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Extractive revenues and government spending

Short- versus long-term considerations

Frederick van der Ploeg and Anthony J. Venables*

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Abstract: The prescription of optimally managing natural resource revenue windfalls by smoothing consumption across generations using an intergenerational sovereign wealth fund that only invests in foreign assets is not appropriate for resource-rich developing economies. It is better for these economies to use their windfalls to boost investment in the domestic economy, especially when they confront capital scarcity and have poor access to international capital markets. However, it is important for such economies to have a parking fund to temporarily 'park' funds until absorption constraints are alleviated, and a stabilization fund to smooth out volatile budgets given the high stochastic volatility of commodity prices, especially if the economy is inflexible and has few other ways of adjusting to shocks.

Keywords: resource curse, managing windfalls, fiscal rules
JEL classification: E60, F43, H21, H63, O11, Q33

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* Both authors: Oxford Centre for the Analysis of Resource Rich Economies, Department of Economics, University of Oxford, Oxford, UK; corresponding author: rick.vanderploeg@economics.ox.ac.uk.

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Information and requests: publications@wider.unu.edu

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1 Introduction

Resource-rich developing countries often display poor growth performance because they fail to save and invest a sufficiently high fraction of their resource revenues. This problem is exacerbated when such countries borrow when world prices of their resources are high, fail to put the extra funds to good use, and then get into serious problems with repayment of principal and debt service when world prices collapse. Some of these countries have put the revenues from their resource windfall into sovereign wealth funds (SWFs), but often their objectives are unclear and their management lacks economic discipline, with many of these funds being run down when there is a change of government or when times get hard. Furthermore, these funds are often raided for political purposes. Also, countries have often invested in international investments with low returns rather than in domestic investments with potentially higher returns, and have sometimes funded SWFs while also issuing expensive sovereign bonds.

This paper offers an analytical treatment of how to best manage revenues from non-renewable natural resources, such as oil, natural gas, and minerals, focusing on two key questions. The first is how much of the windfall should be used for boosting current household consumption or current government spending, and how much should be used for accumulating assets of some sort. This concerns the choice between consumption and saving. The second question is whether saving should be invested in foreign or domestic assets. For the time being we will refer to foreign assets as SWFs. Domestic asset accumulation means investment in the domestic economy, which might be capital spending by either the public or the private sector, and includes expenditures on education and health that build human capital.

We argue that, in capital-poor developing economies, savings rates out of natural resource revenues should be high, and the priority should be to invest them in the domestic economy. The case for high savings rates is motivated both by the temporary nature of revenues from exhaustible resources and by the lack of sufficient finance to undertake capital projects in developing economies. There is a clear opportunity cost to placing funds offshore in SWFs, as the case for an SWF turns on three main reasons.

- The first is to transfer part of the windfall to future generations. However, for developing economies with capital scarcity, this is best done by investing in the domestic economy rather than by accumulating assets abroad.
- Second, 'parking' windfall revenue in SWFs is needed if the efficient time profile of domestic investment does not line up with the time path of the natural resource windfall. Natural resource revenues should be held in SWFs until they can be used most productively in the domestic economy, recognizing that it takes time to build up a high enough quality domestic infrastructure, legal system, educational system, and health system.
- Third, it is necessary to put some of the windfall in a so-called stabilization fund to build up a precautionary buffer to deal with volatility of commodity prices. This is especially important when hedging is too expensive, as most future markets do not exist or are too thin, or hedging is deemed to be inadvisable from a political point of view.

Section 2 sets out our framework and captures the trade-offs between alternative uses of resource revenues. A key feature of our inter-temporal framework, when applied to a developing country, is that the economy starts out with a low capital stock and thus a low level of income per capita. A natural resource windfall enables the economy to speed up the process of economic development, building up the domestic capital stock by investing at least part of the windfall.

Returns to this need to be compared with alternative uses such as consumption or improving the foreign asset position (cutting sovereign debt or building SWFs).

Sections 3 and 4 analyse the long-run saving decision, making the case for using natural resource revenues for investment in the domestic economy, rather than in SWFs. Section 5 investigates the possibility that many developing economies are not—at least in the short run—ready to undertake high-return domestic investments even when they are in dire need of such investments. Further, such economies have experienced booms and busts as domestic spending has moved in line with natural resource revenues, often rising and falling too abruptly. The domestic investment path must be efficient, so it (and spending more broadly) must be decoupled from current revenue flows. To achieve this, natural resource revenues may need to be parked in offshore funds, but this should be temporary, until productive domestic investments can be found and funds can be fruitfully and efficiently invested without destabilizing the domestic economy. Section 6 discusses issues surrounding the volatility of natural resource revenues. This volatility impacts government revenues and the macro economy, and there is a case for building a stabilization fund to cushion these impacts. Section 7 discusses some macroeconomic aspects of managing natural resource revenues when wage and prices are rigid in the short run. Section 8 concludes.

2 Inter-temporal efficiency and the present value budget constraint

Our inter-temporal model has two key ingredients. One is the economy's inter-temporal budget constraint, and the other the efficiency conditions that shape choices within this budget constraint. Starting with the *inter-temporal budget constraint*, we define the wealth of the economy at date t, W_t, as the value of all assets held by the economy:

$$W_t \equiv F_t + PVY_t + PVN_t. \tag{1}$$

Wealth consists of three components. First, net foreign assets, F_t (negative if foreign debt); the rate of return on such assets is r which, for the moment, we take to be constant.[1] Second, the present value of income generated by domestic primary factors minus net investment, that is,

$$PVY_t = \int_t^{\infty} \left[Y(K_\tau) - \dot{K}_\tau \right] e^{-r(\tau - t)} d\tau. \tag{2}$$

The economy's capital stock is K and the production function is $Y(K)$ (depending also on labour, which we hold constant and suppress in the notation).[2] The integral is thus the present value of the economy's output net of investment $\dot{K}$, at all future dates, discounted at rate r to date t. The return on domestic capital is the marginal product, $Y'(K)$, decreasing in K. Third, the value of subsoil assets, that is, the expected present value of the natural resource revenues, net of extraction

[1] If the rate of return is time varying, discount factors in Equations (2) and (3) become $\exp\left[-\int_t^{\tau} r(z) dz \right]$.

[2] All domestic assets are contained either in K or in the form of the production function. Variables can be re-interpreted per capita, to cover the effects of a growing labour force.

costs, which is also commonly referred to as resource wealth. Expected net natural resource revenue at date τ is denoted N_τ so that net natural resource wealth at date t is

$$PVN_t = \int_t^\infty N_\tau e^{-r(\tau-t)}\, d\tau \,. \tag{3}$$

Discovery of natural resource reserves leads to a path of revenues, N_τ, which we take to be exogenous and determined by the size and nature of the resource deposit and by future prices. Thus, we assume that the extraction path of a given field is not given by Hotelling's rule but determined by geological considerations. For oil, the relevant concern is Darcy's law that describes the flow of a fluid through a porous medium. The discovery—or any change in expected future natural revenues, for example, due to a change in the resource price—shifts the expected present value of resource revenues, PVN_t, and hence shifts the natural resource wealth of the economy.

The inter-temporal budget for the economy constrains the time path of future consumption, $C_\tau, \tau \geq t$ by the economy's total wealth. If the economy has access to perfect capital markets, this too can be expressed as a present value constraint, so that

$$W_t = PVC_t \equiv \int_t^\infty C_\tau e^{-r(\tau-t)}\, d\tau \,. \tag{4}$$

Hence, at each date, the present value of consumption, PVC_t, should not exceed the total wealth of the economy consisting of foreign assets plus the discounted value of production income plus natural resource wealth. This implies that the 'no Ponzi games' condition is satisfied.

Intergenerational and inter-temporal efficiency conditions characterize the efficient allocation of wealth between assets and the efficient rate of asset accumulation, that is, the division of income between consumption and saving. The condition for efficient allocation of wealth is simply to hold assets yielding the highest return and, where multiple assets are held, choose quantities of each such that their marginal rates of return are equalized. This means dividing domestic and foreign assets, $K + F$, such that $Y'(K) = r$.

Intergenerational efficiency in the saving/consumption choice depends on the return to investment, r, and the rate at which society trades-off present consumption for future consumption, as measured by the consumption rate of interest. This follows from the often-used Keynes–Ramsey rule. The consumption rate of interest according to this rule consists of the pure rate of time preference, ρ, reflecting inherent impatience and the desire to consume now rather than in the future, and a term that captures changes in society's income through time. To be precise, the Keynes–Ramsey rule implies that the consumption rate of interest is $\rho + \eta g$, where g is the (trend) rate of growth of consumption and the parameter $\eta > 0$ is the inverse of the elasticity of inter-temporal substitution (capturing the rate at which the marginal value of consumption diminishes as individuals become richer).[3] For social decision taking, it is often argued that the pure rate of time preference should be extremely small as the impatience of the present generation gives it no right to consume at the expense of future generations. This argument is especially strong for projects with a long horizon, such as long-term investment projects (e.g. Gollier 2013). The second term, ηg, captures the point that future generations may be richer than the present

[3] The parameter η is also equal to the elasticity of the marginal utility of consumption, $\eta \equiv -CU''(C)/U'(C) > 0$, or the coefficient of relative risk aversion.

generation, so a marginal unit of consumption is less valuable to them than to people currently alive. For a developing country, this can be thought of as saying that, if income is growing ($g > 0$), poverty reduction needs are greater now than they will be in future, implying that greater weight is placed on the present; that is, the future is discounted more heavily especially if intergenerational inequality aversion is large (like an increase in the rate of pure time preference, ρ). Typical values of the parameters for a developing economy are $\rho = 2$ per cent and $\eta = 2$, so if consumption is growing at 3 per cent per annum, then the consumption rate of interest equals $\rho + \eta g = 8$ per cent per annum. Developed economies, by contrast, are no longer catching up and typically have lower trend rates of growth and thus lower rates of interest.

The efficiency condition for inter-temporal savings/consumption decisions, recognizing that the consumption rate of interest must equal the rate of return on investment (see Appendix A), is thus

$$r = \rho + \eta g \text{ or } g = (r - \rho)/\eta. \qquad (5)$$

Given r and ρ, the rate of increase of consumption, g, should be set to satisfy the equation. Thus, given the budget constraint, if the rate of return on assets, r, is high consumption should be relatively low today and high in the future; that is, consumption should grow fast, $g = (r - \rho)/\eta$. The reason is that a lot is saved today in order to make the most of the high rate of return. If the budget constraint is relaxed (e.g. if a windfall of natural resource revenue becomes available) and values of r, ρ, and η remain unchanged, overall consumption increases but the subsequent rate of change of consumption is unaffected.

A final remark is crucial at this point. Natural resource windfalls can last for many decades and sometimes even centuries. It is therefore not appropriate for society to base intergenerational welfare comparisons by using data on inter-temporal savings and consumption choices made by individuals. This is why the Keynes–Ramsey rule, as shown in Equation (5), captures the societal ethical trade-offs between generations today and generations in the future. Following this rule, one should probably use a very small or even zero rate of pure rate of time preference, ρ, as it seems unethical to discount the welfare of future, yet unborn generations. A similar stance has been taken by the Stern (2007) review in the context of climate policy. Furthermore, the coefficient of relative intergenerational inequality aversion should come from deep ethical considerations on how much society is prepared to forsake consumption today to boost consumption of future generations, and should not be derived from estimates of the elasticity of inter-temporal substitution or the inverse of the coefficient of relative risk aversion obtained in the empirical literature on consumption and savings decisions.

3 Benchmark management of windfalls: the permanent income hypothesis

This analysis leads quite naturally to the celebrated permanent income hypothesis (PIH), which has often been used as the basis for advice to resource rich economies. It requires the bold assumption that the economy can borrow or lend internationally at a constant (and exogenous) world rate of interest, which we will denote r^*, and thus that the domestic capital stock adjusts such that $Y'(K) = r = r^*$. This is tantamount to assuming that the economy can borrow from or lend to world capital markets at a relatively low rate and has no shortage of domestic capital because it has invested up to the point where there are no investment opportunities that yield more than the return on foreign assets. Often, a second assumption is made to ensure that the time path of consumption is fully smoothed; namely, that the rate of pure time preference is equal to the rate of interest, so that $\rho = r^*$. The condition for inter-temporal efficiency, $r^* = \rho + \eta g$, given this assumption, is indeed satisfied if aggregate consumption is constant through time, that is, $g = 0$.

Sometimes an alternative, perhaps more realistic, second assumption is made to ensure that the natural resource dividend per capita rather than aggregate consumption is held constant over time (e.g. van den Bremer and van der Ploeg 2013). This requires that the rate of pure time preference is set to a lower value, that is, $\rho = r^* - \eta n$, where n is the rate of population growth, because this ensures from Equation (5) that $g = (r^* - \rho)/\eta = n$ and thus that *growth* in consumption per capita is smoothed. For ease of exposition, we will abstract from population growth in the remainder and suppose that $\rho = r^*$. Many governments in resource-rich developing economies are focused on winning next elections and reaping as much benefit as possible from a natural resource windfall. Such governments may thus employ a much higher pure rate of time preference, especially when their grip on office is not so strong.

Without natural resource revenues, an economy satisfying these assumptions is therefore stationary, with consumption and wealth constant through time. It follows that

$$C = r^* W = Y(K) + r^* F. \tag{6}$$

The first of these equations comes from integrating the budget constraint, Equation (4), given also that the time path of consumption is fully smoothed. The second is the flow budget constraint stating that consumption equals production income given a constant capital stock.[4]

What is the effect of revenues from a current or future natural resource windfall? A discovery of natural resources, which takes place at date $t = 0$, has present value PVN_0. However, it is often the case that revenues may not be earned for a further 5 or 10 years until all exploitation investments have taken place and the pumping or mining starts (e.g. Arezki et al. 2016). With the assumption that r^* is exogenous, the domestic capital stock, K, and hence the level of domestic non-resource output are both fixed by the equation $Y'(K) = r^*$. It follows that a natural resource windfall has no effect on the optimal choice of domestic capital stock or on investment in the domestic economy. Hence, none of the natural resource windfall should be invested in the domestic economy because with access to perfect capital markets the level of domestic investment should already be at its optimal level.

Consumption jumps up on impact of the news of the discovery of new natural resource wealth and well ahead of the revenues pouring in as the news of future resource extraction is fully discounted in efficient markets. Furthermore, with the assumption that $r^* = \rho$ consumption is constant at that new level along the entire optimal consumption path thereafter. Because at the date of discovery wealth jumps by the value of the resource discovery, $\Delta W = PVN_0$, where Δ denotes the discrete change at the date of discovery, the discrete jump in consumption at the time of the discovery is therefore

$$\Delta C = r^* \Delta W = r^* PVN_0. \tag{7}$$

Hence, the level of consumption increases by an amount equal to the annuity value of the natural resource discovery. This increase in consumption occurs at the date of discovery and is maintained in perpetuity thereafter.

[4] Depreciation of capital, just like growth of the labour force, could easily be added to the model.

These are two very strong results indeed. They stem from two very bold assumptions: (i) the domestic capital stock is such that the marginal product of capital equals the world interest; and (ii) $\rho = r^*$, so that the time path of consumption is flat. Neither of these assumptions is appropriate for developing economies, and in the next section we look at the consequence of relaxing both of them. Despite this, the PIH gives some important messages for the fraction of natural resource revenues that should be saved and the path of asset accumulation. We turn to these now.

The time profiles of the flows of income and consumption and of the stocks of assets are shown in Figure 1, with time on the horizontal axis. For simplicity, we assume that the economy has no foreign assets or debt at the date of discovery, $F_0 = 0$. As noted above, wealth jumps up by PVN_0 at the date of discovery, and is constant thereafter. This means that as the resource follows its depletion path (assumed to be exogenous) and PVN declines, so foreign assets, F, increase to exactly offset this, $F = \Delta W - PVN$, as illustrated. This is in line with the Hartwick rule which states that any depletion of subsoil wealth must be mirrored by an equal increase in above-ground wealth.

It is thus clear that the increment in consumption is constant, $\Delta C = r^*PVN_0$, that resource income N declines (exogenously; dashed line in Figure 1), and that interest earned on other assets, r^*F, increases. The sum of these is the post-discovery increment to income, $N + rF$, which, by the flow budget constraint, must equal saving, S, plus the increment to consumption, $N + r^*F = \Delta C + S$, as illustrated by the solid line in Figure 1. These savings all go into foreign assets, $S = \dot{F}$, driving the increase in F illustrated in Figure 1b.

It is convenient to write these relationships one further way. As $N + r^*F = \Delta C + S$ and $\Delta C = r^*PVN_0 = r^*PVN_0 = r^*\Delta W = r^*F + PVN$, savings must satisfy

$$S = N - r^*PVN \text{ or } S/N = 1 - r^*PVN/N. \tag{8}$$

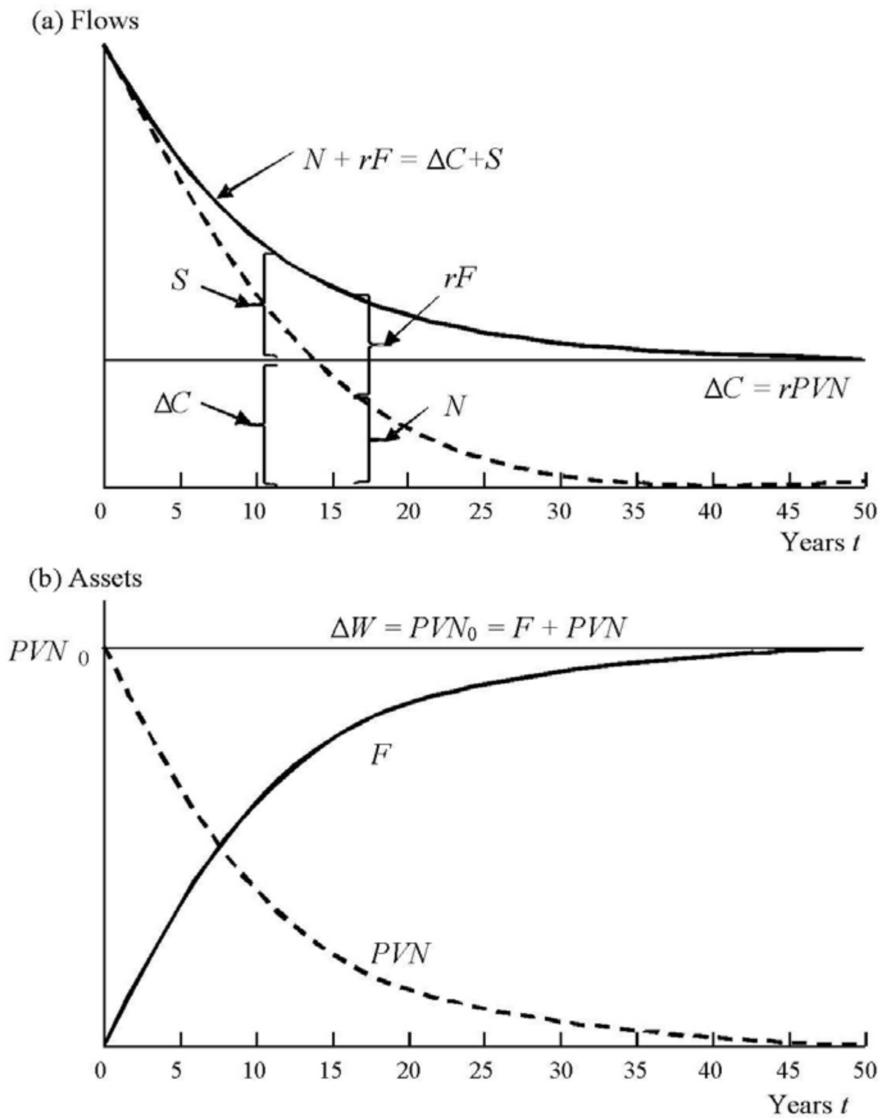
This last equation gives the savings rate out of the flow of resource revenue, S/N, a key policy variable. This depends on PVN/N, the ratio of the present value of resource revenue remaining to the current flow of resource revenue. For an extremely long-lived stock of natural resources, the present value of natural resource revenue remaining, PVN, is largely relative to current resource revenue, so the savings rate is low. The shorter lived a resource discovery (the faster the decline in revenues) the smaller is this ratio and the higher the rate of saving. Of course, the PIH implies that saving out of total resource and non-resource wealth is constant. Some further insights follow from some examples.

The simplest one is where the value of resource revenues declines exponentially at the constant rate κ (as illustrated in Figure 1). In this case, the share of resource revenue saved is

$$S/N = \kappa/(\mathrm{r}^* + \kappa), \tag{9}$$

because the revenue flow is $N_t = N_0 e^{-\kappa t}$ and the present value of natural resource remaining is $PVN_t = N_0 e^{-\kappa t}/(r^* + \kappa)$. This demonstrates clearly that the savings rate should be higher the faster the rate of decline of resource revenues, κ. Thus, for example, if revenues are expected to decline at twice the rate of interest, two-thirds of resource revenue should be saved. The savings rate is constant during the life of the resource, although this is not generally the case, as illustrated in the next example.

Figure 1: Managing resource windfalls according to the permanent income hypothesis (PIH): (a) Flows; (b) assets



Source: Authors' illustration based on own calculations.

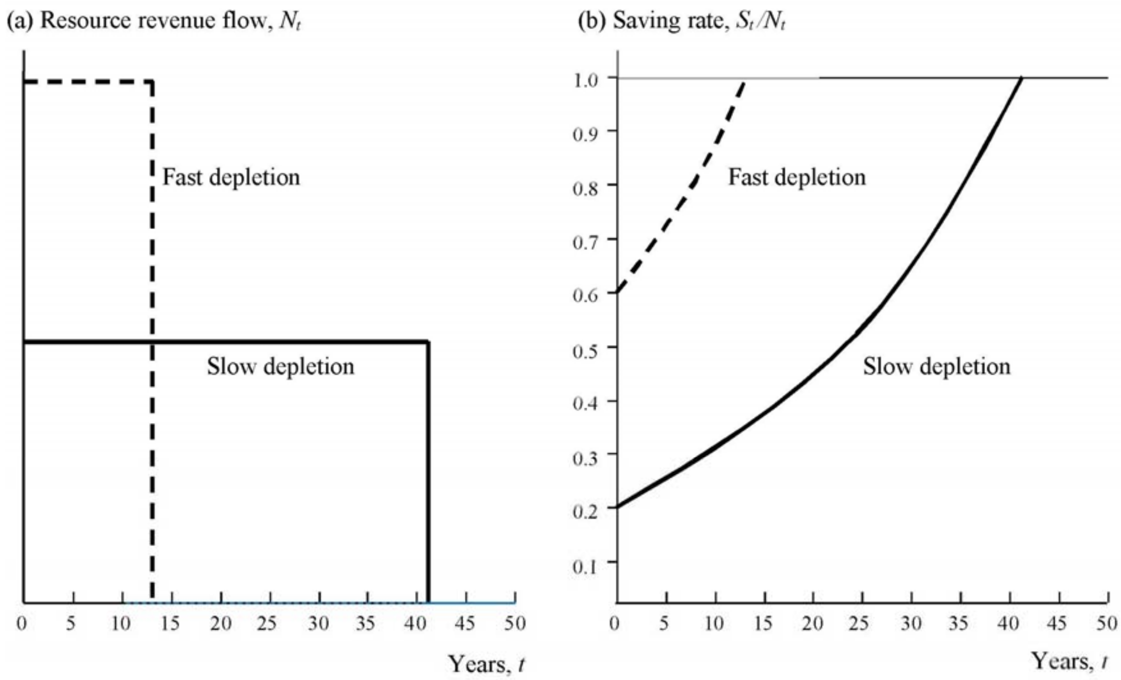
Another example is where revenues from depletion are constant until the point of exhaustion is reached at date T, so $N = \overline{N}$ for $t < T$ and then drops to zero. As reserves are depleted PVN_t falls, so the savings rate rises. The present value is $PVN = \left(1 - e^{r*(T-t)}\right)\overline{N} / r*$ so the savings rate increases during the lifetime of the resource and goes to unity at the day of exhaustion:

$$S / N = 1 - \left(1 - e^{r*(T-t)}\right) e^{r*(T-t)}. \tag{10}$$

The rationale underlying this time profile for the savings rate is that, as time progresses, the stock of the resource left becomes smaller relative to the flow, and thus the windfall becomes more temporary, necessitating more saving.

Figure 2 illustrates this for two natural resource discoveries of equal size. For both, the flow revenue from extraction remains constant over the life of the resource, but the duration and hence the level of the flow differs. Figure 2a shows the two revenue flows, with revenue, *N*, on the vertical axis and time on the horizontal axis. Both profiles, shown by the solid and dashed lines, have the same initial present value, $PVN_0 = 1$ (with $r^* = 0.04$) but that depicted by the solid line depletes the resource slowly so that it lasts for 40 years, whereas that depicted by the dashed line depletes it more rapidly so that it only lasts for 13 years. The optimal savings rates for these depletion profiles are illustrated in Figure 2b, and follow Equation (10).

Figure 2: (a) Resource revenue flow; and (b) optimal savings rates for depletion



Source: Authors' illustration based on own calculations.

In both cases, the savings rate increases over the course of depletion, as the present value, PVN_t, gradually reduces and the flow *N* and discount rate r^* are constant. Because PVN_t goes to zero as exhaustion approaches, the savings rate eventually rises to 100 per cent. The impact of the rate of depletion is dramatic. In the case of slow depletion, the optimal savings rate rises from 20 per cent in the first year to 100 per cent in the final year. Hence, a policy rule that set a *constant* savings rate would be seriously sub-optimal regardless of the level at which it was set. It is important to have a constant savings rate out of the sum of natural resource and all other forms of wealth, not out of the windfall. The savings path for rapid depletion is strikingly higher than for slow depletion, starting at 60 per cent rather than 20 per cent.

In summary, the optimal savings rate out of the current windfall revenue depends upon the extraction path. Our discussion suggests that, first, for a given net present value of a resource discovery, the shorter its duration the higher should be the savings rate, and, second, and more surprisingly, for a range of extraction paths the optimal savings rate should rise over time.

4 Case for investing resource windfalls in the domestic economy

With access to perfect international capital markets and no capital scarcity, the PIH analysis of Section 3 suggests that all savings and resulting asset accumulation after a natural resource windfall has to be placed in foreign assets, an SWF. This result is driven by the assumption that the economy has expanded its capital stock to the point at which the marginal product of capital is equal to the constant and exogenous world interest rate. This is relevant for a high income, fully developed economy with abundant capital and thus investment levels are already optimal. However, most developing countries have only asymmetric access to world capital markets: they can save more readily than they can borrow. Typically, as borrowers they are either entirely cut off or they can only borrow at a substantial premium over the world risk-free interest rate to compensate for perceived high risks of default. To capture this reality we retain the model of Section 2, but now assume that the interest rate faced by the domestic economy is a function of the stock of foreign assets (or debt) that it holds. We denote this $R(F)$, which implies that the economy has to pay a premium on its foreign debt if it is indebted.[5] If the country is not indebted, then it has perfect access to international capital markets and thus the domestic interest rate relevant for investment decisions, r, still equals the world interest rate, r^*. If the country is indebted, then it only has imperfect access to international capital markets and thus the domestic interest rate exceeds the world rate, and is increasing in the level of debt. We capture this with the following specification:

$$\text{If } F \geq 0,\ R(F) = r^* \text{ and if } F < 0,\ R(F) > r^*,\ R'(F) < 0. \tag{11}$$

While $R(F)$ is the rate of interest, the marginal cost of borrowing will exceed this if taking on more debt raises the rates the country has to pay on its existing debt.[6] Thus, the marginal social cost of borrowing is $r(F) = R(F) + FR'(F)$, which we assume is also decreasing in F (i.e. increasing in debt). Capital scarcity of this type fundamentally changes the answers to both our questions in Section 3—the division of resource revenues between consumption and savings, and the assets that savings should be used to acquire. The policy prescriptions will thus be very different from those emanating from the PIH.

4.1 Using the resource windfall for domestic investment and foreign assets

Total assets in the economy, denoted by A, consist of domestic capital K and foreign assets (or debt) F, that is, $A \equiv K + F$. If the economy is asset scarce in the sense that the marginal return on domestic capital satisfies, $Y'(A) > r^*$, all assets should be held as domestic investment and none in an SWF. However, if this economy can borrow, that is, choose $F < 0$, it will do this until the return on domestic capital is driven down to the marginal cost of borrowing,

$$Y'(\mathrm{K}) = Y'(A - F) = r(F). \tag{12}$$

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[5] Akitoby and Stratmann (2008) and van der Ploeg and Venables (2011) document empirical evidence for a positive relationship between foreign debt and the domestic interest rate. They find that the effect of natural resource exports on interest rate spreads is positive, but not significant, suggesting that resources worsen creditworthiness. They also conduct a similar analysis where the cost of borrowing depends on resource revenues. Many of the conclusions are the same, with the main difference being that the cost of borrowing drops, and consumption jumps, immediately at the date of discovery of the new natural resource reserves.

[6] It is possible that a natural resource discovery may reduce the economy's cost of borrowing, as resource revenues are perceived to be a form of collateral for loans. This shifts $R(F)$ downwards.

If some of the windfall revenue is saved, then the economy's assets increase. To see how much of an increase, dA, is allocated to foreign debt reduction, dF, and how much to domestic investment, dK, we totally differentiate Equation (12) to obtain

$$dF = \left\{ \frac{Y''(A-F)}{Y''(A-F)+r'(F)} \right\} dA, \quad dK = \left(\frac{r'(F)}{Y''(A-F)+r'(F)} \right) dA. \tag{13}$$

If the marginal cost of borrowing increases with indebtedness, then $r'(F) < 0$ and the denominators in Equation (13) are always negative. Hence, the bigger the capital scarcity as indicated by the magnitude of $|r'(F)|$, the bigger the fraction of the increment in total assets that is allocated to domestic capital, $dK/dA \in (0,1]$. In the limiting case where the country is completely shut out of capital markets, $r'(F)$ will be equal to minus infinity and the whole of any increase in assets goes to domestic capital formation. At the other extreme, the PIH maintains the assumption that the marginal return or cost of lending and borrowing is given on world capital markets, so $r'(F) = 0$ and hence the domestic capital stock is unaffected by the change in assets, $dK = 0$, and all of the increment in total wealth is allocated to net foreign assets, as we saw in Section 3.

Hence, countries that are capital scarce should use resource windfalls to increase the domestic capital stock (including human capital), and perhaps also pay down foreign debt as investment is sub-optimally low to start off with. The case for increasing domestic investment in resource-rich developing countries is further reinforced if a natural resource boom brings its own direct financing needs. For example, infrastructure projects may need to be brought forward and new downstream projects may also be needed, placing a demand on public funds. There may also be general equilibrium effects. The boom is likely to change the structure of the economy, with some sectors booming and others contracting. The 'Dutch disease' analysis suggests that this will involve a reallocation of economic activity into non-tradable goods and away from non-resource tradables (e.g. Corden and Neary, 1982; van der Ploeg and Venables 2013). If the non-resource traded sector is relatively capital-intensive, this will mean that the capital–labour ratio in the economy as a whole will decrease.

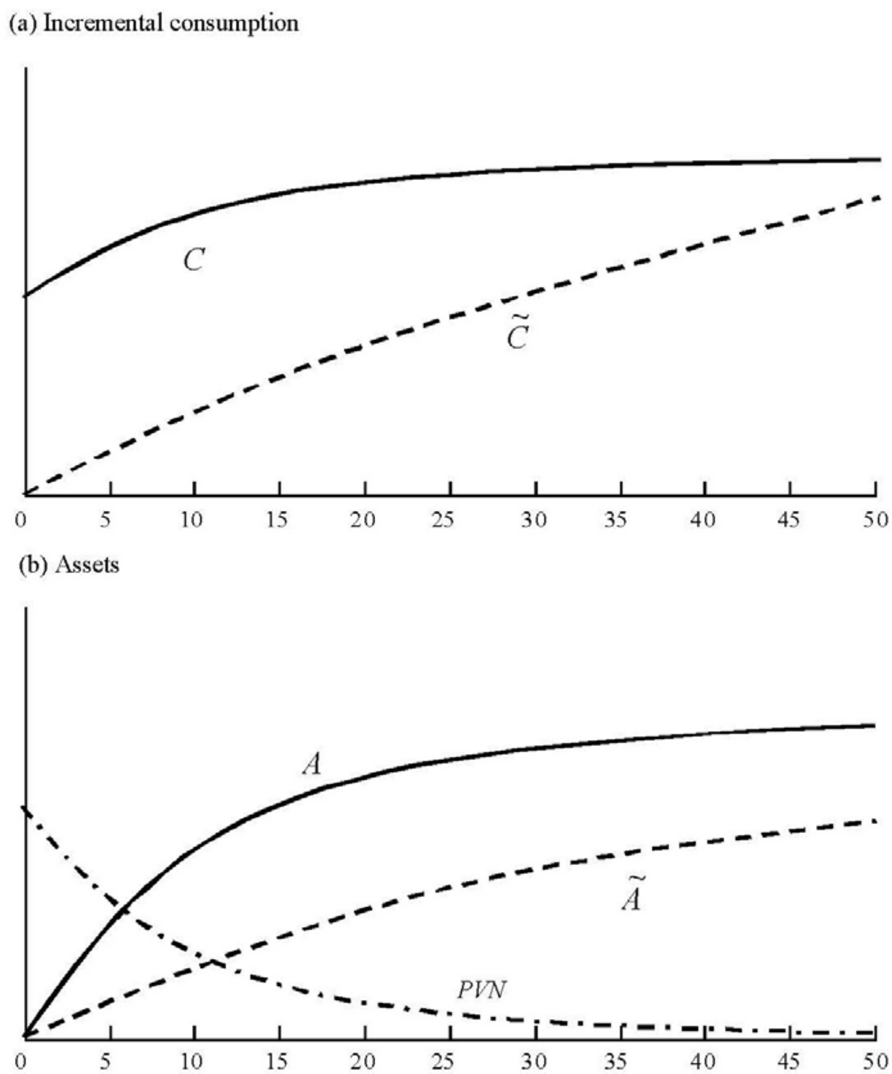
4.2 Capital scarcity and the optimal choice between saving and consumption

We have thus seen that it is not appropriate for a developing economy to set up an SWF and lend to the rest of the world, but it may be appropriate for such an economy to use its newly found natural resource wealth to pay down existing debt as well as to invest in the domestic economy. Now we consider total savings out of windfall revenues and their implications for the growth of domestic income and consumption. The inter-temporal efficiency condition and the ethics therein set out in Section 2 are fairly standard, but it should be realized that it is a very simple model of development. The inter-temporal efficiency condition (4) now becomes

$$g = \frac{1}{\eta}\left[r(F) - \rho \right] > \frac{1}{\eta}\left(r^* - \rho \right) \quad \text{with } Y'(K) = r(F). \tag{14}$$

Because the marginal cost of borrowing satisfies $r(F) > r^*$, the cost of capital and thus the consumer rate of interest will be high so that the time path of consumption is rising through time. The rate of return is now higher than the world interest rate. However, as capital is accumulated and debt is reduced, the rate of return falls and the economy converges to one without capital scarcity. Thus, starting from a low base, capital is accumulated, income rises, and the rate of return falls, ultimately reaching a level similar to that in high-income countries and supporting similar levels of income and consumption. This is illustrated in Figure 3a showing consumption and Figure 3b showing assets; the paths $\tilde{C}$ and $\tilde{A}$ are baseline paths in the absence of natural resource revenues.

Figure 3: Optimal revenue management in a developing economy: (a) Incremental consumption; (b) assets



Source: Authors' illustration based on own calculations.

A natural resource bonanza shifts the inter-temporal budget constraint of the economy and allows both current consumption and saving to increase. This speeds up the rate of asset accumulation. More assets at any date mean a lower marginal cost of borrowing, $r(F)$, and hence a slower rate of increase in consumption. The situation is as illustrated by the solid lines A and C in Figure 3. Consumption jumps up at the date of discovery but then increases less rapidly. Assets are accumulated more rapidly, more than offsetting the decline in the value of natural resources remaining in the crust of the earth. We also observe the following. First, the consumption increment (the difference between the new path of C and the baseline $\tilde{C}$) is largest immediately after the windfall, and then declines. This is in sharp contrast to the results from the PIH where the increment was constant through time. The intuition is that the current generation is poorer than future generations, so the consumption increment is skewed towards this poorer generation. At the same time, room is made for investments to stimulate growth and development.

If the windfall is very large there may be a 'permanent' element that continues in perpetuity, but for small windfalls the consumption paths illustrated in Figure 3 converge (see van der Ploeg and Venables 2011). At the same time as it brings forward consumption, the natural resource windfall also means that accumulated assets are greater at all dates. This brings down the rate of return in the economy as time proceeds. Effectively, the optimal use of the natural resource windfall has the effect of bringing forwards development, as can be seen by comparing the consumption and asset accumulation paths with and without the windfall in Figure 3. The share of resource revenues that should be saved depends, as before, on the precise path of natural resource revenues. However, the argument of this section tends to reinforce the message that savings should increase through time; the additional effect is the front-loading of the consumption increment.

In summary, once the analytical framework is modified to include the key developing country features of capital scarcity and high borrowing costs, the policy messages are significantly different from those of the PIH. The consumption increment is front-loaded whereas natural resource revenues should be used principally for domestic investment, which brings forward the development path of the economy.

5 Absorption constraints and the case for a parking fund

The prioritization of domestic investment rests on there being domestic projects that yield high returns (social as well as private returns), and on projects being delivered in an efficient sequence and manner. Yet, it is possible that—in the short run—because of absorption constraints and other bottlenecks there may be few high-return projects ready to be implemented. This is a commonly observed problem in many newly resource-rich developing economies. More generally, there will inevitably be a mismatch between the time profile of (efficient) investment and the time profile of natural resource revenues. Rather than spending natural revenues on inefficient projects, there is then a case for having a fund that plays the role of 'parking' natural resource revenues until they can be efficiently used in the domestic economy. The case for parking funds until the economy is able to absorb extra spending efficiently involves both micro- and macroeconomic arguments.

A country seeking to scale up investment is likely to encounter numerous bottlenecks. In the public sector, there is unlikely to be a pipeline of good investment projects. There may be a lack of capacity to design and develop projects; project selection and cost benefit processes may be weak; and so too may be the ability to procure, implement, and monitor projects. These problems have to be addressed before effective investments can be undertaken, which implies a strong case for 'investing in investing' (Collier 2010). Sequencing of projects also matters because the return to one project depends on other complementary projects that can be undertaken. For example, the return to private investment may be low, particularly if public investments in infrastructure and other aspects of productive capacity are lagging.[7] More broadly, additional domestic spending will increase demand for both traded and non-traded goods. As demand increases the economy will move up its supply curve of non-traded goods, creating both a quantity and a price response. Because the supply curve is less elastic in the short run than in the long run, upwards price pressure

[7] In terms of Equation (14), $Y''(K)$ is particularly large (negative) in the short run. As the marginal product of capital falls more with domestic investment more should be devoted to foreign assets.

is acute if spending increases too rapidly. This is likely to be particularly true for 'home-grown' capital. Although equipment can usually be imported, structures and human capital require domestic capacity (e.g. in the construction and training/teaching professions), all of which take time to develop (van der Ploeg and Venables 2013).

There need not be market failures associated with these bottlenecks and price responses. It may be merely a feature of the supply side of the economy and the political system that projects take time and have to be done in sequence and that it takes time before the necessary political support is obtained. Nevertheless, this timing dictates the rate at which investment can be increased. If extra investment demand is more likely to be met by a price increase than by a quantity response, it is efficient to slow down the rate of investment until supply can catch up. This suggests that investment should be ramped up quite slowly. It follows that natural resource revenues should be parked in foreign assets until they can be spent in a cost-effective manner on the efficient investment path.

6 Volatility of commodity prices and the case for a stabilization fund

The volatility and unpredictability of commodity prices creates a third argument for using an SWF, namely, to function as a stabilization (or liquidity) fund. The optimal size of such a stabilization fund depends on how the government and the economy respond, ex post, to shocks that occur. Are shocks sufficiently costly that self-insurance through a stabilization fund is necessary, or can they be managed by other means?

An extreme way of doing self-insurance is known as the 'bird-in-hand' rule, according to which all revenues are placed in an SWF, and spending is a fixed share of the SWF (typically, an estimate of the long-run rate of return on assets in the SWF). Essentially, resource revenues are discounted at an infinite rate, so that the spending rule is just a fixed share of above-ground assets and ignores below-ground wealth entirely as done in Norway (e.g. Bjerkholt and Nicolescu 2004; Barnett and Ossowski 2003). Consumption of natural resource windfalls is thus heavily back weighted, and therefore unable to deliver domestic investment which we have argued is efficient for many resource-rich developing economies. The bird-in-the-hand rule is therefore not appropriate for poor countries because it exaggerates the risk that revenues might terminate earlier than expected, while completely ignoring the much larger risks to low-income societies from deferring growth.

A more general strategy to smooth the impact of commodity price volatility is to save when the price of the natural resource is high, and dissave or borrow when the price is low. The foreign asset holdings will fluctuate with the commodity price but the size of the stabilization fund will fluctuate around zero as the expected value of the fund should be zero. However, this view needs to be modified for two reasons. First, owing to asymmetry of borrowing and lending costs, borrowing during a sharp downturn in commodity prices is likely to be expensive, if not impossible. Borrowing rates will be high and, following the fall in prices, many a developing resource exporter will likely be shut out of most borrowing options. Second, commodity and in particular oil price shocks are typically long-lasting and it is difficult to reject the hypothesis that the oil price follows a random walk. If shocks are permanent or very long-lived, it is much less necessary to save for intergenerational reasons but it will be important to build a more sizable stabilization fund (van den Bremer and van der Ploeg 2013). Precautionary savings—that only take into account the utility costs of failing to smooth consumption optimally—stem not from risk aversion but from prudence.

The necessary size of a stabilization fund depends on the costs incurred if, in the event of a commodity price fall, the country is unable or unwilling to borrow. These costs come through both micro- and macroeconomic channels. Precautionary saving only takes into account the volatility costs of failing to smooth consumption optimally, but problems overall are exacerbated by the fact that the fall in revenue principally impacts on government budgets and spending. The consequent and potentially high costs of negative shocks make the case for creating a stabilization fund that gives government the space to undertake countercyclical fiscal policy (e.g. moderate the pace of spending cuts) necessitated by a revenue downturn. Such a fund is even more important in countries with asymmetric borrowing and lending costs. This is because borrowing during a crash in commodity prices is tough.

The management of a stabilization fund, once it is established, opens up a further set of issues. For example, expected rates of return should be adjusted for risk, including political risks such as the possibility that a liquid asset (such as foreign exchange holdings) might be more easily looted by a future government than illiquid assets (such as domestic infrastructure). Further, a diversified portfolio needs to take into account the correlations between returns on the various assets chosen to hedge against commodity price volatility and the risk inherent in the below-the-ground natural resource wealth (van den Bremer et al. 2016).[8] Rates of return should ideally also be calculated to include the full social costs and benefits of investments and to take into account effects of any investment on the domestic macroeconomy, in particular if there is limited absorptive capacity.

Finally, we note that an alternative way of insurance is through hedging strategies as used, for example, by Mexico. However, such strategies typically only offer insurance for a relatively short period. We do not discuss them further as future markets are too thin and costly to make this a reliable strategy in most situations. One issue is that the funds may be so large that they could be seen to manipulate commodity prices, and for many countries such hedging strategies are too risky from a political point of view. For most countries a stabilization fund is a more reliable and pragmatic strategy.

7 Short-run macroeconomic aspects of managing natural resource windfalls

To examine some of the macroeconomic issues, consider again an economy that has perfect access to world capital markets at a nominal rate of interest that is fixed and independent of the natural resource windfall. It is important to point out that using an SWF to smooth consumption across generations typically will also smooth the time path of the real exchange rate or the relative price of non-tradable goods. Here we are also concerned, however, with the nominal wage and price rigidity and the need for appropriate monetary and exchange rate policies.

If, following a resource discovery or price boom, a natural resource exporting country prevents its currency from appreciating, its central bank will have to sell domestic currency dominated assets

[8] Demand for risky assets should be leveraged up and then deleveraged as reserves are depleted. There should be more prudential saving if the oil price cannot be fully diversified in international asset markets. A more pragmatic approach that avoids heavy borrowing to hedge the natural resource wealth in the ground and targets a general equity index such as the FTSE index has to realize that portfolio allocation above the ground accentuates the risk of the natural resource wealth in the ground if returns on exhaustible resources such as oil are positively correlated with stock market returns, as has been the case in recent years. Because such a pragmatic approach avoids hedging with specific financial assets whose returns are negatively correlated with below-ground resource wealth, it has to hold *less* risky assets than suggested by the normal capital asset pricing model and gradually *increase* holdings of risky assets to their normal levels as natural resource wealth is depleted.

and buy foreign currency dominated assets. This way the central bank will accumulate foreign exchange reserves. The effects of this mechanism are similar to those of a de-facto SWF, although they are not equivalent (Wills and van der Ploeg 2014). SWFs accumulate, for example, oil revenues in a foreign currency before they pass through the domestic economy. By contrast, foreign exchange reserves accumulate after the oil revenues have been spent domestically. The counterpart of these reserves is the net asset position of the private sector with the central bank. Therefore, whereas SWFs represent savings by the government, foreign exchange reserves are saved by the private sector. The share of gross domestic product going to private, rather than public, consumption will correspondingly be lower if resource revenues are accumulated in foreign reserves.

The macroeconomic response to a resource discovery also depends on the time lag between discovery and the beginning of production. Despite large increases in investment, empirical findings analysing the effects of giant oil discoveries show that the short-run effect of a natural resource discovery on income is slightly negative until the natural resource production actually commences, whereas the private sector responds immediately to an oil discovery causing the current account to depreciate (Arezki et al. 2016). With forward-looking financial markets, the nominal exchange will appreciate, ahead of the production of natural revenues. As can be shown in a nominal exchange rate overshooting model, this causes unemployment during this anticipation period (Eastwood and Venables 1982), unless offset by project-related spending. In a new Keynesian macroeconomic model the forward-looking component of this inflation will suppress output below its natural level, leading to a recession. If the central bank follows a simple Taylor rule that tightens aggressively against inflation it will exacerbate this recession (Wills 2015b).

Declining resource prices have severe implications, as witnessed by the post-2011 crash in commodity prices. Natural resource exporters will experience a negative demand shock and investors in the mining and exploration sectors will scale back investment and production activity. Households anticipate a drop in future income and will therefore cut spending whereas firms in non-tradable sectors may hold off investment as a result. At the same time, there will be pressure on the government to cut spending. With the drop in demand throughout the economy and nominal wages and prices fixed in the short run, employment and output will fall.

Loosening monetary policy by lowering nominal interest rates should be able to perfectly offset a negative demand shock if such policy is unconstrained, although this will not be possible in many natural resource exporting countries. A negative demand shock will require the price of non-traded goods to fall relative to non-resource traded goods. This will only happen slowly with sticky prices, so for a period output will be below its natural level. A looser monetary policy can, in principle, offset this by inducing households to bring spending forward and the nominal exchange rate to depreciate. However, many resource exporting developing countries with thin financial sectors face constraints on monetary policy so the fall in demand cannot perfectly be offset. As regards the household spending channel, low-income resource exporters typically have underdeveloped financial markets, so changes in interest rates do not necessarily affect household savings behaviour. Poorly developed non-resource export sectors (a product of Dutch disease) take a long time to respond to nominal currency depreciation. Furthermore, 75 per cent of natural resource–

exporting countries operate a nominal peg system for their exchange rate, which is a severe constraint on a monetary policy's ability to respond to demand shocks.[9]

These problems are exacerbated by the fact that much of the impact falls on the government budget. The post-2011 crash required, in some extractives-dependent countries, to be addressed with dramatic government spending cuts and tax hikes. The required adjustments disrupt investment and other spending programmes and are politically tough to implement. However, dramatic drops in oil and other natural resource revenues might make some interventions more politically feasible, such as cutting fuel subsidies or raising oil rent taxes.

8 Conclusion

Savings rates should be high and domestic investment should be the priority for developing countries when managing natural resource windfalls. Management of foreign assets—held perhaps in an SWF—can support this strategy, but it is important that there is clarity on the roles that foreign assets can play.

The first possible role for foreign assets is to put natural resource windfalls into an *intergenerational savings fund* with the purpose of replacing below-ground natural resources with above-ground financial assets. The financial assets are used to generate permanent income for future generations. The desirability of such funds depends principally on the long-run investment opportunities (or lack of them) in the domestic economy, and on political economy. Such a fund should focus on assets with long investment horizons, matching the permanent nature of the fund. Such a fund makes sense for countries with a large domestic infrastructure stock already in place and good access to international capital markets. In such a case, the rate of return on further domestic investment is unlikely to exceed the return that can be made on investments through an SWF. Such a fund also makes sense if it is unlikely to be raided by future governments.

These conditions clearly do not prevail in most developing economies. Capital scarcity means that investment in the domestic economy is a priority. This investment can help place the economy on an accelerated growth path, bringing forward economic growth and consumption benefits. If directed towards the non-traded sector this investment can also limit inflation from resource-financed demand. In developing countries the demographics are such that capital is needed to support jobs for a growing work force, not flows of rent for pensioners. From a political perspective it is also better to sink investments in physical structures and assets rather than leave them in easily drawn-down financial assets. The case for long-run intergenerational funds held in foreign assets is therefore not very strong for developing countries.

To overcome the timing mismatch between receiving resource revenues and investing them efficiently in developing countries, a *parking fund*—the second possible motive for considering investment in foreign assets—is appropriate. The efficient path of investment will depend on the absorptive capacity of the economy, which ideally should be slowly built up through a programme of 'investing in investing'. This requires that savings be parked offshore until they can be efficiently

[9] If the East African Monetary Union goes ahead, this will also severely constrain the ability of the newly resource-rich members of the Union—Kenya, Tanzania, and Uganda—to respond to commodity price shocks (Wills and van der Ploeg 2014).

invested in the domestic economy. This fund should invest in assets with a relatively shorter investment horizon than the intergenerational fund.

The third possible reason is to insulate the economy from short to medium run fluctuations in commodity price revenues. Given the likelihood of such fluctuations, it makes sense to accumulate precautionary buffers in a *stabilization fund*, especially as not all the risk of exposure to volatile resource prices can or will be borne by resource extraction firms or can be hedged away with financial derivatives. Therefore, natural resource exporting countries are well advised to self-insure by using stabilization funds. These funds provide an additional source of interest income when commodity prices crash, and a stock of assets that can finance countercyclical fiscal policy when monetary policy is constrained. Stabilization funds, particularly in developing countries, should hold assets that can be liquidated at short notice if the prices of their commodities fall. Their spending rules must permit fast liquidation, contingent on the resource price, and should also include a mechanism for recapitalizing the fund when the price rises again. The establishment and implementation of such rules is no easy task and even in broadly successful cases such as that of Chile there is ongoing controversy about the specific rules for the accumulation and draw-down of funds (see Solimano and Calderón Guajardo 2017, forthcoming).

These three types of funds held in foreign assets have different purposes, and so need separate legal structures and investment mandates. A developed country's intergenerational fund should invest in long-horizon assets and spend a fixed share of above- and below-ground wealth. A developing country's parking fund should invest in shorter duration assets and have a rising limit for the amount spent each year, with this constraint used for domestic investment. Stabilization funds should focus on liquid assets and have spending and recapitalization rules tied to the commodity price. All funds should design their portfolios with subsoil assets in mind—hedging resource-price volatility at the start of the windfall and rebalancing towards a diversified portfolio as the resource is exhausted. It may well make sense to manage these funds from within a single government body, but they should remain independent in line with their original aims with an investment mandate and spending rule that is appropriate for their purpose.

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Appendix A: Optimization problem in presence of capital scarcity

The social planner's problem is to choose consumption, *C*, and investment, *I*, in the face of exponentially declining oil revenues, *N*, to maximize utility,

$$J(F,K,t)=\max_{C,I}\left[\int_t^{\infty}U(C_t)\exp\left(-\rho(\tau-t)\right)d\tau\right], \tag{A1}$$

subject to the constraints describing foreign asset accumulation,

$$\dot{F}_t=r(F_t)F_t+N_t+Y(K_t)-C_t-I_t, \tag{A2}$$

accumulation of physical capital,

$$\dot{K}_t=I_t-\delta K_t, \tag{A3}$$

and the time path of natural resource revenues,

$$\dot{N}_t=N_0\exp(-\kappa t) \tag{A4}$$

(cf. van der Ploeg and Venables 2011; Wills 2015a). We incorporate capital scarcity by assuming that interest rates may be elastic with respect to foreign assets *F*, $r(F_t)>r, r'(F_t)<0$ for $F_t<0$ and $r(F_t)=r$ otherwise. Output is produced using domestic capital and a fixed stock of labour, $Y_t(K_t)$ and $\bar{L}=1$, and it is sold at the constant world price $P^*=1$. The Hamiltonian for this dynamic optimization problem is

$$H\equiv U(C)+\lambda_1\left[r(F)F+N+Y(K)-C-I\right]+\lambda_2\left[I-\delta K\right]+\lambda_3\left[N_0e^{-\kappa t}\right]. \tag{A5}$$

This has the optimality conditions

$$\begin{aligned}
&H_C=U'(C)-\lambda_1=0,\\
&H_1=-\lambda_1+\lambda_2=0,\\
&\rho\lambda_1-\dot{\lambda}_1=H_F=\lambda_1\left[r'(F)F+r(F)\right],\\
&\rho\lambda_2-\dot{\lambda}_2=H_K=\lambda_1Y'(K)-\lambda_2\delta,\quad\text{and}\\
&\rho\lambda_3-\dot{\lambda}_3=H_N=\lambda_1,
\end{aligned} \tag{A6}$$

and the transversality condition $\lim_{t\to\infty}\left[\exp(-rt)\lambda_{1,t}F_t\right]=0$. The Euler equation or Keynes–Ramsey rule can be derived from the first-order conditions as follows:

$$g=\frac{1}{\eta}\left[r(F)+r'(F)F-\rho\right], \tag{A7}$$

where g denotes the rate of growth of consumption. Therefore, consumption will be delayed as long as the country faces a premium on its debt ($F < 0$), so the rate of consumption will be rising with time. The decision of whether to use the income that is not consumed for domestic investment or foreign savings follows from equating the marginal utility of consumption to that of an extra unit of foreign assets or domestic capital,

$$U'(C_t) = \lambda_1 = \lambda_2 \Rightarrow r(F) + r'(F)F = Y'(K) - \delta. \tag{A8}$$

Since with capital scarcity the left-hand side is a negative function of F, we establish that the optimal stock of capital rises as the stock of foreign debt is curbed.