analytical solution (see Eq. (18)), the largest welfare gains from the subsidy occur when the CO_2 price constraint binds at low levels. In the absence of any carbon price, subsidizing clean energy can achieve 11 percent of the maximum reduction in CO_2 and improvement in welfare achievable under the first-best carbon pricing level. When the CO_2 price rises, the welfare and emissions performance improvements from the clean energy subsidy decline. This is because the optimal subsidy level decreases as the damages associated with emissions are steadily internalized by the carbon price. In all cases, a non-zero subsidy improves overall welfare unless the carbon price equals the full social cost of carbon.

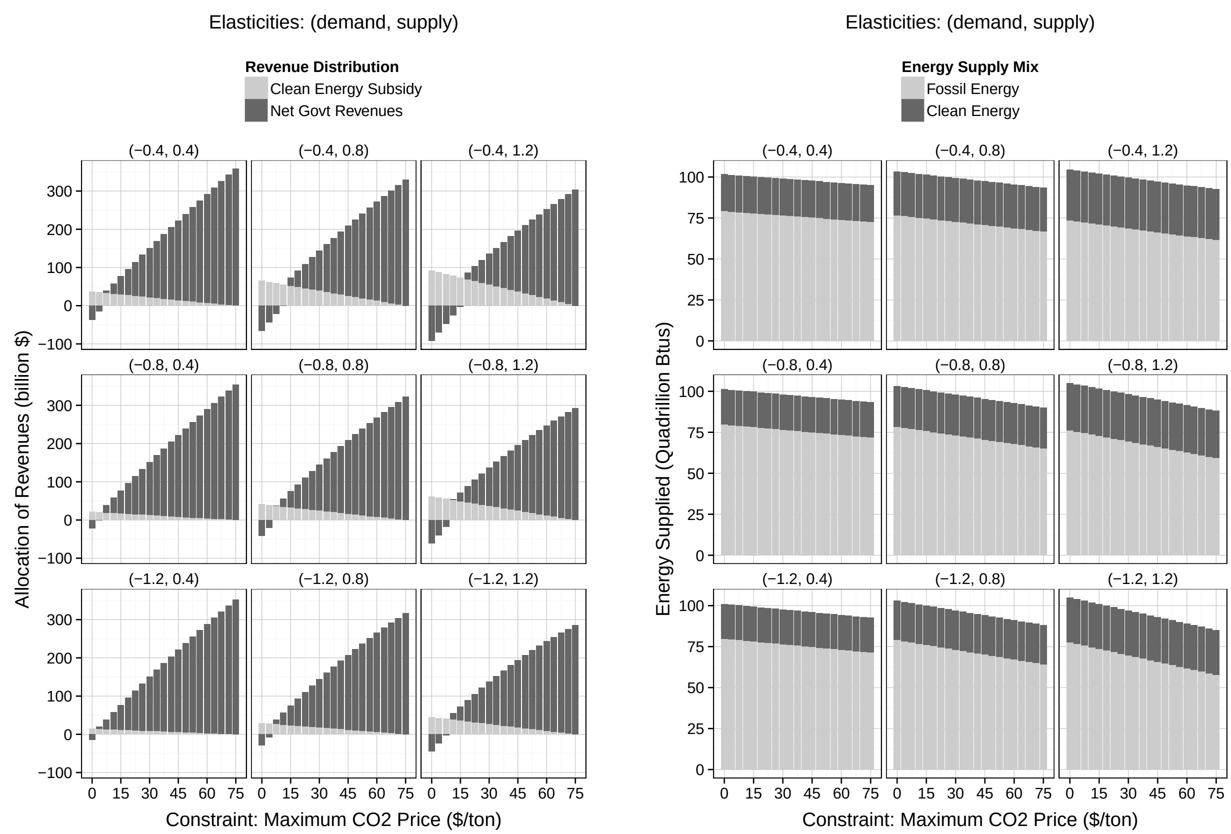
As Figure 5a illustrates, the share of revenues required to fund a clean energy subsidy when carbon prices are directly constrained depends on the elasticities of supply and demand. The more elastic supply is, the more responsive fossil energy production is to changes in energy prices induced by the subsidy, and thus the more effective the subsidy is at driving substitution of clean for dirty fuels, as illustrated in Figure 5b. In contrast, the more elastic demand is, the greater the deadweight loss due to overconsumption of energy driven by the subsidy. When demand is more elastic, the optimal subsidy level is reduced. In all cases, carbon pricing revenues are insufficient to cover the full costs of the clean energy subsidy when the carbon price level is constrained at low levels, resulting in a net decline in government revenues (i.e., necessitating an increase in taxation or reduction in spending elsewhere to compensate). If a revenue neutrality constraint were binding, the effectiveness of this subsidy strategy would be reduced if the carbon price is constrained to very low levels (i.e. below \$10 per ton in these scenarios). Once the carbon price rises sufficiently, the welfare-maximizing carbon price level and subsidy strategy under this constraint yields positive net revenues, which can be used for other purposes.

Figure 3. Improvement in performance due to clean energy subsidy under a binding CO_2 price constraint, compared to the constrained case with no subsidy (Initial elasticity of demand and supply: $-0.8, 0.8$)



(a) CO_2 emissions **(b)** Total welfare

Figure 4. Sensitivity to alternative values for price elasticities of supply an demand under a binding CO_2 price constraint



(a) Disposition of government revenues **(b)** Energy supply

The direct constraint on CO_2 prices is in many ways the most challenging constraint to overcome via the strategic use of carbon pricing revenues. Subsidizing clean energy in this case does not *relax* the constraint itself, but merely compensates for the low carbon price by delivering additional abatement. However, this abatement comes at the cost of economic distortions due to the subsidy, delivering relatively modest improvements in overall welfare. By contrast, under the other constraints, use of revenues not only generates additional abatement but also directly relaxes the constraint itself, allowing for higher carbon prices to be achieved than would otherwise be possible and thus moving closer to the first-best solution.

4.2 Constraint on the energy price increase

Under a constraint on the allowable energy price increase, employing carbon pricing revenues to subsidize clean energy enables a significantly higher price of CO_2, as demonstrated in Figure 5. As clean energy subsidies reduce final energy prices, *ceteris paribus*, deploying revenues to subsidize clean energy alternatives effectively relaxes the constraint on the energy price increase. For example, using clean energy subsidies to offset the rising costs of energy enables a carbon price of $35 per ton CO_2 even when only a negligible increase in final energy prices is permitted. In addition, as in the carbon price constraint case above, the clean energy subsidy drives additional abatement that would not be achieved via the carbon price alone, further improving overall welfare. These benefits again trade off against the deadweight loss due to distortions induced by the clean energy production subsidy.

In combination, the carbon price and clean energy subsidy deliver much greater CO_2 reductions than a carbon price alone, especially when the energy price increase is constrained at low levels (Figure 7a). Nearly two-thirds of the optimal reduction in CO_2 emissions can be achieved without increasing energy prices at all (assuming initial elasticities of demand and supply of -0.8 and 0.8 respectively), and employing revenues to fund clean energy subsidies improves the environmental performance of the policy intervention until the full social cost of carbon is internalized. Overall welfare improves similarly when revenues are used to subsidized clean energy production, achieving two-thirds of the optimal welfare gain even when no increase in energy prices is permitted, rising to nearly 90 percent when a 5 percent increase in final energy prices is tolerated (Figure 7b).

As in the case of limits on the CO_2 price increase described above, the price elasticities of supply and demand influence the required level of clean energy subsidy for a given allowable level of energy price increase. Larger supply elasticities translate into a higher optimal share of clean energy deployment at a rising subsidy expenditure (Figure 8a). At the same time, funding clean energy enables a higher politically-feasible carbon price, which in turn raises overall government revenues, making this policy strategy revenue positive under all cases. Total revenues collected decrease as the elasticity of supply rises, as fossil producers decrease output more sharply rather than pay the required CO_2 charge. The subsidy requirement decreases as price elasticity of demand rises, as higher demand elasticities exacerbate the deadweight loss from overconsumption driven by the subsidy. The optimal, welfare-maximizing subsidy level for a given level of the CO_2 price thus decreases as demand elasticity increases.

Figure 5. CO_2 price achieved under binding constraint on energy price increases, with and without employing revenues to subsidize clean energy (Initial elasticity of demand and supply: $-0.8, 0.8$)

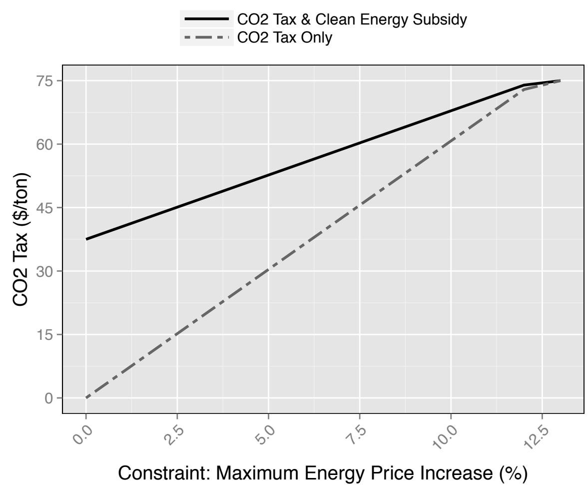
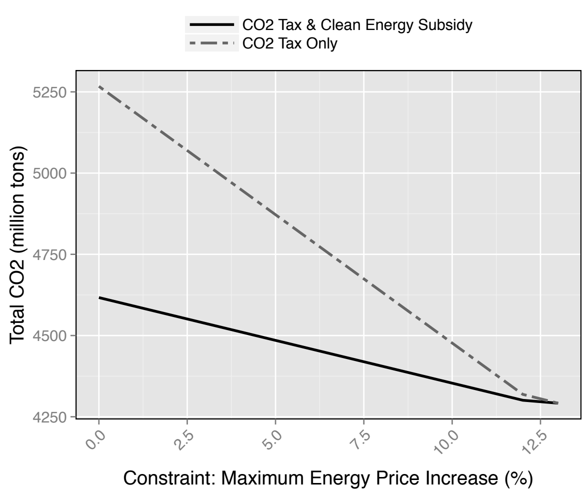
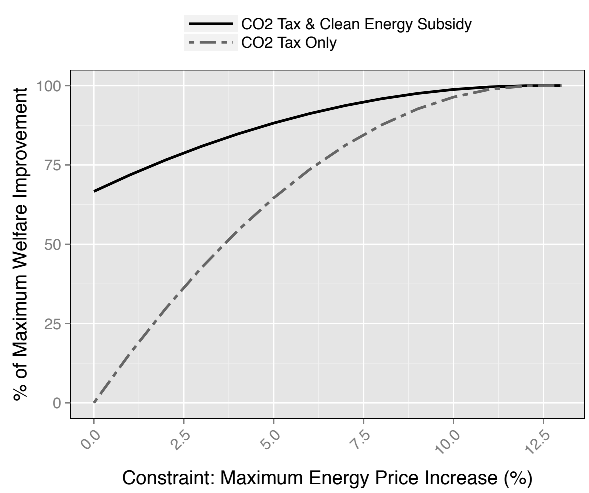


Figure 6. Improvement in performance due to clean energy subsidy under a binding constraint on energy price increases (Initial elasticity of demand and supply: $-0.8, 0.8$)

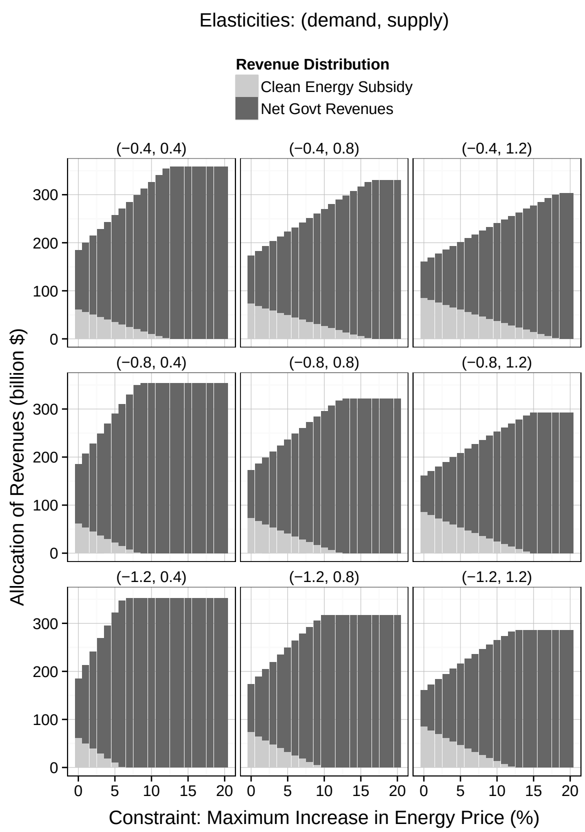


(a) CO_2 emissions

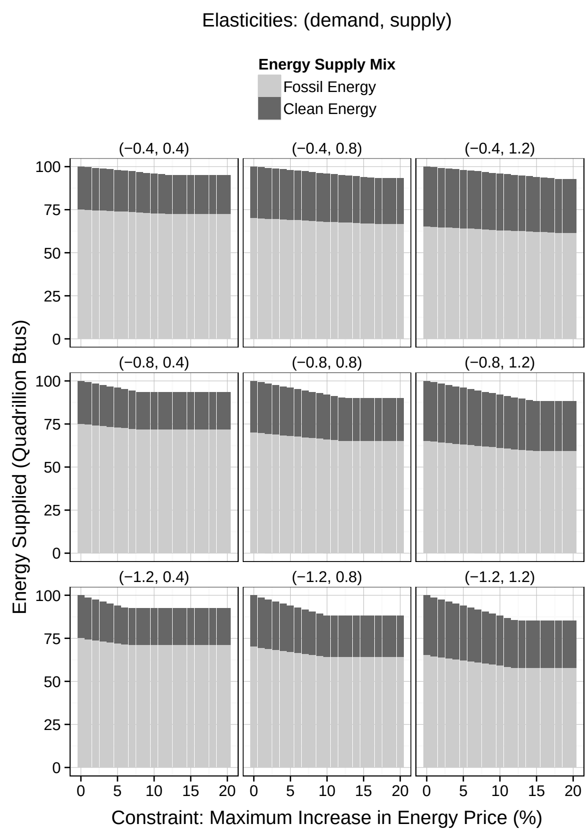


(b) Total welfare

Figure 7. Sensitivity to alternative values for price elasticities of supply and demand under a binding constraint on energy price increases



(a) Disposition of government revenues

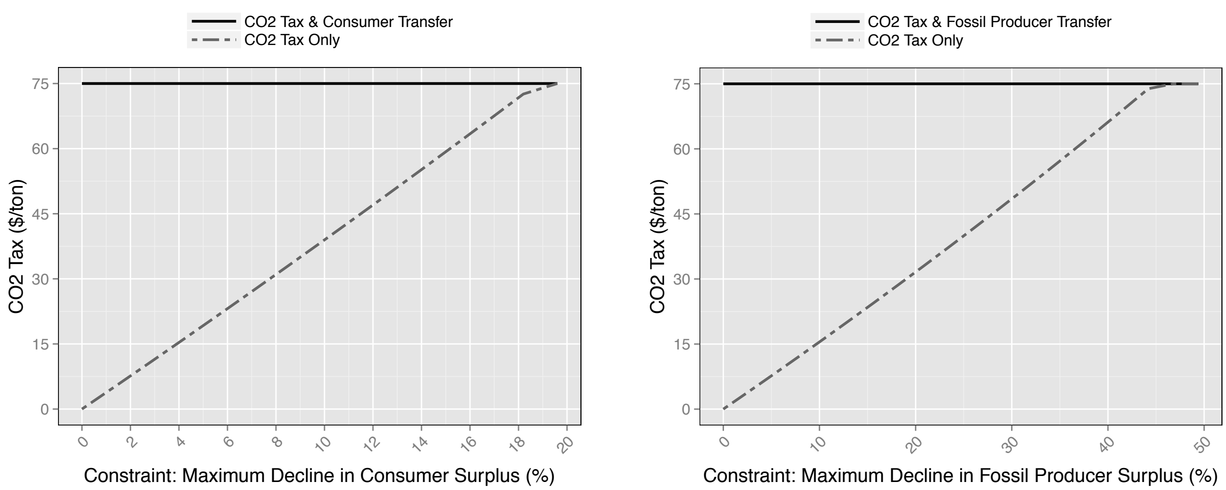


(b) Energy supply

4.3 Constraints on energy consumer and fossil producer surplus loss

We turn to focus on constraints on the loss of energy consumer surplus and fossil energy producer surplus. Unlike the prior cases, where political constraints continue to prevent achievement of the first-best CO_2 pricing level, redistributing carbon revenues as lump-sum transfers allows the private surplus losses for energy consumers or fossil producers to be fully offset. As a result, under constraints on consumer or producer surplus loss, the strategic use of revenues makes the optimal carbon price immediately feasible, provided transfers are frictionless (see Figures 9b, 9a). In contrast, if compensatory transfers are not employed, the available CO_2 price rises linearly under this form of constraint as the allowable consumer or producer surplus loss increases.

Figure 8. Achievable CO_2 price under binding constraints on energy consumer or fossil energy producer surplus loss, with and without compensatory transfers (Initial elasticity of demand and supply: $-0.8, 0.8$)

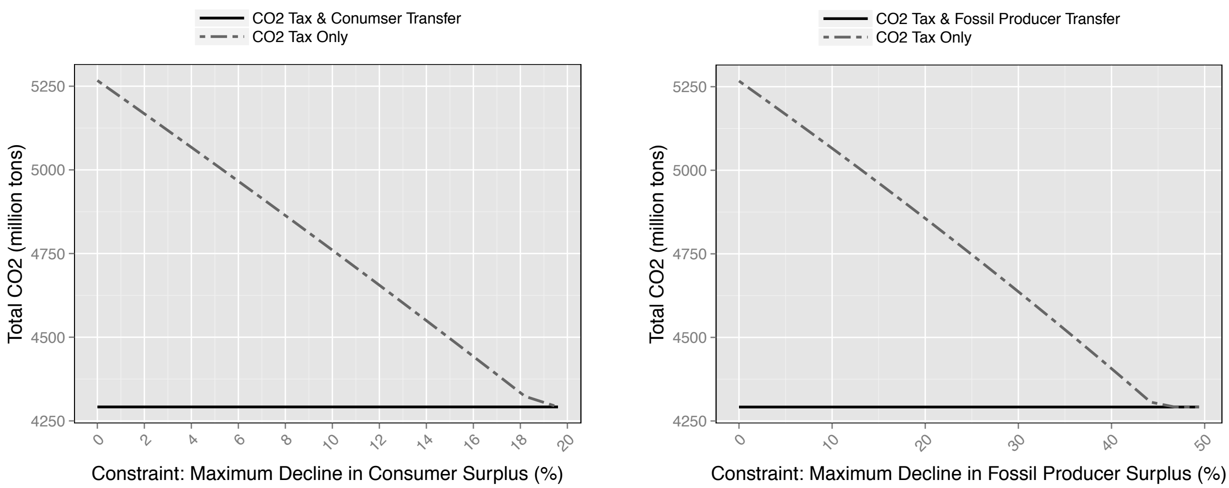


(a) Energy consumer surplus loss constraint

(b) Fossil producer surplus loss constraint

Compensatory transfers to energy consumers or fossil energy producers similarly enables the optimal level of CO_2 emissions abatement (see Figures 10b, 10a), even when welfare losses are tightly constrained. When transfers are possible, the externality can be fully internalized for the levels of consumer and producer surplus loss we consider here, maximizing welfare for all values of the constraint (Figures 11b, 11a)).

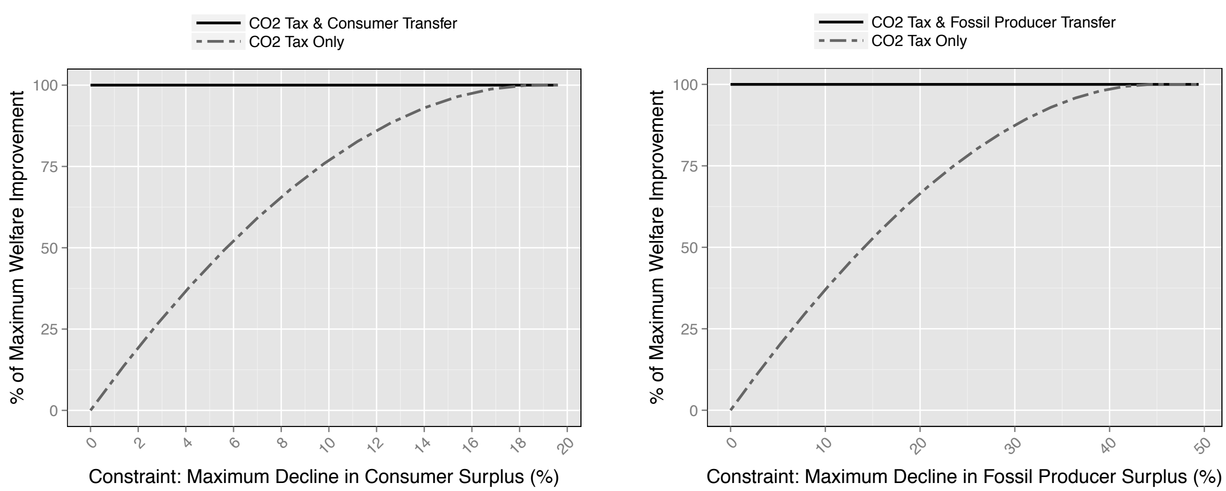
Figure 9. Total CO_2 emissions under binding constraints on energy consumer or fossil energy producer surplus loss, with and without compensatory transfers (Initial elasticity of demand and supply: $-0.8, 0.8$)



(a) Energy consumer surplus loss constraint

(b) Fossil producer surplus loss constraint

Figure 10. Total welfare under binding constraints on energy consumer or fossil energy producer surplus loss, with and without compensatory transfers (Initial elasticity of demand and supply: $-0.8, 0.8$)

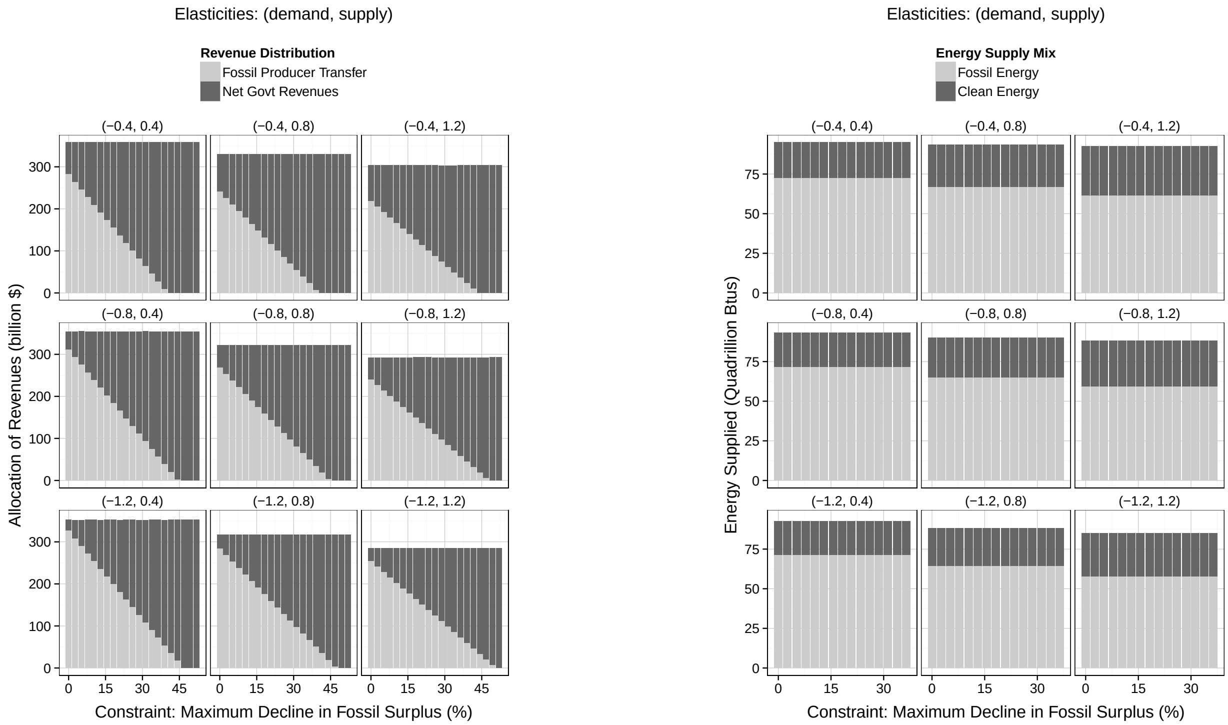


(a) Energy consumer surplus loss constraint

(b) Fossil producer surplus loss constraint

Turning to the elasticity sensitivity cases, patterns observed are similar to the scenarios above, with the main difference again being that the optimal CO_2 price is achievable over all values of the constraints. Total energy supplied falls as the elasticity of supply increases, with larger supply elasticities favoring clean deployment as it is not penalized directly by the CO_2 price (Figures 12b, 13b). Increasing the price elasticity of demand induces greater demand response at the expense of clean energy deployment, similar to previous cases. While a relatively modest portion of revenues is required to achieve the optimal CO_2 price under the constraint on consumer surplus loss (Figure 13a), fossil energy producer surplus losses, and thus the compensating revenues required, are more substantial under carbon pricing policies. Strict constraints on the permissible reduction in producer surplus thus necessitates a substantial fraction of carbon revenues be allocated to fund compensatory transfers (Figure 12a). If producers exhibit loss aversion and do not equally value a dollar of surplus loss and a dollar of compensatory transfers (i.e. if $\phi_f < 1.0$), complying with strict constraints on producer surplus loss may necessitate more revenues than generated by the carbon pricing policy. Furthermore, in both constraint cases, as supply and demand grow more elastic, compensating transfers remain important over a wider range of allowable consumer or producer surplus losses. This is because higher elasticity implies greater surplus losses for a given carbon price level.

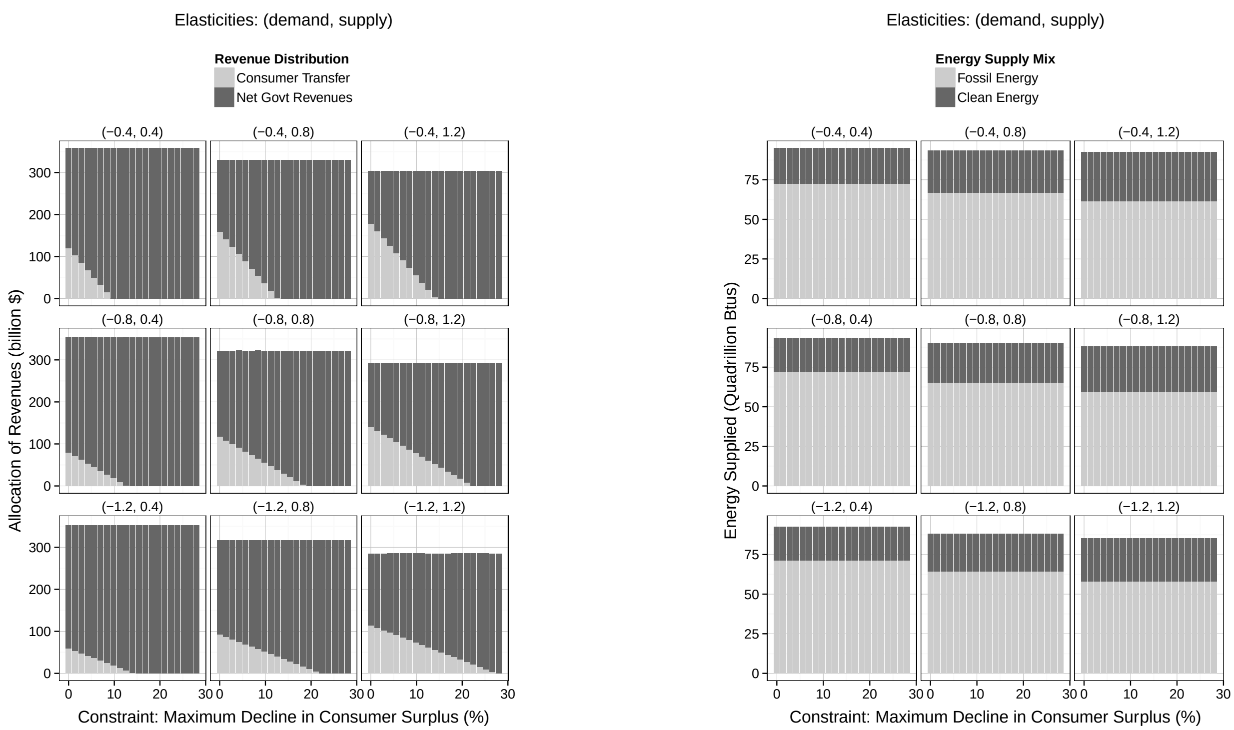
Figure 11. Sensitivity to alternative values for price elasticities of supply an demand under a binding constraint on fossil energy producer surplus losses



(a) Disposition of government revenues

(b) Energy supply

Figure 12. Sensitivity to alternative values for price elasticities of supply an demand under a binding constraint on energy consumer surplus losses



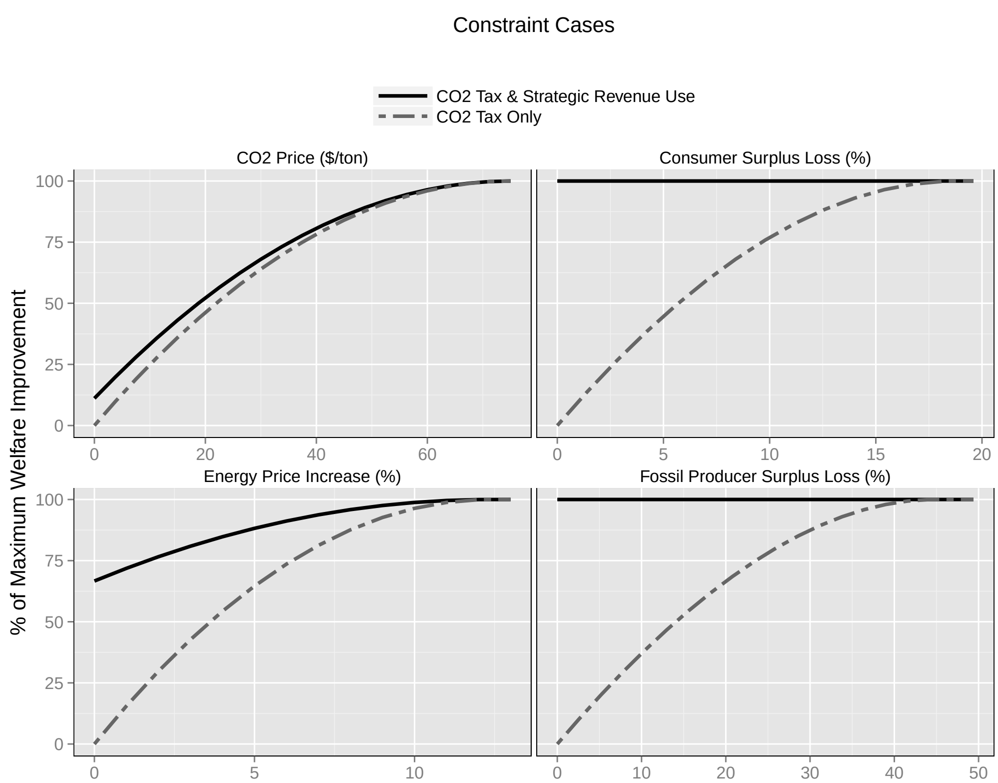
(a) Disposition of government revenues

(b) Energy supply

4.4 Comparison across scenarios

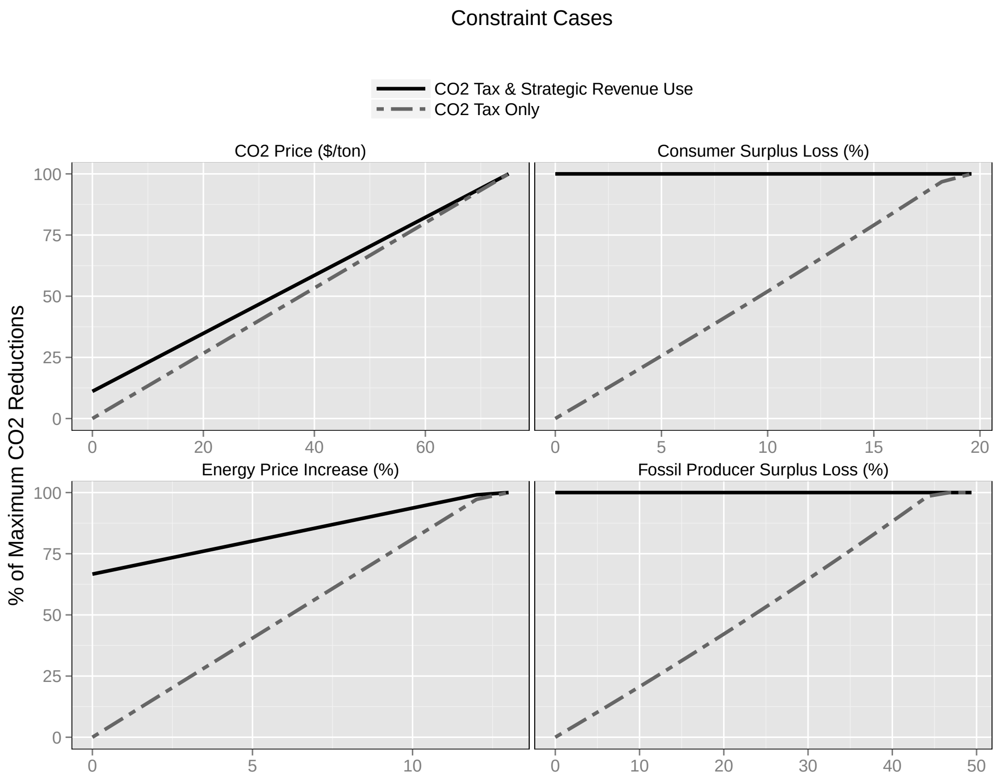
Comparing across the four constraint cases, it is clear that strategically using revenues to compensate for or relax binding political constraints improves economic and environmental performance in all cases (Figures 13, 14). That said, improvements are modest in the case of the direct constraint on CO_2 prices. In addition, both the constraints on carbon price level and energy price increases result in second-best outcomes whenever the political economy constraint is binding, regardless of the use of revenues. Of these two cases, the welfare gain associated with allowing clean energy subsidies in the presence of limits on energy price increases is particularly large, with two-thirds of available welfare improvement achievable even when no increase in the energy price is permitted.

Figure 13. Total welfare gain under four political constraint scenarios



In contrast to the cases with constraints imposed on the CO_2 price and energy price, the ability to use revenues to directly offset private surplus losses allows the full climate externality to be internalized via the CO_2 price under constraints on fossil producer or consumer surplus losses. In other words, revenue recycling achieves the first-best outcome for CO_2 reductions and welfare gains under all constraint levels. Importantly, this outcome depends on lump-sum transfers being frictionless and consumers and producers exhibiting no loss aversion, two assumptions which in practice may be unrealistic. These results thus raise two important questions: 1) what is the real loss, if any, due to frictions or overhead, which would reduce the efficiency of transfers, and 2)

Figure 14. Total CO_2 emissions under four political constraint scenarios



what is the perceived loss, if any, to consumers and producers, given that recipients may demand more than a dollar of compensation to cover each dollar of foregone surplus? Our framework provides a way to consider transfer inefficiency in calculations of deadweight loss, which will have implications for the optimal CO_2 price, CO_2 emissions abatement, and distribution of welfare impacts. We will explore these implications in future work.

As Figure 15 illustrates, the distribution of welfare under the four cases differs significantly. As one might expect, consumers and fossil producers are best off under the respective cases where political constraints motivate direct transfers to offset any surplus losses they incur due to policy intervention. At the same time, consumers are almost equally well off when revenues are used to subsidize clean energy in the face of a constraint on energy price increases. Here, clean energy subsidies drive incremental substitution of clean for dirty energy and keep final energy prices low, insulating consumers from welfare losses. Similarly, as total reductions in fossil energy use are modest under the case where the CO_2 price is directly constrained, fossil producers are nearly as well off in this case as they are under the direct constraint on fossil producer losses. Political constraints on the carbon price or energy price increases may therefore be interpreted as the indirect expression of concern about producer or consumer surplus losses, respectively, particularly in cases where consumers and producers exhibit significant loss aversion, and thus view compensatory payments in an inferior light.

Figure 15. Disposition of welfare under four political constraint scenarios

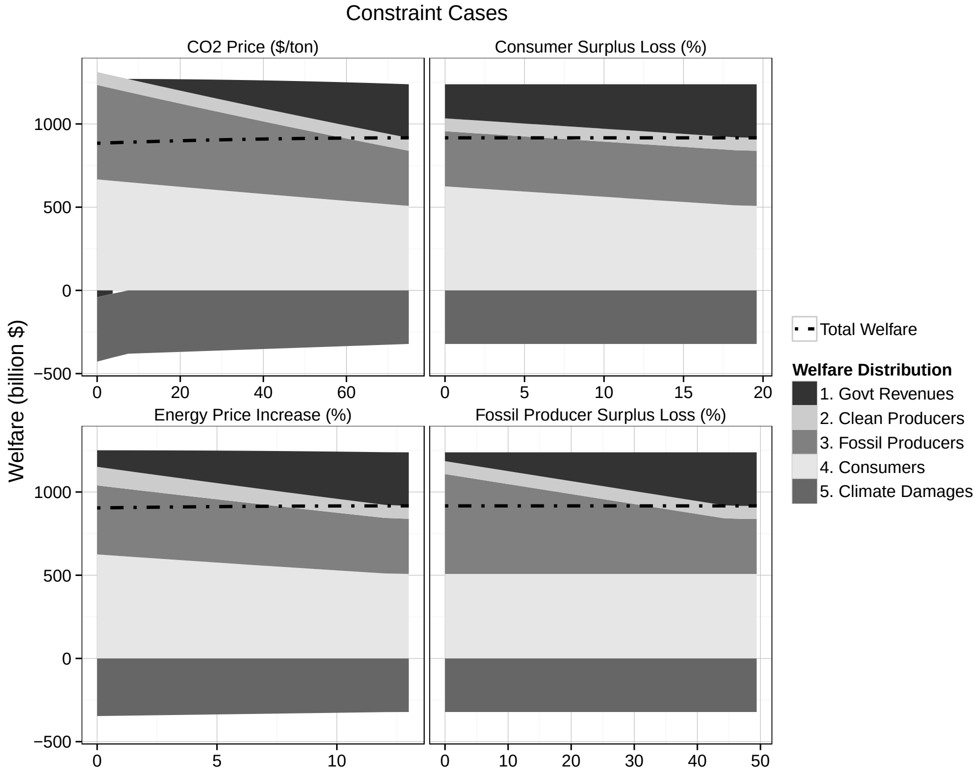
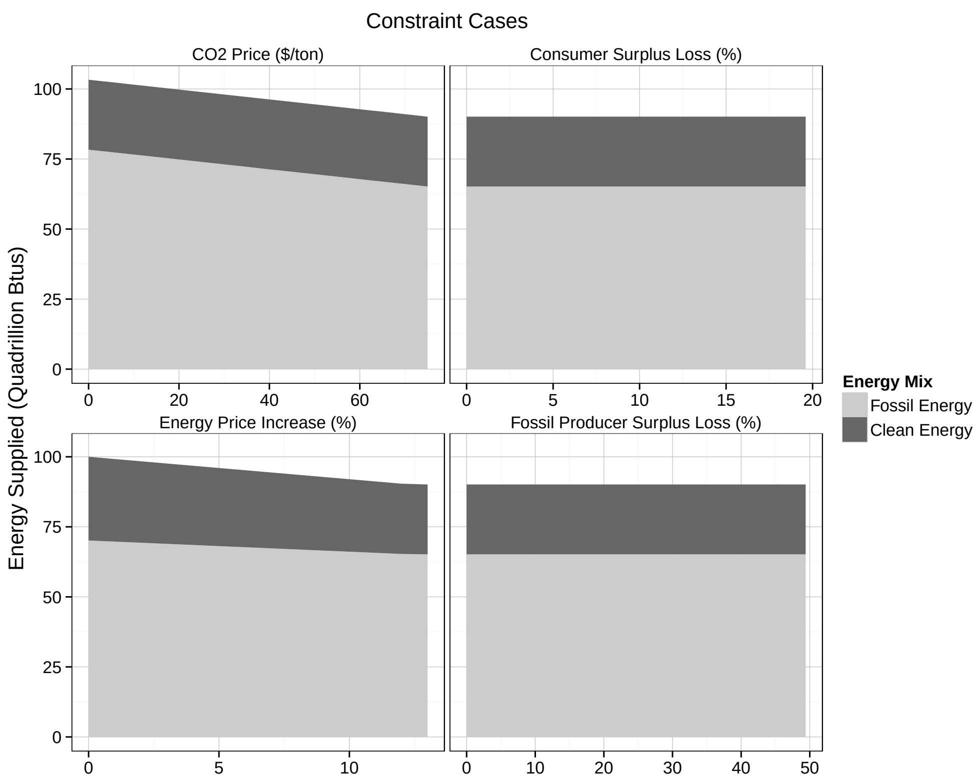


Figure 16. Energy supply under four political constraint scenarios



The growth of clean energy producer surplus is most significant under the energy price constraint (Figure 15), which mobilizes the greatest support for clean energy production, resulting in a transfer from consumers and fossil producers to clean energy producers. As Figure 16 illustrates, clean energy market share is highest under this case. If the size and relative economic importance of clean energy production sectors positively affects the political durability of climate mitigation policy and increases tolerance for future increases in carbon prices, combining a carbon price with subsidies for clean energy producers may yield additional dynamic benefits. Similarly, additional deployment of clean energy in the near-term could drive learning-by-doing, economies of scale, or induced research and innovation, decreasing the cost of clean energy supply in the future, although the magnitude of these benefits is uncertain. Over time, the result would be greater mitigation at a given cost, an important dynamic benefit to consider.

5. CONCLUSION AND IMPLICATIONS FOR POLICY-MAKING AND RESEARCH

Global experience to date suggest that the distributional impacts of carbon pricing policies on energy producers and consumers make it difficult to legislate CO_2 price levels needed to fully internalize the climate change externality. This reality points to two important ongoing agendas for research: one aimed at improving on existing estimates of the social cost of carbon and evaluating the impacts of fully internalizing these damages through CO_2 pricing, and another that starts from the presently feasible set of alternatives, taking political constraints as binding in the near-term, and evaluates options for improving welfare and expanding this feasible set over time. In the latter case, the goal is to identify policy designs that are not too distant from the efficient frontier and that alter the relative influence of actors in ways that support gradual convergence toward a socially optimal CO_2 charge. Although methods for arriving at the SCC are still hotly debated, as long as prevailing CO_2 prices remain below the lower end of the SCC range, as they do in many CO_2 pricing systems at present, focusing on political constraints is important to answering the question: where do we start today?

In this analysis, we have systematically investigated the impact of four different political economy constraints on carbon pricing, focusing on how the stringency of the constraints affect the welfare gain associated with alternative uses of CO_2 price revenues. We find that in all cases, without the ability to use revenues in ways that increase abatement or offset private surplus losses, the optimal CO_2 price is beyond reach. We show that compensating for a direct constraint on the CO_2 price delivers modest gains, because the benefits associated with additional abatement are offset by deadweight loss resulting from over-consumption induced by subsidies. In this respect, a constraint on the absolute level of the CO_2 price constitutes the most restrictive case. By contrast, greater welfare gains are possible under a constraint on energy price increases, as carbon pricing revenues can be used to subsidize clean energy and keep final energy prices low, allowing a higher carbon price to be achieved than would otherwise be possible. Indeed, when revenues are deployed to subsidize clean energy, a substantial CO_2 price is possible even if no increase in final energy prices is tolerated at all. Finally, using revenues to offset consumer and producer surplus loss supports a return to optimal CO_2 price levels and a first-best solution—provided compensatory transfers are frictionless and consumers and producers do not exhibit loss aversion.

While the analysis above develops intuition about how constraints function individually and in an idealized context, reality is inevitably more complex. An important question for decision-makers and political scientists involves establishing which political economy constraints bind in the jurisdiction in question and through what mechanisms they operate. In practice, multiple political economy constraints may bind at the same time—e.g., a high CO_2 price may be unavailable because covered parties are concerned about the resulting energy price increase, or the magnitude of the impact on consumer and producer surplus, or all of the above. In the face of multiple political economy constraints, one potential solution would be to dip into government budgets to further subsidize CO_2 abatement or to offset reductions in consumer and producer surplus. However, this option requires careful consideration of the opportunity cost of channeling additional funds to relieve political economy constraints, as potential second-best solutions will compete with each other, and with other possible uses of public funds, for available government revenues. Ultimately, the political feasibility of this path is constrained by public decision-making on appropriate spending priorities, and the nature of the climate change problem is such that near-term public investments with more concrete benefits may be preferred.

Our analysis clearly shows that it is possible to achieve the first-best CO_2 price if revenues can be used to offset consumer and producer surplus losses. In reality, however, none of the transfers discussed here are likely to be frictionless. It is important therefore to also understand the real and perceived value of these transfers to recipients and the general equilibrium implications of changes in government revenues. Transfers to support clean energy subsidies may also have associated frictions, which will magnify the relative inefficiency of the subsidy. On the other hand, more targeted subsidies which only apply to supra-marginal suppliers could reduce the overall revenues required to drive clean energy adoption and associated mitigation, an important consideration in cases that subsidy programs entail additional efficiency losses (i.e. due to foregone opportunities to reduce other distortionary taxes). The nature and magnitude of these frictions and their efficiency implications will be specific to particular contexts, increasing the importance of understanding and quantifying their impact on interests and incentives.

The main objective of this exercise was to put an analytical framework around the question of how we can get started down a relatively efficient path to a lower carbon world. The answer will be different, depending on the unique political economy of the climate issue across nations and regions. We conclude by briefly illustrating the guidance this framework would offer under different prevailing constraints.

First, if the prevailing constraint is the unwillingness of energy consumers to bear the burden of higher energy prices and associated surplus loss, our results suggest that as a starting point, returning revenues in ways that make energy cheaper (i.e. via clean energy subsidies) and directly offsetting any remaining losses to consumers is a potential solution. This strategy may be most viable in jurisdictions without significant domestic fossil energy production sectors, where the major concern is the impact of climate mitigation policies on household incomes and the economic competitiveness of domestic industries.

Second, to the extent that influential fossil energy producers and industrial energy consumers are aligned in opposition to CO_2 pricing, neutralizing opposition from industrial energy consumers by subsidizing clean energy adoption and keeping final energy prices low could remove a major barrier to CO_2 pricing, while allowing the CO_2 price to rise to a meaningful level. Remaining resistance from the fossil energy industry could then be addressed through transfer payments—either taken from CO_2 price revenues or elsewhere in the government budget. This strategy may be most viable in jurisdictions with strong domestic fossil energy sectors and relatively large energy-intensive industrial sectors, such as steel, aluminum, concrete, or pulp and paper production.

Under either case, if structural changes relax political constraints over time, CO_2 prices could rise toward the full social cost of carbon. The dynamic impacts of near-term policy decisions on political constraints over time is thus an additional key consideration worthy of future research further. For example, encouraging near-term deployment of clean energy to an extent that realizes benefits from scale economies, learning, and a growing clean energy constituency with a strong interest in its own continued survival and growth could have significant impacts on the political durability of climate policy over time.

These scenarios offer illustrative paths by which the costs of a clean energy transition, which will inevitably create winners and losers, could be smoothed over time, gradually nudging the possible in the direction of the optimal.

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APPENDIX A

This Appendix drives the optimal clean energy subsidy level, σ^* when political constraints directly limit the CO_2 price, τ below the full social cost of carbon, η.

From (7), we can see that the imposition of a subsidy reduces the aggregate supply curve as follows:

$$\Delta MC_t(\sigma) = \frac{-\sigma\beta_t}{\beta_c} \tag{28}$$

$$\text{where: } \beta_t = \frac{\beta_f\beta_c}{\beta_f + \beta_c}$$

From the equilibrium market clearing constraints (11), we know $MB(q^*) = MC_t(q^*)$. Substituting (1) and (7), we can then solve for the equilibrium total energy production/consumption:

$$\alpha_d + \beta_d q^* = \alpha_t + \beta_t q^* \tag{29}$$

$$q^* = \frac{\alpha_t - \alpha_d}{\beta_d - \beta_t}$$

$$\text{where: } \alpha_t = \left(\frac{\alpha_c - \sigma}{\beta_c} + \frac{\alpha_f + \tau\rho_f}{\beta_f}\right)\left(\frac{\beta_f\beta_c}{\beta_f + \beta_c}\right)$$

The impact of a clean energy subsidy on equilibrium total energy consumption is thus:

$$\Delta q^*(\sigma) = \frac{-\sigma\beta_t}{\beta_c(\beta_d - \beta_t)} \tag{30}$$

We also know that in equilibrium:

$$p^* = MB(q^*) = \alpha_d + \beta_d q^* \tag{31}$$

Combining this with (30) yields the effect of the subsidy on equilibrium market clearing prices:

$$\Delta p^*(\sigma) = \beta_d\frac{-\sigma\beta_t}{\beta_c(\beta_d - \beta_t)} = \frac{-\sigma\beta_t\beta_d}{\beta_c(\beta_d - \beta_t)} \tag{32}$$

Combining (3), (5), (11), and (32) we derive the impact of the clean energy subsidy on fossil and clean energy production/consumption:

$$\Delta q_f(\sigma) = \frac{-\sigma\beta_t\beta_d}{\beta_c\beta_f(\beta_d - \beta_t)} \tag{33}$$

$$\Delta q_c(\sigma) = \frac{\sigma}{\beta_c} - \frac{\sigma\beta_t\beta_d}{\beta_c^2(\beta_d - \beta_t)} \tag{34}$$

We can now combine (15) and (33) to derive the reduction in deadweight loss achieved by a given clean energy subsidy level and the marginal change in this deadweight loss with respect to changes in the subsidy:

$$\Delta DWL_{CO_2}(\sigma) = (\eta - \tau)\rho \frac{-\sigma\beta_t\beta_d}{\beta_c\beta_f(\beta_d - \beta_t)} = -\sigma(\eta - \tau)\rho \frac{\beta_t\beta_d}{\beta_c\beta_f(\beta_d - \beta_t)} \tag{35}$$

$$\frac{\partial}{\partial\sigma}\Delta DWL_{CO_2}(\sigma) = -(\eta - \tau)\rho \frac{\beta_t\beta_d}{\beta_c\beta_f(\beta_d - \beta_t)} \tag{36}$$

We then substitute (30), (32), (33) and (34) into (17) and derive the aggregate deadweight loss due to the multiple market distortions caused by the clean energy subsidy as well as the partial derivative with respect to changes in subsidy levels:

$$\begin{aligned}
DWL_\sigma(\sigma) &= \frac{1}{2}\Big[\Big|\frac{-\sigma\beta_t\beta_d}{\beta_c(\beta_d - \beta_t)}\Big|\left(\frac{-\sigma\beta_t}{\beta_c(\beta_d - \beta_t)}\right) + \Big|\frac{-\sigma\beta_t\beta_d}{\beta_c(\beta_d - \beta_t)}\Big|\Big|\frac{-\sigma\beta_t\beta_d}{\beta_c\beta_f(\beta_d - \beta_t)}\Big| \\
&\quad + \beta_c\left(\frac{\sigma}{\beta_c} - \frac{\sigma\beta_t\beta_d}{\beta_c^2(\beta_d - \beta_t)}\right)^2\Big] \\
&= \frac{1}{2}\Big[\left(\frac{\sigma\beta_t\beta_d}{\beta_c(\beta_d - \beta_t)}\right)\left(\frac{-\sigma\beta_t}{\beta_c(\beta_d - \beta_t)}\right) + \left(\frac{\sigma\beta_t\beta_d}{\beta_c(\beta_d - \beta_t)}\right)\left(\frac{\sigma\beta_t\beta_d}{\beta_c\beta_f(\beta_d - \beta_t)}\right) \\
&\quad + \beta_c\left(\frac{\sigma^2}{\beta_c^2} - \frac{2\sigma^2\beta_t\beta_d}{\beta_c^3(\beta_d - \beta_t)} + \frac{\sigma^2\beta_t^2\beta_d^2}{\beta_c^4(\beta_d - \beta_t)^2}\right)\Big] \\
&= \frac{1}{2}\Big[\left(\frac{\sigma\beta_t\beta_d}{\beta_c(\beta_d - \beta_t)}\right)\left(\frac{-\sigma\beta_t}{\beta_c(\beta_d - \beta_t)}\right) + \left(\frac{\sigma\beta_t\beta_d}{\beta_c(\beta_d - \beta_t)}\right)\left(\frac{\sigma\beta_t\beta_d}{\beta_c\beta_f(\beta_d - \beta_t)}\right) \\
&\quad + \beta_c\left(\frac{\sigma^2}{\beta_c^2} - \frac{2\sigma^2\beta_t\beta_d}{\beta_c^3(\beta_d - \beta_t)} + \frac{\sigma^2\beta_t^2\beta_d^2}{\beta_c^4(\beta_d - \beta_t)^2}\right)\Big] \\
&= \frac{1}{2}\Big[\frac{-\sigma^2\beta_t^2\beta_d}{\beta_c^2(\beta_d - \beta_t)^2} + \frac{\sigma^2\beta_t^2\beta_d^2}{\beta_c^2\beta_F(\beta_d - \beta_t)^2} + \frac{\sigma^2}{\beta_c} - \frac{2\sigma^2\beta_t\beta_d}{\beta_c^2(\beta_d - \beta_t)} + \frac{\sigma^2\beta_t^2\beta_d^2}{\beta_c^3(\beta_d - \beta_t)^2}\Big] \\
&= \frac{\sigma^2}{2}\Big[\frac{-\beta_t^2\beta_d}{\beta_c^2(\beta_d - \beta_t)^2} + \frac{\beta_t^2\beta_d^2}{\beta_c^2\beta_f(\beta_d - \beta_t)^2} + \frac{1}{\beta_c} - \frac{2\beta_t\beta_d}{\beta_c^2(\beta_d - \beta_t)} + \frac{\beta_t^2\beta_d^2}{\beta_c^3(\beta_d - \beta_t)^2}\Big]
\end{aligned} \tag{37}$$

$$\begin{aligned}
&\frac{\partial}{\partial\sigma}DWL_\sigma(\sigma) = \\
&\qquad \sigma\Big[\frac{-\beta_t^2\beta_d}{\beta_c^2(\beta_d - \beta_t)^2} + \frac{\beta_t^2\beta_d^2}{\beta_c^2\beta_f(\beta_d - \beta_t)^2} + \frac{1}{\beta_c} - \frac{2\beta_t\beta_d}{\beta_c^2(\beta_d - \beta_t)} + \frac{\beta_t^2\beta_d^2}{\beta_c^3(\beta_d - \beta_t)^2}\Big]
\end{aligned} \tag{38}$$

Finally, using (36) and (38), we can now solve for σ^* satisfying the first-order condition (18):

$$\sigma^*\Big[\frac{-\beta_t^2\beta_d}{\beta_c^2(\beta_d-\beta_t)^2}+\frac{\beta_t^2\beta_d^2}{\beta_c^2\beta_f(\beta_d-\beta_t)^2}+\frac{1}{\beta_c}-\frac{2\beta_t\beta_d}{\beta_c^2(\beta_d-\beta_t)}+\frac{\beta_t^2\beta_d^2}{\beta_c^3(\beta_d-\beta_t)^2}\Big]$$
$$-(\eta-\tau)\rho\frac{\beta_t\beta_d}{\beta_c\beta_f(\beta_d-\beta_t)}=0$$

$$\sigma^*\Big[\frac{-\beta_t^2\beta_d}{\beta_c^2(\beta_d-\beta_t)^2}+\frac{\beta_t^2\beta_d^2}{\beta_c^2\beta_f(\beta_d-\beta_t)^2}+\frac{1}{\beta_c}-\frac{2\beta_t\beta_d}{\beta_c^2(\beta_d-\beta_t)}+\frac{\beta_t^2\beta_d^2}{\beta_c^3(\beta_d-\beta_t)^2}\Big]$$
$$=(\eta-\tau)\rho\frac{\beta_t\beta_d}{\beta_c\beta_f(\beta_d-\beta_t)}$$

$$\sigma^*=(\eta-\tau)\rho\frac{\beta_t\beta_d}{\beta_c\beta_f(\beta_d-\beta_t)}\Big[\frac{-\beta_t^2\beta_d}{\beta_c^2(\beta_d-\beta_t)^2}$$
$$+\frac{\beta_t^2\beta_d^2}{\beta_c^2\beta_f(\beta_d-\beta_t)^2}+\frac{1}{\beta_c}-\frac{2\beta_t\beta_d}{\beta_c^2(\beta_d-\beta_t)}+\frac{\beta_t^2\beta_d^2}{\beta_c^3(\beta_d-\beta_t)^2}\Big]^{-1} \tag{39}$$

APPENDIX B

This Appendix drives the optimal CO_2 tax level, τ^* and the optimal clean energy subsidy level, σ^* when political constraints limit the maximum increase in energy price to $p \leq p^0(1+\overline{\Delta p})$. In the case this constraint is binding, then:

$$p^* = p^0(1+\overline{\Delta p}) \tag{40}$$

We also know that the market clearing constraints must bind (41) and that energy supply from clean and fossil suppliers must sum to the total quantity of energy supplies (42):

$$p^* = \alpha_d + \beta_d q^* = \alpha_f + \tau\rho_f + \beta_f q_f^* = \alpha_c - \sigma + \beta_c q_c^* \tag{41}$$

$$q^* = q_f^* + q_c^* \tag{42}$$

From these constraints, we can derive the following:

$$q^* = \frac{p^* - \alpha_d}{\beta_d} \tag{43}$$

$$\frac{\partial q^*}{\partial \tau} = 0 \tag{44}$$

$$q_f^* = \frac{p^* - \alpha_f - \tau\rho_f}{\beta_f} \tag{45}$$

$$\frac{\partial q_f^*}{\partial \tau} = \frac{-\rho_f}{\beta_f} \tag{46}$$

$$q_c^* = q^* - q_f^* \tag{47}$$

$$\frac{\partial q_c^*}{\partial \tau} = \frac{\rho_f}{\beta_f} \tag{48}$$

$$\sigma = \alpha_c + \beta_c q_c^* - \alpha_f - \tau\rho_f - \beta_f q_f^* \tag{49}$$

$$= \alpha_c + \beta_c(q^* - q_f^*) - \alpha_f - \tau\rho_f - \beta_f q_f^* \tag{50}$$

$$\frac{\partial \sigma}{\partial \tau} = \beta_c \frac{\rho_f}{\beta_f} - \rho_f - \beta_f \frac{-\rho_f}{\beta_f} = \frac{\rho_f}{\beta_f}\beta_c \tag{51}$$

Finally, we know that the optimal values τ^*, σ^* maximize total welfare, $W(\tau^*, \sigma^*)$, as defined in (10). Substituting for σ^* using (50), we obtain a function $W(\tau^*)$, and recalling the partial derivatives above, we solve the first-order condition maximizing welfare:

$$\frac{\partial W}{\partial \tau} = \frac{\partial}{\partial \tau}\left[CS + PS_f + PS_c + R - E\right] = 0 \tag{52}$$

Finding the partial derivatives piece by piece...

$$\frac{\partial}{\partial \tau}CS = \frac{\partial}{\partial \tau}\left[\alpha_d q^* + \frac{1}{2}\beta_d (q^*)^2 - p^* q^*\right] = 0 \tag{53}$$

$$\begin{aligned}
\frac{\partial}{\partial \tau}PS_f &= \frac{\partial}{\partial \tau}\left[p^* q_f^* - \alpha_f q_f^* - \tau \rho_f q_f^* - \frac{1}{2}\beta_f (q_f^*)^2\right] \\
&= p^* \frac{\partial}{\partial \tau}(q_f^*) - \alpha_f \frac{\partial}{\partial \tau}(q_f^*) - \rho_f \frac{\partial}{\partial \tau}(\tau q_f^*) - \frac{1}{2}\beta_f \frac{\partial}{\partial \tau}[(q_f^*)^2] \\
&= p^* \frac{\partial}{\partial \tau}(q_f^*) - \alpha_f \frac{\partial}{\partial \tau}(q_f^*) - \rho_f \left(q_f^* \frac{\partial}{\partial \tau}(\tau) + \tau \frac{\partial}{\partial \tau}(q_f^*)\right) - \frac{1}{2}\beta_f \left(2 q_f^* \frac{\partial}{\partial \tau}(q_f^*)\right) \\
&= p^* \frac{\partial}{\partial \tau}(q_f^*) - \alpha_f \frac{\partial}{\partial \tau}(q_f^*) - \rho_f q_f^* - \rho_f \tau \frac{\partial}{\partial \tau}(q_f^*) - \beta_f q_f^* \frac{\partial}{\partial \tau}(q_f^*) \\
&= \left(p^* - \alpha_f - \rho_f \tau - \beta_f q_f^*\right) \frac{\partial}{\partial \tau}(q_f^*) - \rho_f q_f^* \\
&= \left(p^* - \alpha_f - \rho_f \tau - \beta_f q_f^*\right) \frac{-\rho_f}{\beta_f} - \rho_f q_f^* \\
&= \left(p^* - \alpha_f - \rho_f \tau\right) \frac{-\rho_f}{\beta_f} - \beta_f q_f^* \frac{-\rho_f}{\beta_f} - \rho_f q_f^* \\
&= \left(p^* - \alpha_f - \rho_f \tau\right) \frac{-\rho_f}{\beta_f} + \rho_f q_f^* - \rho_f q_f^* \\
&= \left(p^* - \alpha_f - \rho_f \tau\right) \frac{-\rho_f}{\beta_f} = \left(p^* - \alpha_f\right) \frac{-\rho_f}{\beta_f} - \rho_f \tau \frac{-\rho_f}{\beta_f} \\
&= \frac{\rho_f^2}{\beta_f}\tau - \frac{\rho_f}{\beta_f}\left(p^* - \alpha_f\right) = \frac{\rho_f^2}{\beta_f^2}\beta_f \tau - \frac{\rho_f}{\beta_f}\left(p^* - \alpha_f\right)
\end{aligned} \tag{54}$$

$$
\begin{aligned}
\frac{\partial}{\partial \tau} PS_c &= \frac{\partial}{\partial \tau}\left[p^* q_c^* - \alpha_c q_c^* + \sigma q_c^* - \frac{1}{2}\beta_c (q_c^*)^2\right] \\
&= p^* \frac{\partial}{\partial \tau}(q_c^*) - \alpha_c \frac{\partial}{\partial \tau}(q_c^*) + \frac{\partial}{\partial \tau}(\sigma q_c^*) - \frac{1}{2}\beta_c \frac{\partial}{\partial \tau}[(q_c^*)^2] \\
&= p^* \frac{\partial}{\partial \tau}(q_c^*) - \alpha_c \frac{\partial}{\partial \tau}(q_c^*) + \left(q_c^* \frac{\partial}{\partial \tau}(\sigma) + \sigma \frac{\partial}{\partial \tau}(q_c^*)\right) - \frac{1}{2}\beta_c \left(2 q_c^* \frac{\partial}{\partial \tau}(q_c^*)\right) \\
&= p^* \frac{\partial}{\partial \tau}(q_c^*) - \alpha_c \frac{\partial}{\partial \tau}(q_c^*) + q_c^* \frac{\partial}{\partial \tau}(\sigma) + \sigma \frac{\partial}{\partial \tau}(q_c^*) - \beta_c q_c^* \frac{\partial}{\partial \tau}(q_c^*) \\
&= \left(p^* - \alpha_c + \sigma - \beta_c q_c^*\right) \frac{\partial}{\partial \tau}(q_c^*) + q_c^* \frac{\partial}{\partial \tau}(\sigma) \\
&= \left(p^* - \alpha_c + \sigma - \beta_c q_c^*\right) \frac{\rho_f}{\beta_f} + q_c^* \frac{\rho_f}{\beta_f}\beta_c \\
&= \left(p^* - \alpha_c + \sigma\right) \frac{\rho_f}{\beta_f} - q_c^* \frac{\rho_f}{\beta_f}\beta_c + q_c^* \frac{\rho_f}{\beta_f}\beta_c \\
&= \left(p^* - \alpha_c + \sigma\right) \frac{\rho_f}{\beta_f} \\
&= \left(p^* - \alpha_c + \alpha_c + \beta_c (q^* - q_f^*) - \alpha_f - \tau\rho_f - \beta_f q_f^*\right) \frac{\rho_f}{\beta_f} \\
&= \left(p^* + \beta_c q^* - \beta_c q_f^* - \alpha_f - \tau\rho_f - \beta_f q_f^*\right) \frac{\rho_f}{\beta_f} \\
&= \left(p^* - \alpha_f + \beta_c q^* - \beta_c q_f^* - \beta_f \left(\frac{p^* - \alpha_f - \tau\rho_f}{\beta_f}\right) - \tau\rho_f\right) \frac{\rho_f}{\beta_f} \\
&= \left(p^* - \alpha_f + \beta_c q^* - \beta_c q_f^* - p^* + \alpha_f + \tau\rho_f - \tau\rho_f\right) \frac{\rho_f}{\beta_f} \\
&= \left(\beta_c q^* - \beta_c q_f^*\right) \frac{\rho_f}{\beta_f} \\
&= \left(q^* - q_f^*\right) \frac{\rho_f}{\beta_f}\beta_c \\
&= \left(\frac{p^* - \alpha_d}{\beta_d} - \frac{p^* - \alpha_f - \tau\rho_f}{\beta_f}\right) \frac{\rho_f}{\beta_f}\beta_c \\
&= \left(\frac{p^* - \alpha_d}{\beta_d} - \frac{p^* - \alpha_f}{\beta_f} + \frac{\tau\rho_f}{\beta_f}\right) \frac{\rho_f}{\beta_f}\beta_c \\
&= \frac{\rho_f^2}{\beta_f^2}\beta_c \tau + \frac{\rho_f}{\beta_f}\left(\frac{p^* - \alpha_d}{\beta_d} - \frac{p^* - \alpha_f}{\beta_f}\right)\beta_c \qquad (55)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \tau} R &= \frac{\partial}{\partial \tau}\left[\phi_g(\tau\rho_f q_f^* - \sigma q_c^*)\right] \\
&= \phi_g \left[\rho_f \frac{\partial}{\partial \tau}(\tau q_f^*) - \frac{\partial}{\partial \tau}(\sigma q_c^*)\right] \\
&= \phi_g \left[\rho_f \left(q_f^* \frac{\partial}{\partial \tau}(\tau) + \tau \frac{\partial}{\partial \tau}(q_f^*)\right) - \left(q_c^* \frac{\partial}{\partial \tau}(\sigma) + \sigma \frac{\partial}{\partial \tau}(q_c^*)\right)\right] \\
&= \phi_g \left[\rho_f q_f^* + \rho_f \tau \frac{\partial}{\partial \tau}(q_f^*) - q_c^* \frac{\partial}{\partial \tau}(\sigma) - \sigma \frac{\partial}{\partial \tau}(q_c^*)\right] \\
&= \phi_g \left[\rho_f q_f^* + \rho_f \tau \frac{-\rho_f}{\beta_f} - q_c^* \frac{\rho_f}{\beta_f}\beta_c - \sigma \frac{\rho_f}{\beta_f}\right] \\
&= \phi_g \left[\frac{\rho_f}{\beta_f}\beta_f q_f^* - \frac{\rho_f}{\beta_f}(\rho_f \tau + q_c^* \beta_c + \sigma)\right] \\
&= \frac{\rho_f}{\beta_f}\phi_g \left[\beta_f q_f^* - (\rho_f \tau + q_c^* \beta_c + \sigma)\right] \\
&= \frac{\rho_f}{\beta_f}\phi_g \left[\beta_f q_f^* - (\tau\rho_f + \beta_c(q^* - q_f^*) + \alpha_c + \beta_c(q^* - q_f^*) - \alpha_f - \tau\rho_f - \beta_f q_f^*)\right] \\
&= \frac{\rho_f}{\beta_f}\phi_g \left[\beta_f q_f^* - (2\beta_c(q^* - q_f^*) - \beta_f q_f^* + \alpha_c - \alpha_f)\right] \\
&= \frac{\rho_f}{\beta_f}\phi_g \left[\beta_f q_f^* - 2\beta_c q^* + 2\beta_c q_f^* + \beta_f q_f^* - \alpha_c + \alpha_f)\right] \\
&= \frac{\rho_f}{\beta_f}\phi_g \left[2(\beta_f + \beta_c) q_f^* - 2\beta_c q^* - \alpha_c + \alpha_f\right] \\
&= \frac{\rho_f}{\beta_f}2\phi_g \left[(\beta_f + \beta_c) q_f^* - \beta_c q^* + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right] \\
&= \frac{\rho_f}{\beta_f}2\phi_g \left[(\beta_f + \beta_c)\left(\frac{p^* - \alpha_f - \tau\rho_f}{\beta_f}\right) - \beta_c\left(\frac{p^* - \alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right] \\
&= \frac{\rho_f}{\beta_f}2\phi_g \left[(\beta_f + \beta_c)\left(\frac{p^* - \alpha_f}{\beta_f}\right) - (\beta_f + \beta_c)\left(\frac{\tau\rho_f}{\beta_f}\right) - \beta_c\left(\frac{p^* - \alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right] \\
&= \frac{\rho_f}{\beta_f}2\phi_g \left[(\beta_f + \beta_c)\left(\frac{p^* - \alpha_f}{\beta_f}\right) - \beta_c\left(\frac{p^* - \alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right] - \frac{\rho_f}{\beta_f}2\phi_g\,(\beta_f + \beta_c)\left(\frac{\tau\rho_f}{\beta_f}\right) \\
&= -\frac{\rho_f^2}{\beta_f^2}2\phi_g\,(\beta_f + \beta_c)\,\tau + \frac{\rho_f}{\beta_f}2\phi_g \left[(\beta_f + \beta_c)\left(\frac{p^* - \alpha_f}{\beta_f}\right) - \beta_c\left(\frac{p^* - \alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right]
\end{aligned}
\tag{56}
$$

$$\frac{\partial}{\partial \tau}E = \frac{\partial}{\partial \tau}\left[\eta \rho_f q_f^*\right] = \eta \rho_f \frac{\partial}{\partial \tau} q_f^* = \eta \rho_f \frac{-\rho_f}{\beta_f}$$

$$= -\frac{\rho_f}{\beta_f}\rho_f \eta \tag{57}$$

Next, using (53-57), we return to the first-order condition (52):

$$\frac{\partial W}{\partial \tau} = \frac{\partial}{\partial \tau}[CS + PS_f + PS_c + R - E] = 0$$

$$0 + \frac{\rho_f^2}{\beta_f^2}\beta_f \tau - \frac{\rho_f}{\beta_f}(p^* - \alpha_f) + \frac{\rho_f^2}{\beta_f^2}\beta_c \tau + \frac{\rho_f}{\beta_f}\left(\frac{p^* - \alpha_d}{\beta_d} - \frac{p^* - \alpha_f}{\beta_f}\right)\beta_c$$
$$- \frac{\rho_f^2}{\beta_f^2}2\phi_g(\beta_f + \beta_c)\tau + \frac{\rho_f}{\beta_f}2\phi_g\left[(\beta_f + \beta_c)\left(\frac{p^* - \alpha_f}{\beta_f}\right) - \beta_c\left(\frac{p^* - \alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right]$$
$$+ \frac{\rho_f}{\beta_f}\rho_f \eta = 0$$

$$\frac{\rho_f^2}{\beta_f^2}\beta_f \tau + \frac{\rho_f^2}{\beta_f^2}\beta_c \tau - \frac{\rho_f^2}{\beta_f^2}2\phi_g(\beta_f + \beta_c)\tau =$$
$$\frac{\rho_f}{\beta_f}(p^* - \alpha_f) - \frac{\rho_f}{\beta_f}\left(\frac{p^* - \alpha_d}{\beta_d} - \frac{p^* - \alpha_f}{\beta_f}\right)\beta_c$$
$$- \frac{\rho_f}{\beta_f}2\phi_g\left[(\beta_f + \beta_c)\left(\frac{p^* - \alpha_f}{\beta_f}\right) - \beta_c\left(\frac{p^* - \alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right] - \frac{\rho_f}{\beta_f}\rho_f \eta$$

$$\frac{\rho_f^2}{\beta_f^2}\left[\beta_f + \beta_c - 2\phi_g(\beta_f + \beta_c)\right]\tau = \frac{\rho_f}{\beta_f}\Bigg((p^* - \alpha_f) - \left(\frac{p^* - \alpha_d}{\beta_d} - \frac{p^* - \alpha_f}{\beta_f}\right)\beta_c$$
$$- 2\phi_g\left[(\beta_f + \beta_c)\left(\frac{p^* - \alpha_f}{\beta_f}\right) - \beta_c\left(\frac{p^* - \alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f - \alpha_c}{2}\right)\right] - \rho_f \eta\Bigg)$$

$$\frac{\rho_f}{\beta_f}(1-2\phi_g)(\beta_f+\beta_c)\tau = p^* - \alpha_f - \left(\frac{p^*-\alpha_d}{\beta_d} - \frac{p^*-\alpha_f}{\beta_f}\right)\beta_c$$

$$- 2\phi_g\left[(\beta_f+\beta_c)\left(\frac{p^*-\alpha_f}{\beta_f}\right) - \beta_c\left(\frac{p^*-\alpha_d}{\beta_d}\right) + \left(\frac{\alpha_f-\alpha_c}{2}\right)\right] - \rho_f\eta$$

$$\frac{\rho_f}{\beta_f}(1-2\phi_g)(\beta_f+\beta_c)\tau = \beta_f\frac{p^*-\alpha_f}{\beta_f} - \beta_c\frac{p^*-\alpha_d}{\beta_d} + \beta_c\frac{p^*-\alpha_f}{\beta_f}$$

$$- 2\phi_g\beta_f\frac{p^*-\alpha_f}{\beta_f} - 2\phi_g\beta_c\frac{p^*-\alpha_f}{\beta_f} + 2\phi_g\beta_c\frac{p^*-\alpha_d}{\beta_d} - 2\phi_g\frac{\alpha_f-\alpha_c}{2} - \rho_f\eta$$

$$\frac{\rho_f}{\beta_f}(1-2\phi_g)(\beta_f+\beta_c)\tau = (1-2\phi_g)\beta_f\frac{p^*-\alpha_f}{\beta_f} - (1-2\phi_g)\beta_c\frac{p^*-\alpha_d}{\beta_d} + (1-2\phi_g)\beta_c\frac{p^*-\alpha_f}{\beta_f}$$

$$- \phi_g(\alpha_f-\alpha_c) - \rho_f\eta$$

$$\frac{\rho_f}{\beta_f}(1-2\phi_g)(\beta_f+\beta_c)\tau = (1-2\phi_g)\left(\beta_f\frac{p^*-\alpha_f}{\beta_f} - \beta_c\frac{p^*-\alpha_d}{\beta_d} + \beta_c\frac{p^*-\alpha_f}{\beta_f}\right) - \phi_g(\alpha_f-\alpha_c) - \rho_f\eta$$

$$\tau^* = \frac{\beta_f}{\rho_f}\frac{(1-2\phi_g)\left(\beta_f\frac{p^*-\alpha_f}{\beta_f} - \beta_c\frac{p^*-\alpha_d}{\beta_d} + \beta_c\frac{p^*-\alpha_f}{\beta_f}\right) - \phi_g(\alpha_f-\alpha_c) - \rho_f\eta}{(1-2\phi_g)(\beta_f+\beta_c)}$$

$$= \frac{\beta_f^2\frac{p^*-\alpha_f}{\beta_f} - \beta_f\beta_c\frac{p^*-\alpha_d}{\beta_d} + \beta_f\beta_c\frac{p^*-\alpha_f}{\beta_f}}{\rho_f(\beta_f+\beta_c)} - \frac{\beta_f\phi_g(\alpha_f-\alpha_c)}{\rho_f(1-2\phi_g)(\beta_f+\beta_c)} - \frac{\beta_f\eta}{(1-2\phi_g)(\beta_f+\beta_c)}$$

$$= \frac{(\beta_f+\beta_c)(p^*-\alpha_f) - \beta_f\beta_c(\beta_d^{-1})(p^*-\alpha_d)}{\rho_f(\beta_f+\beta_c)} - \frac{\beta_f\phi_g(\alpha_f-\alpha_c)}{\rho_f(1-2\phi_g)(\beta_f+\beta_c)} - \frac{\beta_f\eta}{(1-2\phi_g)(\beta_f+\beta_c)}$$

$$\tau^* = \frac{(p^* - \alpha_f)}{\rho_f} - \frac{\beta_f\beta_c(p^* - \alpha_d)}{\rho_f\beta_d(\beta_f + \beta_c)} - \frac{\beta_f\phi_g(\alpha_f - \alpha_c)}{\rho_f(1 - 2\phi_g)(\beta_f + \beta_c)} - \frac{\beta_f\eta}{(1 - 2\phi_g)(\beta_f + \beta_c)}$$

$$= \frac{(p^* - \alpha_f)}{\rho_f} - \frac{\beta_f}{(\beta_f + \beta_c)}\left(\frac{\beta_c}{\beta_d}\frac{(p^* - \alpha_d)}{\rho_f} + \frac{\phi_g}{(1 - 2\phi_g)}\frac{(\alpha_f - \alpha_c)}{\rho_f} + \frac{1}{(1 - 2\phi_g)}\eta\right) \tag{58}$$

Expanding (50) and substituting (58) yields the optimal subsidy level:

$$\begin{aligned}
\sigma^* &= \alpha_c + \beta_c(q^* - q_f^*) - \alpha_f - \tau^*\rho_f - \beta_f q_f^* \\
&= \alpha_c + \beta_c q^* - \alpha_f - \tau^*\rho_f - (\beta_f + \beta_c)q_f^* \\
&= \alpha_c + \beta_c\frac{(p^* - \alpha_d)}{\beta_d} - \alpha_f - \tau^*\rho_f - (\beta_f + \beta_c)\frac{(p^* - \alpha_f - \tau^*\rho_f)}{\beta_f} \\
&= \alpha_c - \alpha_f + \frac{\beta_c}{\beta_d}(p^* - \alpha_d) - \frac{(\beta_f + \beta_c)}{\beta_f}(p^* - \alpha_f) + \frac{(\beta_f + \beta_c)}{\beta_f}\tau^*\rho_f - \tau^*\rho_f \\
&= \alpha_c - \alpha_f + \frac{\beta_c}{\beta_d}(p^* - \alpha_d) - \frac{\beta_c}{\beta_f}(p^* - \alpha_f) - (p^* - \alpha_f) + \frac{\beta_c}{\beta_f}\tau^*\rho_f + \tau^*\rho_f - \tau^*\rho_f \\
&= \alpha_c - \alpha_f + \frac{\beta_c}{\beta_d}(p^* - \alpha_d) - \left(\frac{\beta_c}{\beta_f} + 1\right)(p^* - \alpha_f) + \frac{\beta_c}{\beta_f}\rho_f\tau^* \\
&= \alpha_c - \alpha_f + \frac{\beta_c}{\beta_d}(p^* - \alpha_d) - \left(\frac{\beta_c}{\beta_f} + 1\right)(p^* - \alpha_f) \\
&\quad + \frac{\beta_c}{\beta_f}\rho_f\left[\frac{(p^* - \alpha_f)}{\rho_f} - \frac{\beta_f}{(\beta_f + \beta_c)}\left(\frac{\beta_c}{\beta_d}\frac{(p^* - \alpha_d)}{\rho_f} + \frac{\phi_g}{(1 - 2\phi_g)}\frac{(\alpha_f - \alpha_c)}{\rho_f} + \frac{1}{(1 - 2\phi_g)}\eta\right)\right] \\
&= \alpha_c - \alpha_f + \frac{\beta_c}{\beta_d}(p^* - \alpha_d) - \left(\frac{\beta_c}{\beta_f} + 1\right)(p^* - \alpha_f) \\
&\quad + \frac{\beta_c}{\beta_f}(p^* - \alpha_f) - \frac{\beta_c}{(\beta_f + \beta_c)}\left(\frac{\beta_c}{\beta_d}(p^* - \alpha_d) + \frac{\phi_g}{(1 - 2\phi_g)}(\alpha_f - \alpha_c) + \frac{1}{(1 - 2\phi_g)}\eta\rho_f\right)
\end{aligned}$$

$$\sigma^* = \left(1 + \frac{\beta_c}{(\beta_f + \beta_c)} \frac{\phi_g}{(1 - 2\phi_g)}\right)(\alpha_c - \alpha_f) + \left(1 - \frac{\beta_c}{(\beta_f + \beta_c)}\right)\frac{\beta_c}{\beta_d}(p^* - \alpha_d)$$
$$- (p^* - \alpha_f) - \frac{\beta_c}{(\beta_f + \beta_c)} \frac{1}{(1 - 2\phi_g)} \eta \rho_f \tag{59}$$

In the case where $\phi_g = 1.0$, this yields:

$$\tau^* = \frac{(p^* - \alpha_f)}{\rho_f} - \frac{\beta_f}{(\beta_f + \beta_c)}\left(\frac{\beta_c}{\beta_d}\frac{(p^* - \alpha_d)}{\rho_f} - \frac{(\alpha_f - \alpha_c)}{\rho_f} - \eta\right) \tag{60}$$

$$\sigma^* = \left(1 - \frac{\beta_c}{(\beta_f + \beta_c)}\right)(\alpha_c - \alpha_f) + \left(1 - \frac{\beta_c}{(\beta_f + \beta_c)}\right)\frac{\beta_c}{\beta_d}(p^* - \alpha_d) - (p^* - \alpha_f) + \frac{\beta_c}{(\beta_f + \beta_c)}\eta\rho_f \tag{61}$$